ELECTROSTATICS - I
– Electrostatic Force

1. **Frictional Electricity**
2. **Properties of Electric Charges**
3. **Coulomb's Law**
4. **Coulomb's Law in Vector Form**
5. **Units of Charge**
6. **Relative Permittivity or Dielectric Constant**
7. **Continuous Charge Distribution**

   i) Linear Charge Density

   ii) Surface Charge Density

   iii) Volume Charge Density

Frictional Electricity:

Frictional electricity is the electricity produced by rubbing two suitable bodies and transfer of electrons from one body to other.

Electrons in glass are loosely bound in it than the electrons in silk. So, when glass and silk are rubbed together, the comparatively loosely bound electrons from glass get transferred to silk.
As a result, glass becomes positively charged and silk becomes negatively charged.

Electrons in fur are loosely bound in it than the electrons in ebonite. So, when ebonite and fur are rubbed together, the comparatively loosely bound electrons from fur get transferred to ebonite.
As a result, ebonite becomes negatively charged and fur becomes positively charged.

It is very important to note that the electrification of the body (whether positive or negative) is due to transfer of electrons from one body to another.

i.e. If the electrons are transferred from a body, then the deficiency of electrons makes the body positive.

If the electrons are gained by a body, then the excess of electrons makes the body negative.

If the two bodies from the following list are rubbed, then the body appearing early in the list is positively charges whereas the latter is negatively charged.

Fur, Glass, Silk, Human body, Cotton, Wood, Sealing wax, Amber, Resin, Sulphur, Rubber, Ebonite.

Column I (+ve Charge)	Column II (-ve Charge)
Glass	Silk
Wool, Flannel	Amber, Ebonite, Rubber, Plastic
Ebonite	Polythene
Dry hair	Comb

Properties of Charges:

1. There exists only two types of charges, namely positive and negative.
2. Like charges repel and unlike charges attract each other.
3. Charge is a scalar quantity.
4. Charge is additive in nature. eg. +2 C + 5 C – 3 C = +4 C
5. Charge is quantized.

   i.e. Electric charge exists in discrete packets rather than in continuous amount.

   It can be expressed in integral multiples fundamental electronic charge ($e = 1.6 \times 10^{-19}$ C)

   $$q = \pm ne \qquad \text{where } n = 1, 2, 3, \ldots\ldots\ldots\ldots$$

6. Charge is conserved.

   i.e. The algebraic sum of positive and negative charges in an isolated system remains constant.

   eg. When a glass rod is rubbed with silk, negative charge appears on the silk and an equal amount of positive charge appear on the glass rod. The net charge on the glass-silk system remains zero before and after rubbing.

   It does not change with velocity also.

Note: Recently, the existence of quarks of charge ⅓ e and ⅔ e has been postulated. If the quarks are detected in any experiment with concrete practical evidence, then the minimum value of ‘quantum of charge’ will be either ⅓ e or ⅔ e. However, the law of quantization will hold good.

Coulomb’s Law – Force between two point electric charges:

The electrostatic force of interaction (attraction or repulsion) between two point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the line joining the two charges.

Strictly speaking, Coulomb’s law applies to stationary point charges.

$F \propto q_1 q_2$

$F \propto 1 / r^2$

or $F \propto \dfrac{q_1 q_2}{r^2}$ or $F = k \dfrac{q_1 q_2}{r^2}$

where k is a positive constant of proportionality called electrostatic force constant or Coulomb constant.

In vacuum, $k = \dfrac{1}{4\pi\varepsilon_0}$ where ε_0 is the permittivity of free space

In medium, $k = \dfrac{1}{4\pi\varepsilon}$ where ε is the absolute electric permittivity of the dielectric medium

The dielectric constant or relative permittivity or specific inductive capacity or dielectric coefficient is given by

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

∴ In vacuum, $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^2}$

In medium, $F = \dfrac{1}{4\pi\varepsilon_0\varepsilon_r} \dfrac{q_1 q_2}{r^2}$

$$\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$$

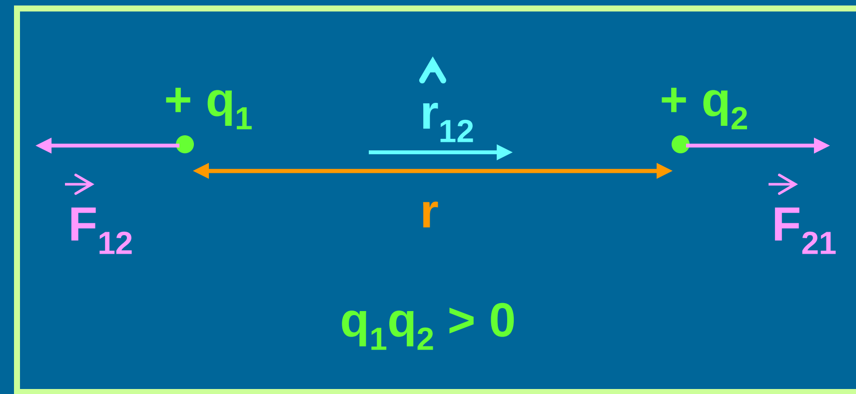
$\dfrac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^9\ N\ m^2\ C^{-2}$ or $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\ m^2\ C^{-2}$

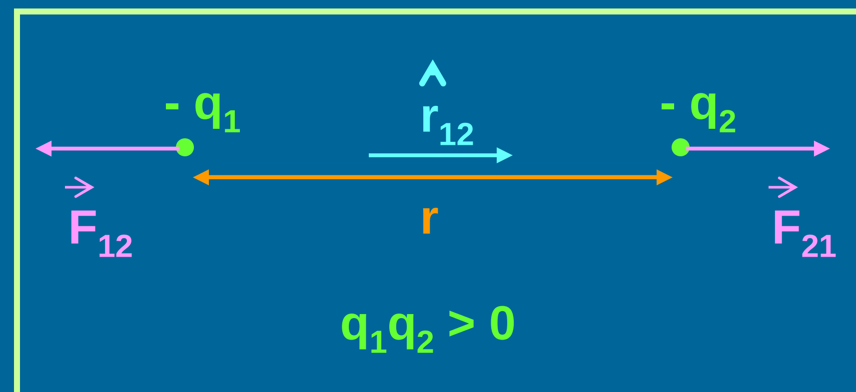
Coulomb's Law in Vector Form:

In vacuum, for $q_1 q_2 > 0$,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

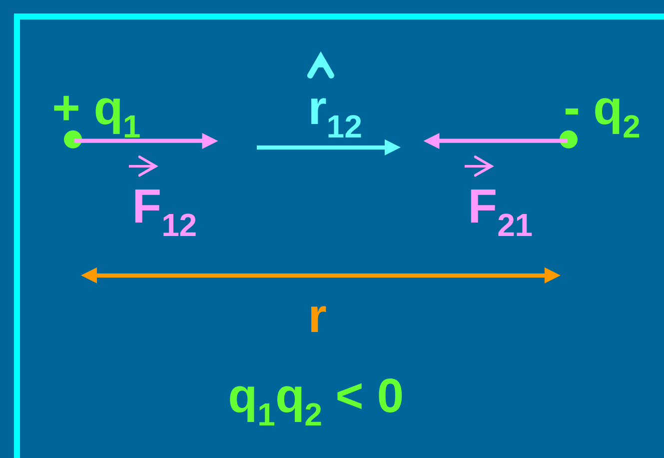




In vacuum, for $q_1 q_2 < 0$,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$\therefore$ $\vec{F}_{12} = -\vec{F}_{21}$ (in all the cases)



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21}$$

Note: The cube term of the distance is simply because of vector form.

Otherwise the law is 'Inverse Square Law' only.

Units of Charge:

In SI system, the unit of charge is coulomb (C).

One coulomb of charge is that charge which when placed at rest in vacuum at a distance of one metre from an equal and similar stationary charge repels it and is repelled by it with a force of 9×10^9 newton.

In cgs electrostatic system, the unit of charge is 'statcoulomb' or 'esu of charge'.

In cgs electrostatic system, $k = 1 / K$ where K is 'dielectric constant'.

For vacuum, $K = 1$.

$$\therefore \quad F = \frac{q_1 q_2}{r^2}$$

If $q_1 = q_2 = q$ (say), $r = 1$ cm and $F = 1$ dyne, then $q = \pm 1$ statcoulomb.

In cgs electromagnetic system, the unit of charge is 'abcoulomb' or 'emu of charge'.

1 emu of charge = c esu of charge

1 emu of charge = 3×10^{10} esu of charge

1 coulomb of charge = 3×10^{9} statcoulomb

1 abcoulomb = 10 coulomb

Relative Permittivity or Dielectric Constant or Specific Inductive Capacity or Dielectric Coefficient:

The dielectric constant or relative permittivity or specific inductive capacity or dielectric coefficient is given by the ratio of the absolute permittivity of the medium to the permittivity of free space.

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

The dielectric constant or relative permittivity or specific inductive capacity or dielectric coefficient can also be defined as the ratio of the electrostatic force between two charges separated by a certain distance in vacuum to the electrostatic force between the same two charges separated by the same distance in that medium.

$$K = \varepsilon_r = \frac{F_v}{F_m}$$

Dielectric constant has no unit.

Continuous Charge Distribution:

Any charge which covers a space with dimensions much less than its distance away from an observation point can be considered a point charge.

A system of closely spaced charges is said to form a continuous charge distribution.

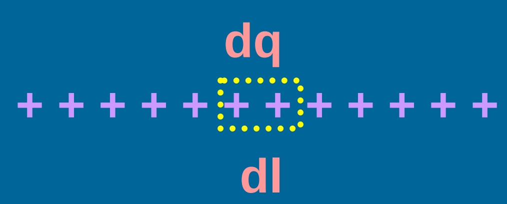
It is useful to consider the density of a charge distribution as we do for density of solid, liquid, gas, etc.

(i) Line or Linear Charge Density (λ):

If the charge is distributed over a straight line or over the circumference of a circle or over the edge of a cuboid, etc, then the distribution is called 'linear charge distribution'.

Linear charge density is the charge per unit length. Its SI unit is C / m.

$$\lambda = \frac{q}{l} \quad \text{or} \quad \lambda = \frac{dq}{dl}$$



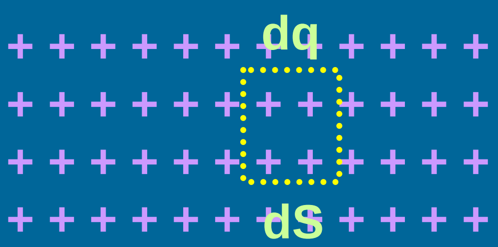
Total charge on line l, $$q = \int_l \lambda \, dl$$

(ii) Surface Charge Density (σ):

If the charge is distributed over a surface area, then the distribution is called 'surface charge distribution'.

Surface charge density is the charge per unit area. Its SI unit is C / m².

$$\sigma = \frac{q}{S} \quad \text{or} \quad \sigma = \frac{dq}{dS}$$



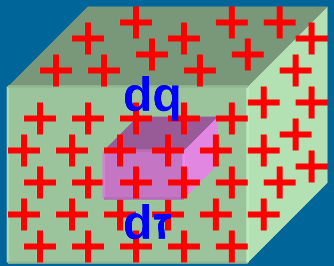
Total charge on surface S, $$q = \int_S \sigma \, dS$$

(iii) Volume Charge Density (ρ):

If the charge is distributed over a volume, then the distribution is called 'volume charge distribution'.

Volume charge density is the charge per unit volume. Its SI unit is C / m³.

$$\rho = \frac{q}{\tau} \quad \text{or} \quad \rho = \frac{dq}{d\tau}$$



Total charge on volume τ, $$q = \int_\tau \rho \, d\tau$$

ELECTROSTATICS - II : Electric Field

1. Electric Field
2. Electric Field Intensity or Electric Field Strength
3. Electric Field Intensity due to a Point Charge
4. Superposition Principle
5. Electric Lines of Force
   i) Due to a Point Charge
   ii) Due to a Dipole
   iii) Due to a Equal and Like Charges
   iv) Due to a Uniform Field
6. Properties of Electric Lines of Force
7. Electric Dipole
8. Electric Field Intensity due to an Electric Dipole
9. Torque on an Electric Dipole
10. Work Done on an Electric Dipole

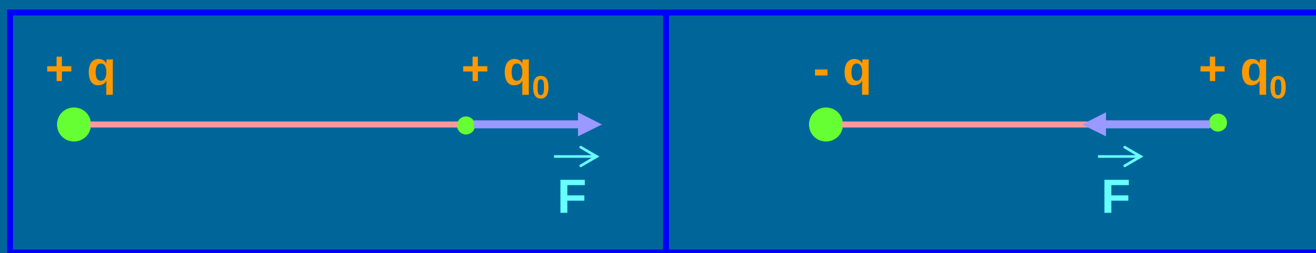
Electric Field:

Electric field is a region of space around a charge or a system of charges within which other charged particles experience electrostatic forces.

Theoretically, electric field extends upto infinity but practically it is limited to a certain distance.

Electric Field Strength or Electric Field Intensity or Electric Field:

Electric field strength at a point in an electric field is the electrostatic force per unit positive charge acting on a vanishingly small positive test charge placed at that point.



q – Source charge, q_0 – Test charge, F – Force & E - Field

$$\vec{E} = \underset{\Delta q \to 0}{\mathrm{Lt}} \frac{\vec{F}}{\Delta q} \quad \text{or} \quad \vec{E} = \frac{\vec{F}}{q_0} \quad \text{or} \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

The test charge is considered to be vanishingly small because its presence should not alter the configuration of the charge(s) and thus the electric field which is intended to be measured.

Note:

1. Since q_0 is taken positive, the direction of electric field ($\vec{E}$) is along the direction of electrostatic force ($\vec{F}$).
2. Electrostatic force on a negatively charged particle will be opposite to the direction of electric field.
3. Electric field is a vector quantity whose magnitude and direction are uniquely determined at every point in the field.
4. SI unit of electric field is newton / coulomb ($N\ C^{-1}$).

Electric Field due to a Point Charge:

Force exerted on q_0 by q is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q\ q_0}{r^2} \hat{r}$$

or $$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q\ q_0}{r^3} \vec{r}$$

Electric field strength is $$\vec{E} = \frac{\vec{F}}{q_0}$$

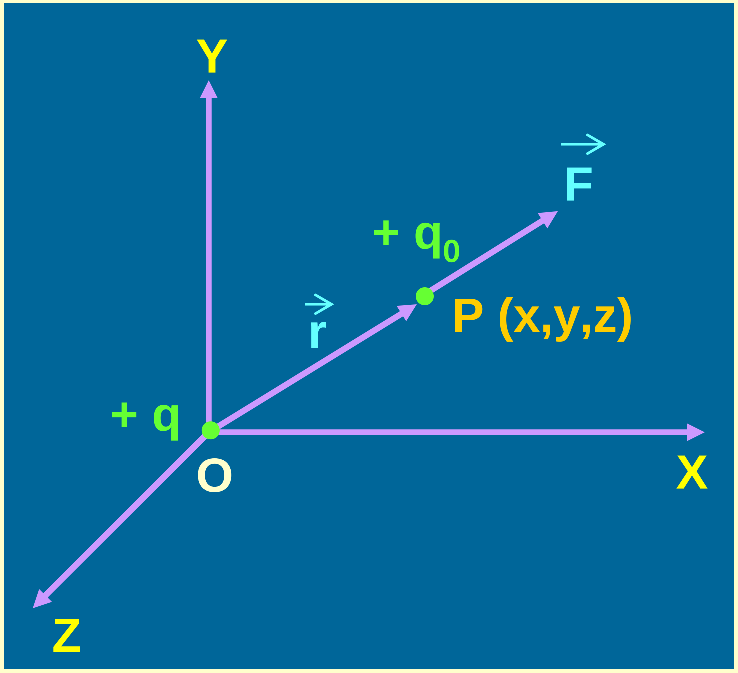
$$\therefore \quad \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \vec{r}$$

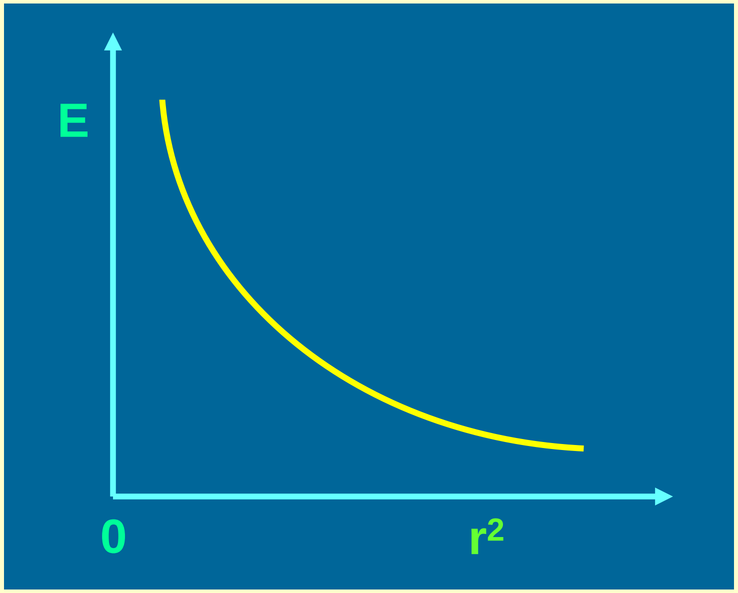
or $$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

The electric field due to a point charge has spherical symmetry.

If q > 0, then the field is radially outwards.

If q < 0, then the field is radially inwards.





Electric field in terms of co-ordinates is given by

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2 + y^2 + z^2)^{3/2}} (x\hat{i} + y\hat{j} + z\hat{k})$$

Superposition Principle:

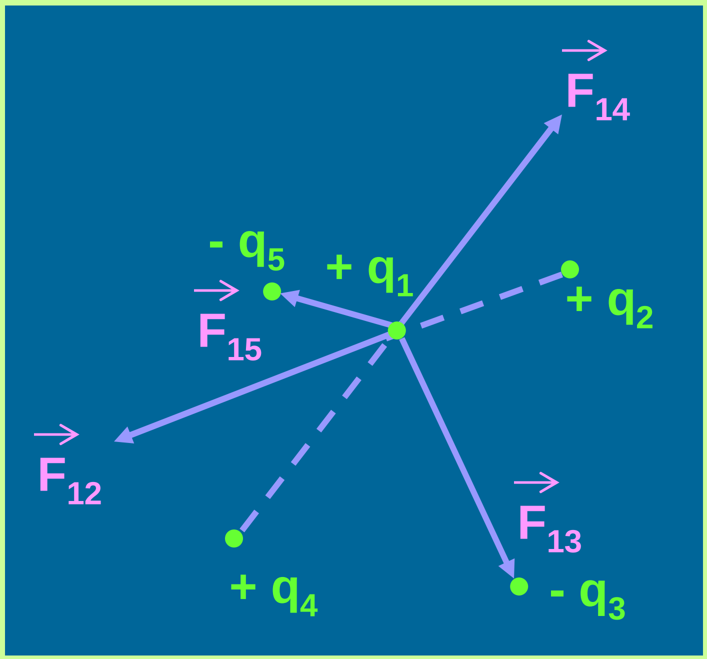
The electrostatic force experienced by a charge due to other charges is the vector sum of electrostatic forces due to these other charges as if they are existing individually.

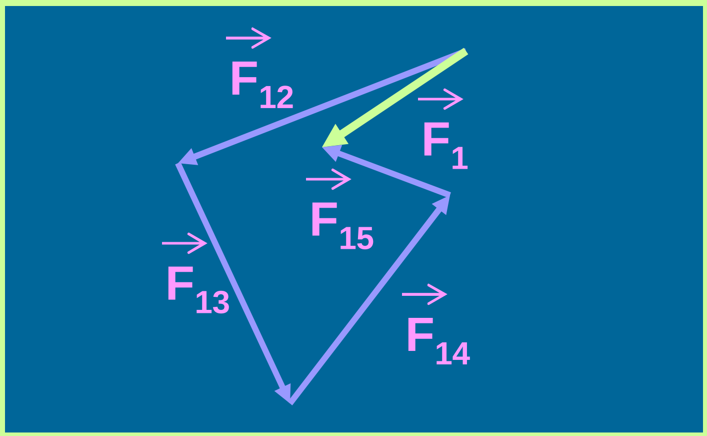
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15}$$

$$\vec{F}_a(\vec{r}_a) = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{b=1 \\ b\neq a}}^{N} q_a q_b \frac{\vec{r}_a - \vec{r}_b}{|\vec{r}_a - \vec{r}_b|^3}$$

In the present example, a = 1 and b = 2 to 5.

If the force is to be found on 2nd charge, then a = 2 and b = 1 and 3 to 5.





Note:

The interactions must be on the charge which is to be studied due to other charges.

The charge on which the influence due to other charges is to be found is assumed to be floating charge and others are rigidly fixed.

For eg. 1st charge (floating) is repelled away by q_2 and q_4 and attracted towards q_3 and q_5.

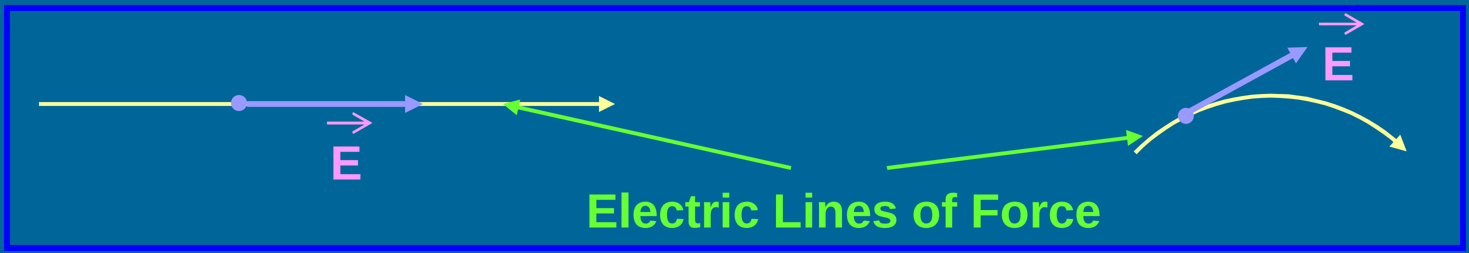
The interactions between the other charges (among themselves) must be ignored. i.e. F_{23}, F_{24}, F_{25}, F_{34}, F_{35} and F_{45} are ignored.

Superposition principle holds good for electric field also.

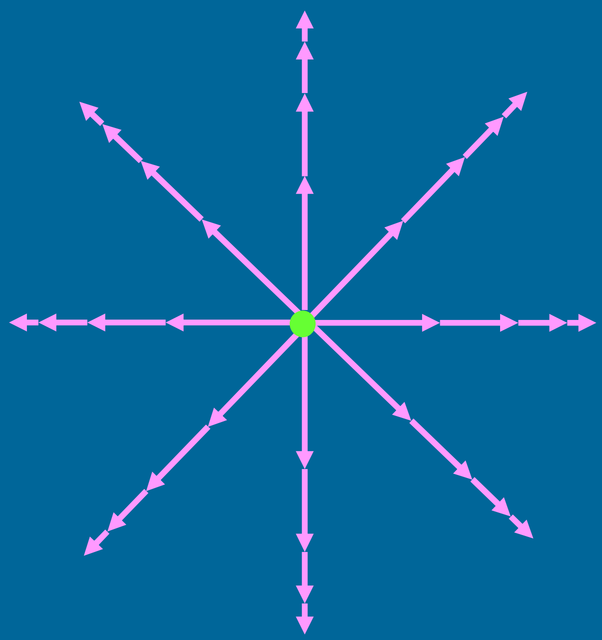
Electric Lines of Force:

An electric line of force is an imaginary straight or curved path along which a unit positive charge is supposed to move when free to do so in an electric field.

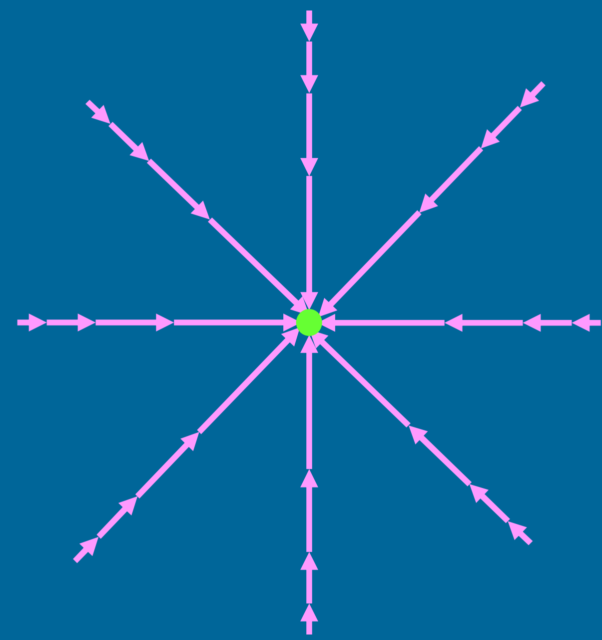
Electric lines of force do not physically exist but they represent real situations.



1. Electric Lines of Force due to a Point Charge:



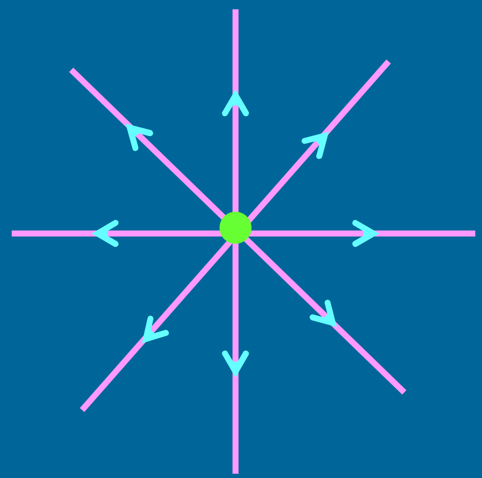
$q > 0$

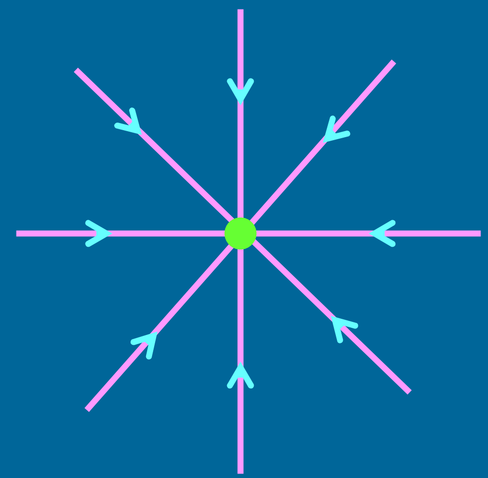


$q < 0$

a) Representation of electric field in terms of field vectors:

The size of the arrow represents the strength of electric field.

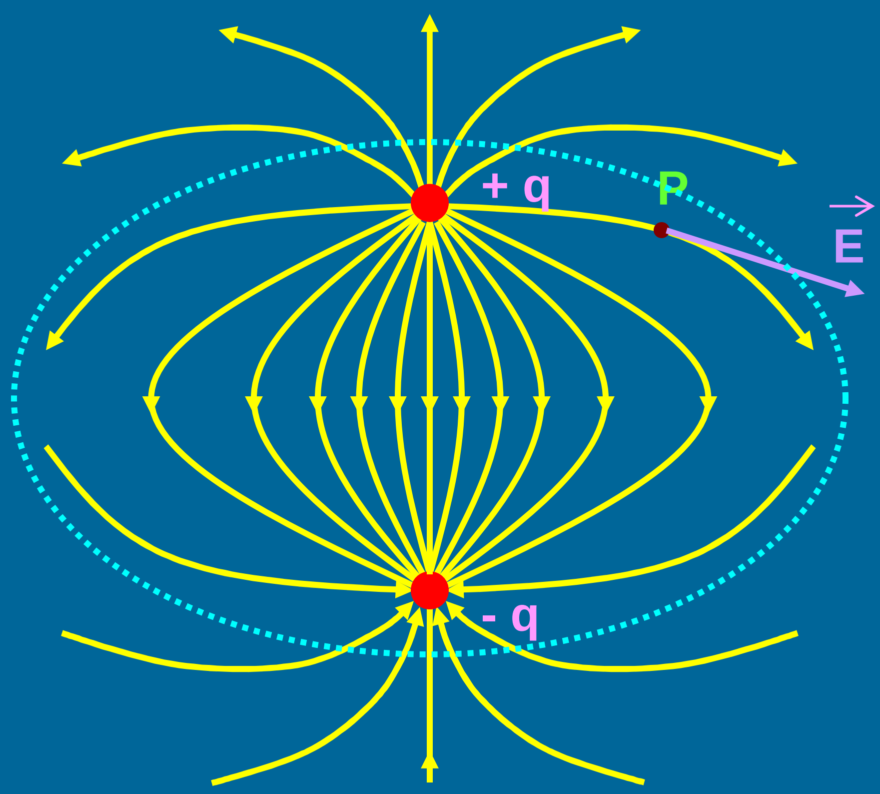




b) Representation of electric field in terms of field lines

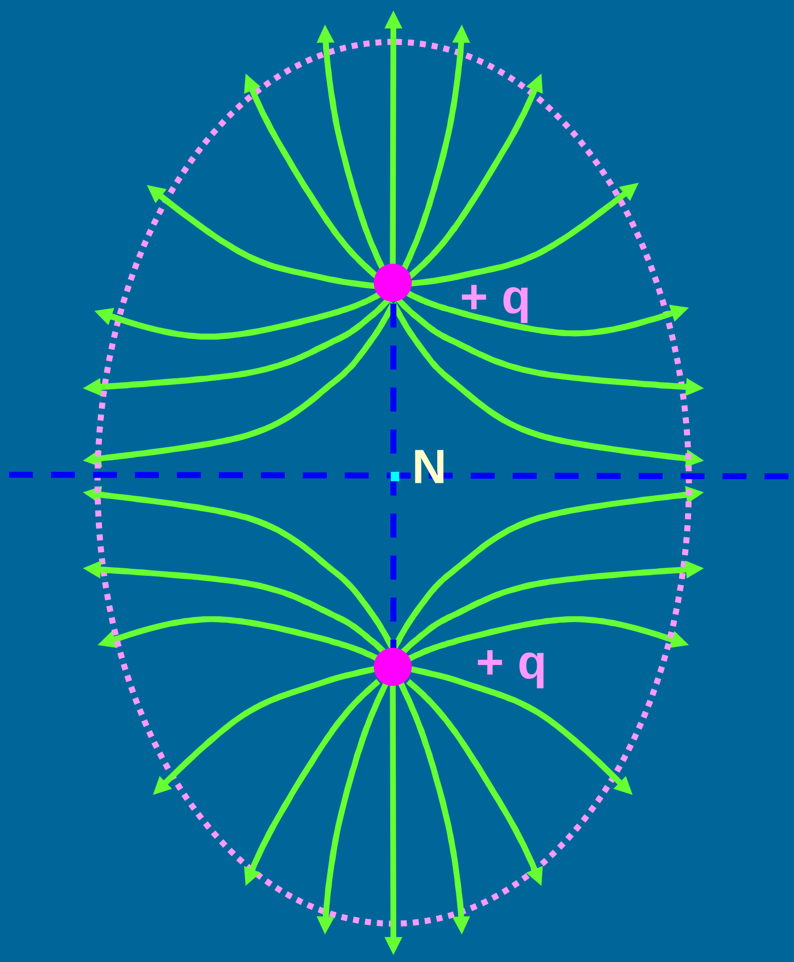
(Easy way of drawing)

2. Electric Lines of Force due to a pair of Equal and Unlike Charges: (Dipole)



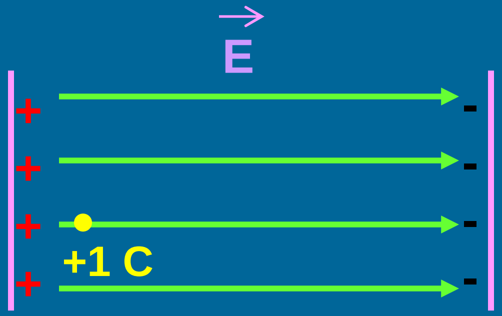
Electric lines of force contract lengthwise to represent attraction between two unlike charges.

3. Electric Lines of Force due to a pair of Equal and Like Charges:



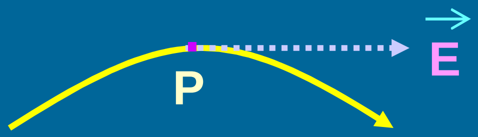
Electric lines of force exert lateral (sideways) pressure to represent repulsion between two like charges.

4. Electric Lines of Force due to a Uniform Field:



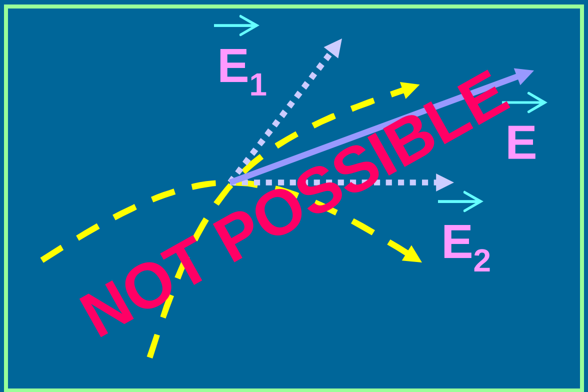
Properties of Electric Lines of Force or Field Lines:

1. The electric lines of force are imaginary lines.
2. A unit positive charge placed in the electric field tends to follow a path along the field line if it is free to do so.
3. The electric lines of force emanate from a positive charge and terminate on a negative charge.
4. The tangent to an electric field line at any point gives the direction of the electric field at that point.

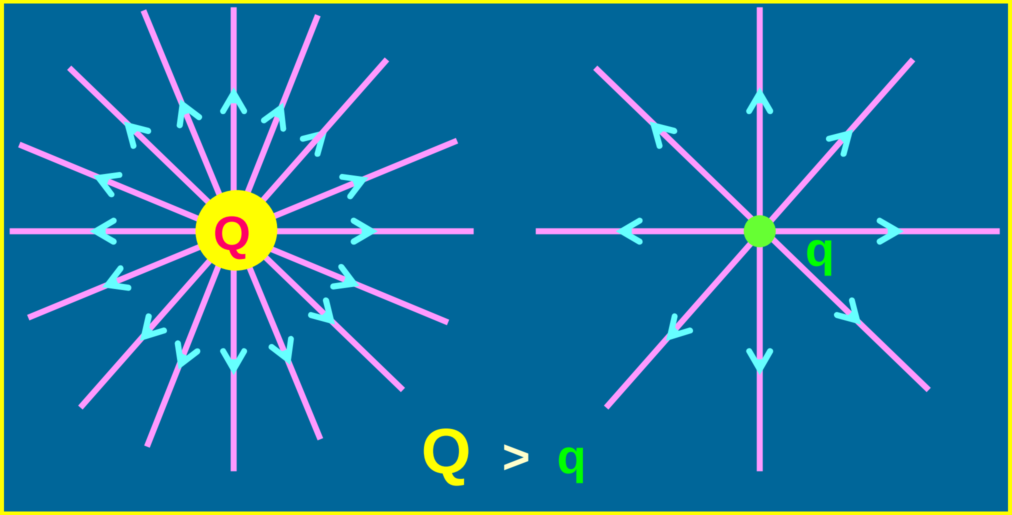


5. Two electric lines of force can never cross each other. If they do, then at the point of intersection, there will be two tangents. It means there are two values of the electric field at that point, which is not possible.

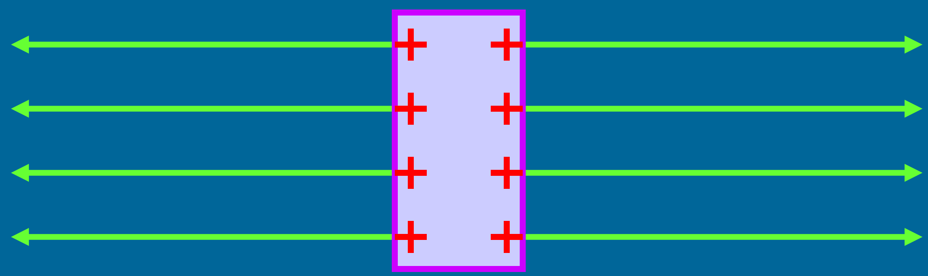
   Further, electric field being a vector quantity, there can be only one resultant field at the given point, represented by one tangent at the given point for the given line of force.

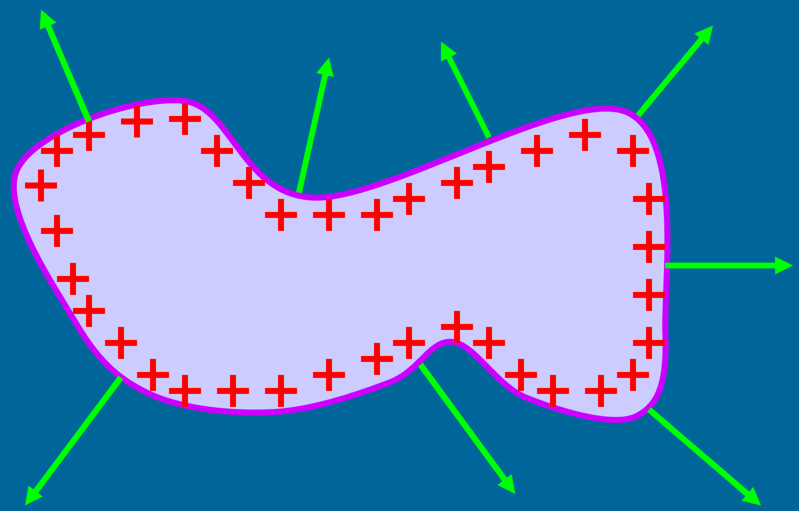


6. Electric lines of force are closer (crowded) where the electric field is stronger and the lines spread out where the electric field is weaker.



7. Electric lines of force are perpendicular to the surface of a positively or negatively charged body.





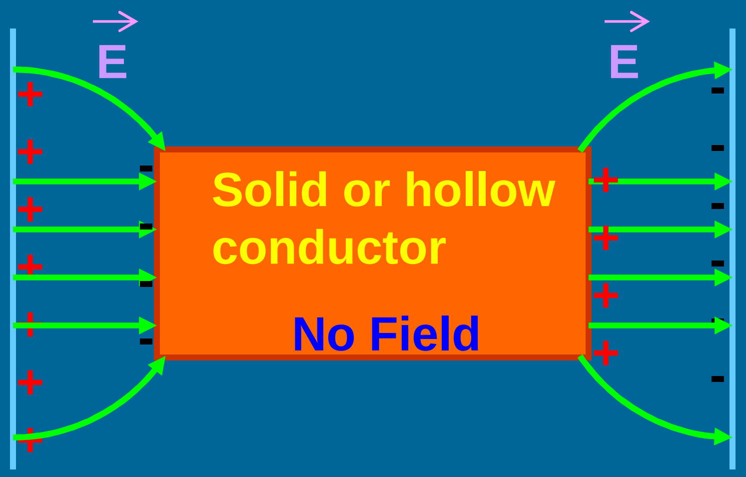
8. Electric lines of force contract lengthwise to represent attraction between two unlike charges.

9. Electric lines of force exert lateral (sideways) pressure to represent repulsion between two like charges.

10. The number of lines per unit cross – sectional area perpendicular to the field lines (i.e. density of lines of force) is directly proportional to the magnitude of the intensity of electric field in that region.

$$\frac{\Delta N}{\Delta A} \propto E$$

11. Electric lines of force do not pass through a conductor. Hence, the interior of the conductor is free from the influence of the electric field.



(Electrostatic Shielding)

12. Electric lines of force can pass through an insulator.

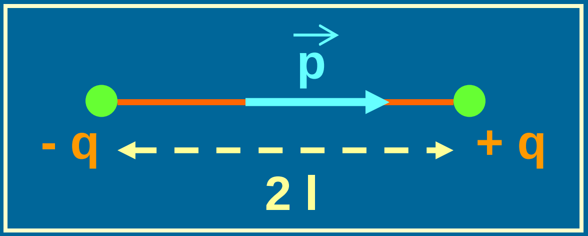
Electric Dipole:

Electric dipole is a pair of equal and opposite charges separated by a very small distance.

The electric field produced by a dipole is known as dipole field.

Electric dipole moment is a vector quantity used to measure the strength of an electric dipole.

$$\vec{p} = (q \times 2l)\ \hat{l}$$



The magnitude of electric dipole moment is the product of magnitude of either charge and the distance between the two charges.

The direction is from negative to positive charge.

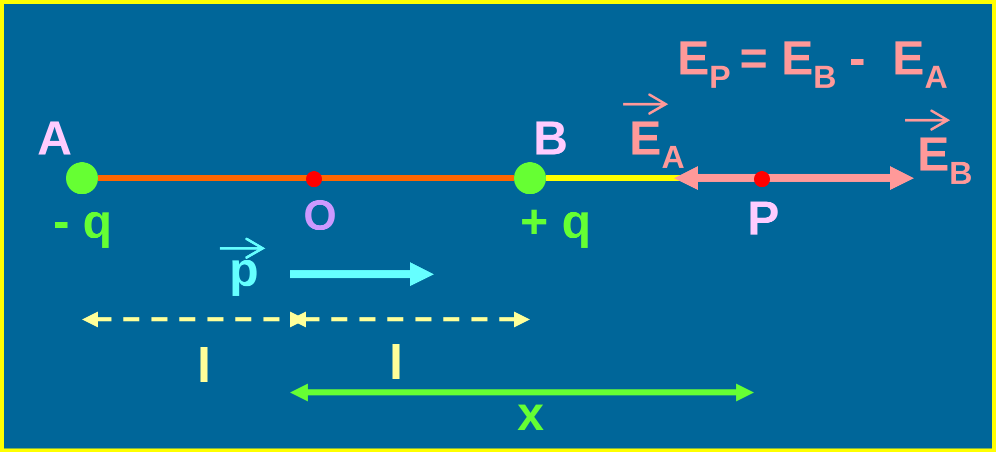
The SI unit of ‘p’ is ‘coulomb metre (C m)’.

Note:

An ideal dipole is the dipole in which the charge becomes larger and larger and the separation becomes smaller and smaller.

Electric Field Intensity due to an Electric Dipole:

i) At a point on the axial line:



Resultant electric field intensity at the point P is $\vec{E}_P = \vec{E}_A + \vec{E}_B$

The vectors $\vec{E}_A$ and $\vec{E}_B$ are collinear and opposite.

$$\therefore \quad |\vec{E}_P| = |\vec{E}_B| - |\vec{E}_A|$$

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x + l)^2} \hat{i}$$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x - l)^2} \hat{i}$$

$$|\vec{E}_P| = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{(x - l)^2} - \frac{q}{(x + l)^2} \right]$$

$$|\vec{E}_P| = \frac{1}{4\pi\varepsilon_0} \frac{2 (q \,.\, 2l)\, x}{(x^2 - l^2)^2}$$

$$|\vec{E}_P| = \frac{1}{4\pi\varepsilon_0} \frac{2\, p\, x}{(x^2 - l^2)^2}$$

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0} \frac{2\, p\, x}{(x^2 - l^2)^2} \hat{i}$$

If l << x, then

$$E_P \approx \frac{2\, p}{4\pi\varepsilon_0 \; x^3}$$

The direction of electric field intensity at a point on the axial line due to a dipole is always along the direction of the dipole moment.

ii) At a point on the equatorial line:

Resultant electric field intensity at the point Q is

$$\vec{E}_Q = \vec{E}_A + \vec{E}_B$$

The vectors $\vec{E}_A$ and $\vec{E}_B$ are acting at an angle 2θ.

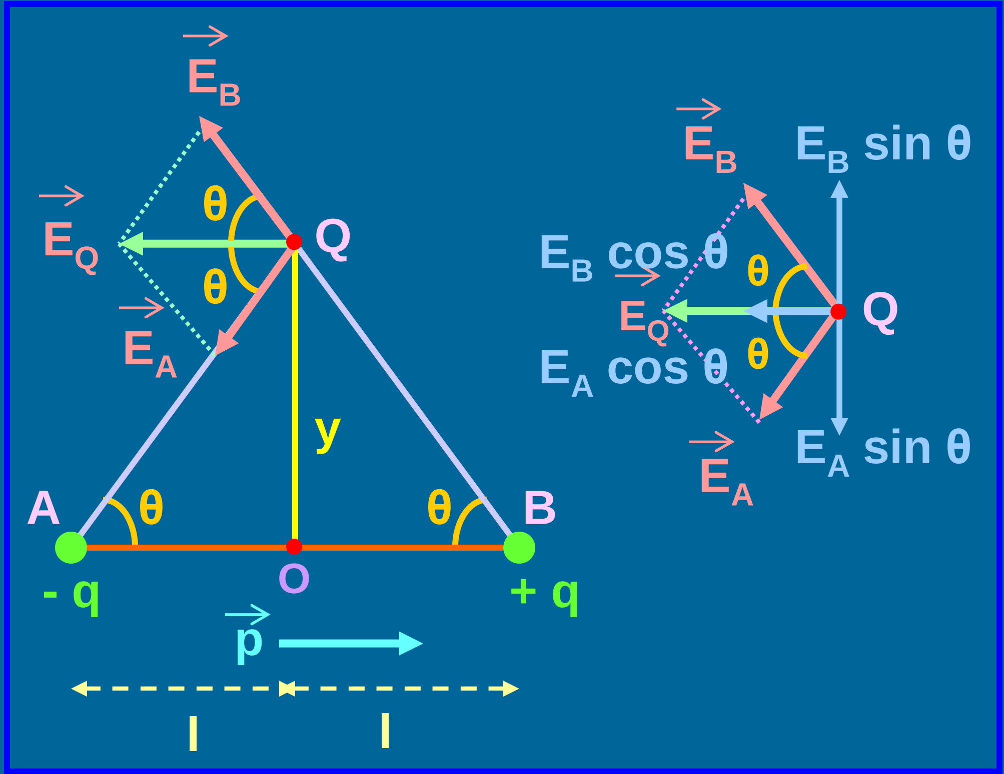
$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2 + l^2)} \hat{i}$$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2 + l^2)} \hat{i}$$

The vectors $E_A \sin\theta$ and $E_B \sin\theta$ are opposite to each other and hence cancel out.

The vectors $E_A \cos\theta$ and $E_B \cos\theta$ are acting along the same direction and hence add up.

$$\therefore \quad E_Q = E_A \cos\theta + E_B \cos\theta$$



$$E_Q = \frac{2}{4\pi\varepsilon_0} \frac{q}{(x^2 + l^2)} \frac{l}{(x^2 + l^2)^{1/2}}$$

$$E_Q = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot 2l}{(x^2 + l^2)^{3/2}}$$

$$\boxed{E_Q = \frac{1}{4\pi\varepsilon_0} \frac{p}{(x^2 + l^2)^{3/2}}}$$

$$\vec{E}_Q = \frac{1}{4\pi\varepsilon_0} \frac{p}{(x^2 + l^2)^{3/2}} (-\hat{i})$$

If l << y, then

$$E_Q \approx \frac{p}{4\pi\varepsilon_0 \; y^3}$$

The direction of electric field intensity at a point on the equatorial line due to a dipole is parallel and opposite to the direction of the dipole moment.

If the observation point is far away or when the dipole is very short, then the electric field intensity at a point on the axial line is double the electric field intensity at a point on the equatorial line.

i.e. If l << x and l << y, then $E_P = 2\,E_Q$

Torque on an Electric Dipole in a Uniform Electric Field:

The forces of magnitude pE act opposite to each other and hence net force acting on the dipole due to external uniform electric field is zero. So, there is no translational motion of the dipole.

However the forces are along different lines of action and constitute a couple. Hence the dipole will rotate and experience torque.

Torque = Electric Force x ⊥ distance

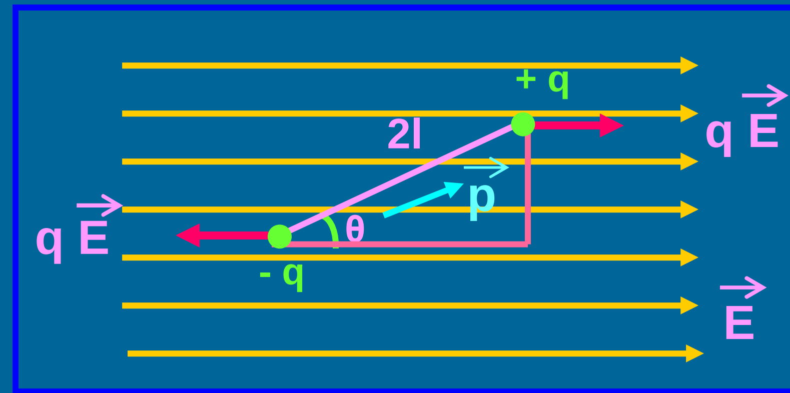
$$t = q E (2l \sin \theta)$$

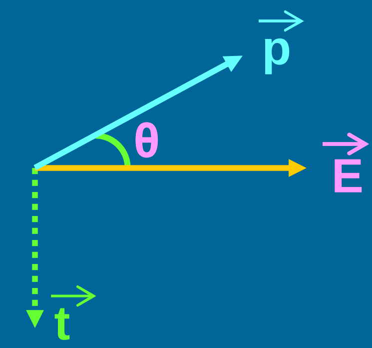
$$= p E \sin \theta$$

$$\vec{t} = \vec{p} \times \vec{E}$$

Direction of Torque is perpendicular and into the plane containing $\vec{p}$ and $\vec{E}$.

SI unit of torque is newton metre (Nm).





Case i: If θ = 0°, then t = 0.

Case ii: If θ = 90°, then t = pE (maximum value).

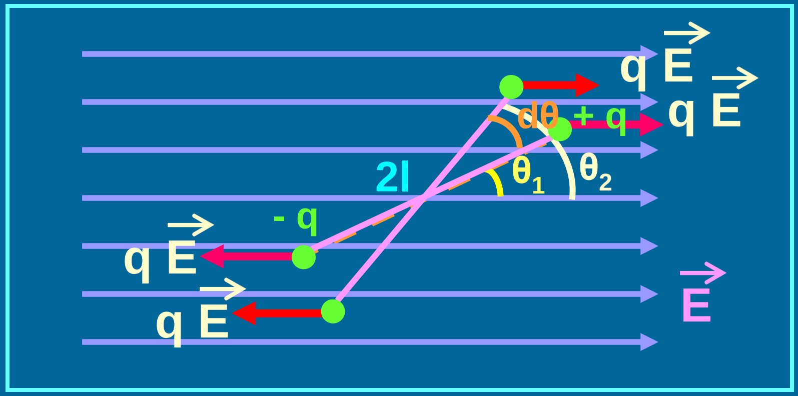
Case iii: If θ = 180°, then t = 0.

Work done on an Electric Dipole in Uniform Electric Field:

When an electric dipole is placed in a uniform electric field, it experiences torque and tends to allign in such a way to attain stable equilibrium.

$dW = \tau d\theta$

$= p E \sin\theta \, d\theta$



$$W = \int_{\theta_1}^{\theta_2} p E \sin\theta \, d\theta$$

$$W = p E (\cos\theta_1 - \cos\theta_2)$$

If Potential Energy is arbitrarily taken zero when the dipole is at 90°, then P.E in rotating the dipole and inclining it at an angle θ is

Potential Energy $U = - p E \cos\theta$

Note: Potential Energy can be taken zero arbitrarily at any position of the dipole.

Case i: If $\theta = 0°$, then $U = - pE$ (Stable Equilibrium)

Case ii: If $\theta = 90°$, then $U = 0$

Case iii: If $\theta = 180°$, then $U = pE$ (Unstable Equilibrium)

ELECTROSTATICS - III

- Electrostatic Potential and Gauss's Theorem

1. Line Integral of Electric Field
2. Electric Potential and Potential Difference
3. Electric Potential due to a Single Point Charge
4. Electric Potential due to a Group of Charges
5. Electric Potential due to an Electric Dipole
6. Equipotential Surfaces and their Properties
7. Electrostatic Potential Energy
8. Area Vector, Solid Angle, Electric Flux
9. Gauss's Theorem and its Proof
10. Coulomb's Law from Gauss's Theorem
11. Applications of Gauss's Theorem:

    Electric Field Intensity due to Line Charge, Plane Sheet of Charge and Spherical Shell

Line Integral of Electric Field (Work Done by Electric Field):

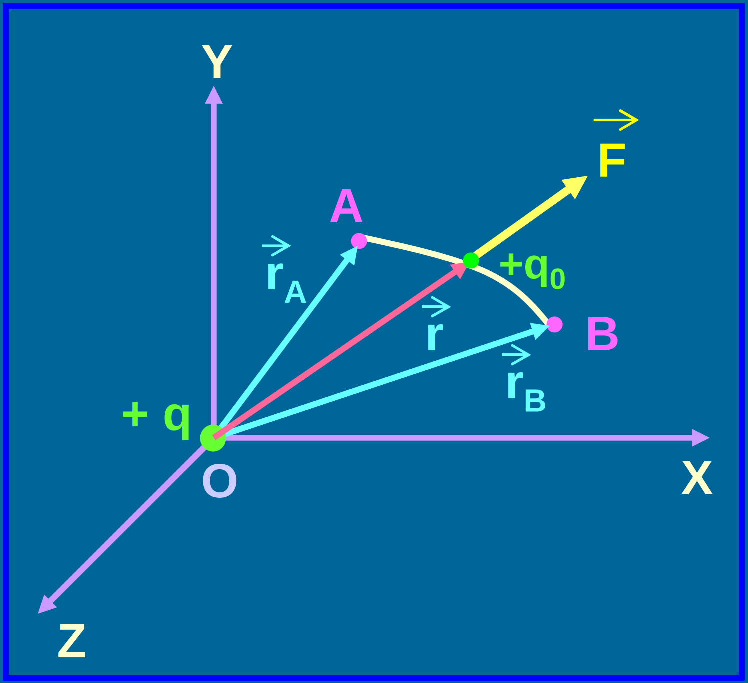
Negative Line Integral of Electric Field represents the work done by the electric field on a unit positive charge in moving it from one point to another in the electric field.

$$W_{AB} = \int dW = -\int_A^B \vec{E} \cdot \vec{dl}$$

Let q_0 be the test charge in place of the unit positive charge.

The force $\vec{F} = +q_0\vec{E}$ acts on the test charge due to the source charge +q.

It is radially outward and tends to accelerate the test charge. To prevent this acceleration, equal and opposite force $-q_0\vec{E}$ has to be applied on the test charge.



Total work done by the electric field on the test charge in moving it from A to B in the electric field is

$$W_{AB} = \int dW = -\int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$W_{AB} = \int dW = - \int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

1. The equation shows that the work done in moving a test charge q_0 from point A to another point B along any path AB in an electric field due to +q charge depends only on the positions of these points and is independent of the actual path followed between A and B.
2. That is, the line integral of electric field is path independent.
3. Therefore, electric field is 'conservative field'.
4. Line integral of electric field over a closed path is zero. This is another condition satisfied by conservative field.

$$\oint_A^B \vec{E} \cdot \vec{dl} = 0$$

Note:

Line integral of only static electric field is independent of the path followed. However, line integral of the field due to a moving charge is not independent of the path because the field varies with time.

Electric Potential:

Electric potential is a physical quantity which determines the flow of charges from one body to another.

It is a physical quantity that determines the degree of electrification of a body.

Electric Potential at a point in the electric field is defined as the work done in moving (without any acceleration) a unit positive charge from infinity to that point against the electrostatic force irrespective of the path followed.

$$W_{AB} = -\int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad \text{or} \quad \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

According to definition, $r_A = \infty$ and $r_B = r$

(where r is the distance from the source charge and the point of consideration)

$$\therefore \frac{W_{\infty B}}{q_0} = \frac{q}{4\pi\varepsilon_0 r} = V \qquad \therefore \boxed{V = \frac{W_{\infty B}}{q_0}}$$

SI unit of electric potential is volt (V) or J C^{-1} or Nm C^{-1}.

Electric potential at a point is one volt if one joule of work is done in moving one coulomb charge from infinity to that point in the electric field.

Electric Potential Difference:

Electric Potential Difference between any two points in the electric field is defined as the work done in moving (without any acceleration) a unit positive charge from one point to the other against the electrostatic force irrespective of the path followed.

$$W_{AB} = -\int_A^B \vec{E} \cdot \vec{dl} = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad \text{or} \quad \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

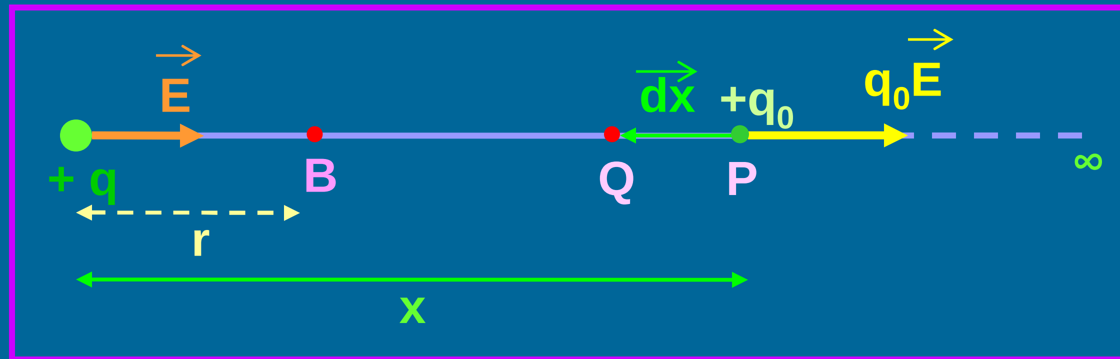
$$\frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r_B} - \frac{q}{4\pi\varepsilon_0}\frac{1}{r_A} = V_B - V_A$$

$$\therefore \quad \boxed{V_B - V_A = \Delta V = \frac{W_{AB}}{q_0}}$$

1. Electric potential and potential difference are scalar quantities.
2. Electric potential at infinity is zero.
3. Electric potential near an isolated positive charge ($q > 0$) is positive and that near an isolated negative charge ($q < 0$) is negative.
4. cgs unit of electric potential is stat volt. 1 stat volt = 1 erg / stat coulomb

Electric Potential due to a Single Point Charge:

Let $+q_0$ be the test charge placed at P at a distance x from the source charge +q.



The force $F = +q_0E$ is radially outward and tends to accelerate the test charge.

To prevent this acceleration, equal and opposite force $-q_0E$ has to be applied on the test charge.

Work done to move q_0 from P to Q through 'dx' against q_0E is

$$dW = \vec{F} \cdot \vec{dx} = q_0\vec{E} \cdot \vec{dx}$$ or $$dW = q_0E\,dx\cos 180° = -q_0E\,dx$$

$$dW = -\frac{q\,q_0}{4\pi\varepsilon_0\,x^2}\,dx \quad \because \quad E = \frac{q}{4\pi\varepsilon_0\,x^2}$$

Total work done to move q_0 from A to B (from ∞ to r) is

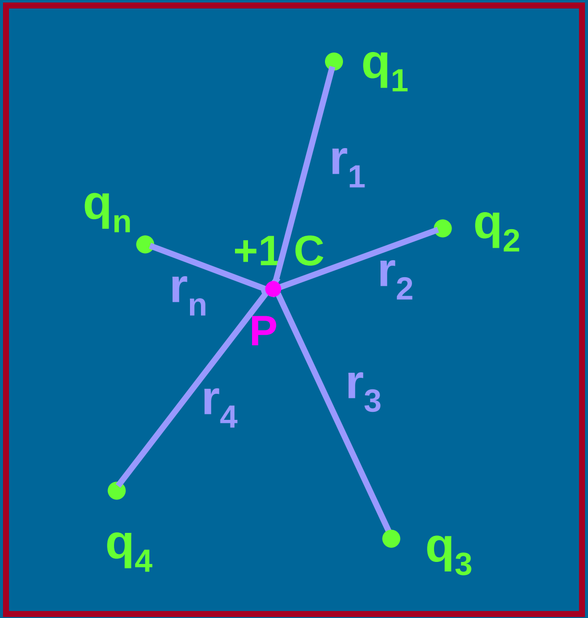
$$W_{\infty B} = \int_{\infty}^{B} dW = -\int_{\infty}^{r} \frac{q\,q_0}{4\pi\varepsilon_0\,x^2}\,dx = -\frac{q\,q_0}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{1}{x^2}\,dx$$

$$\frac{W_{\infty B}}{q_0} = \frac{q}{4\pi\varepsilon_0\,r}$$

$$V = \frac{q}{4\pi\varepsilon_0\,r}$$

Electric Potential due to a Group of Point Charges:

The net electrostatic potential at a point in the electric field due to a group of charges is the algebraic sum of their individual potentials at that point.



$$V_P = V_1 + V_2 + V_3 + V_4 + \ldots\ldots\ldots\ldots + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r} - \vec{r}_i|}$$

(in terms of position vector)

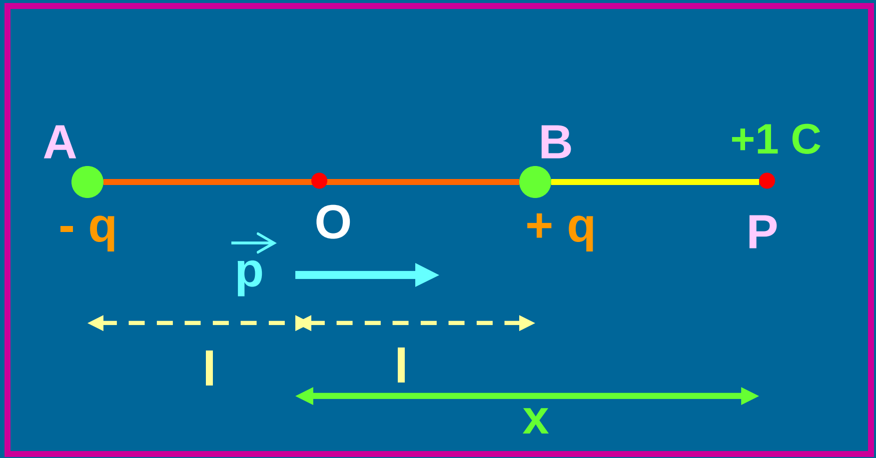
1. Electric potential at a point due to a charge is not affected by the presence of other charges.
2. Potential, $V \propto 1 / r$ whereas Coulomb's force $F \propto 1 / r^2$.
3. Potential is a scalar whereas Force is a vector.
4. Although V is called the potential at a point, it is actually equal to the potential difference between the points r and ∞.

Electric Potential due to an Electric Dipole:

i) At a point on the axial line:

$$V_{P_{q+}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x - l)}$$

$$V_{P_{q-}} = \frac{1}{4\pi\varepsilon_0} \frac{-q}{(x + l)}$$



$$V_P = V_{P_{q+}} + V_{P_{q-}}$$

$$V_P = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{(x - l)} - \frac{1}{(x + l)} \right]$$

$$V_P = \frac{1}{4\pi\varepsilon_0} \frac{q \, . \, 2l}{(x^2 - l^2)}$$

$$V_P = \frac{1}{4\pi\varepsilon_0} \frac{p}{(x^2 - l^2)}$$

ii) At a point on the equatorial line:

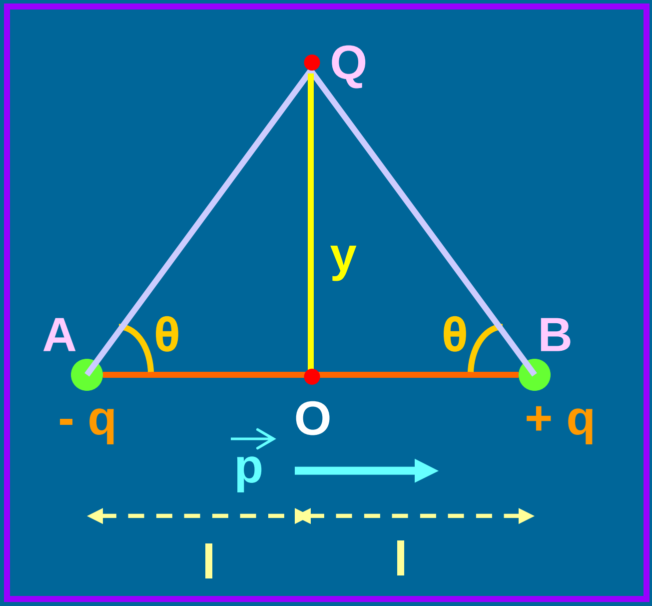
$$V_{Q\,q+} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{BQ}$$

$$V_{Q\,q-} = \frac{1}{4\pi\varepsilon_0}\,\frac{-q}{AQ}$$

$$V_Q = V_{P\,q+} + V_{P\,q-}$$

$$V_Q = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{BQ} - \frac{1}{AQ}\right]$$

$$\boxed{V_Q = 0} \qquad \because \quad BQ = AQ$$

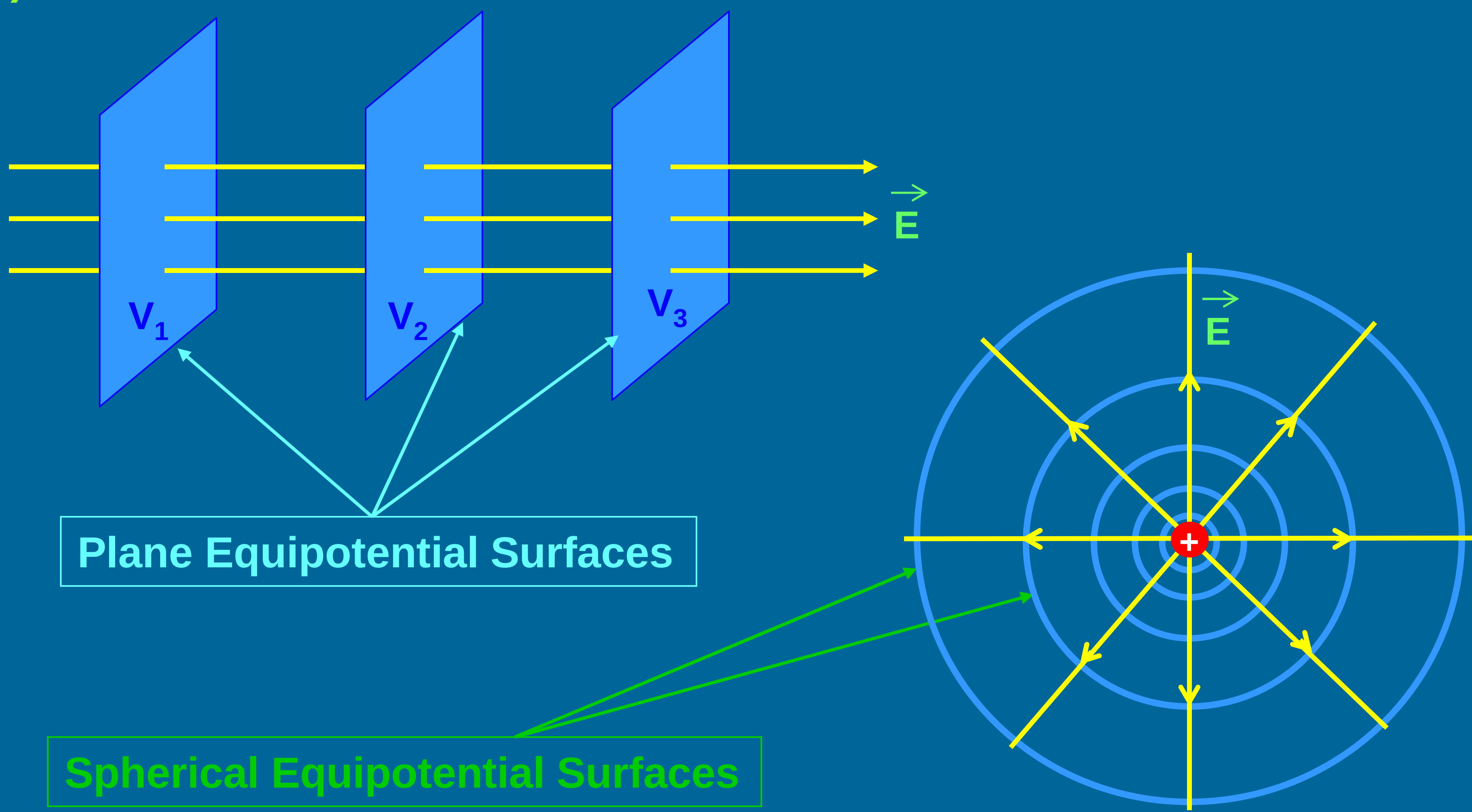


The net electrostatic potential at a point in the electric field due to an electric dipole at any point on the equatorial line is zero.

Equipotential Surfaces:

A surface at every point of which the potential due to charge distribution is the same is called equipotential surface.

i) For a uniform electric field:



ii) For an isolated charge:

Properties of Equipotential Surfaces:

1. No work is done in moving a test charge from one point to another on an equipotential surface.

$$V_B - V_A = \Delta V = \frac{W_{AB}}{q_0}$$

If A and B are two points on the equipotential surface, then $V_B = V_A$.

$$\therefore \quad \frac{W_{AB}}{q_0} = 0 \qquad \text{or} \qquad W_{AB} = 0$$

2. The electric field is always perpendicular to the element dl of the equipotential surface.

Since no work is done on equipotential surface,

$$W_{AB} = -\int_A^B \vec{E} \cdot \vec{dl} = 0 \qquad \text{i.e.} \qquad E \;\, dl \cos\theta = 0$$

As $E \neq 0$ and $dl \neq 0$, $\cos\theta = 0$ or $\theta = 90°$

3. Equipotential surfaces indicate regions of strong or weak electric fields.

Electric field is defined as the negative potential gradient.

$$\therefore \quad E = -\frac{dV}{dr} \quad \text{or} \quad dr = -\frac{dV}{E}$$

Since dV is constant on equipotential surface, so

$$dr \propto \frac{1}{E}$$

If E is strong (large), dr will be small, i.e. the separation of equipotential surfaces will be smaller (i.e. equipotential surfaces are crowded) and vice versa.

4. Two equipotential surfaces can not intersect.

If two equipotential surfaces intersect, then at the points of intersection, there will be two values of the electric potential which is not possible.

(Refer to properties of electric lines of force)

Note:

Electric potential is a scalar quantity whereas potential gradient is a vector quantity.

The negative sign of potential gradient shows that the rate of change of potential with distance is always against the electric field intensity.

Electrostatic Potential Energy:

The work done in moving a charge q from infinity to a point in the field against the electric force is called electrostatic potential energy.

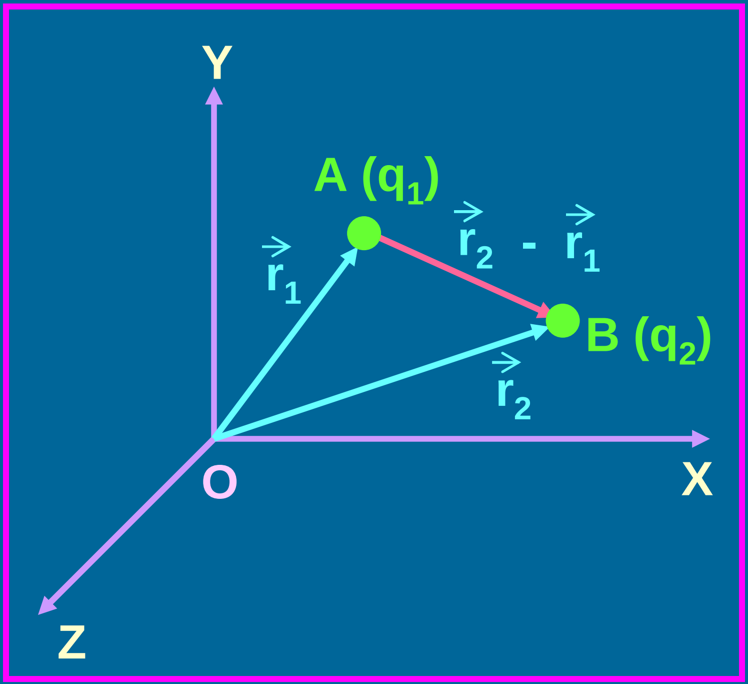
$$W = q\,V$$

i) Electrostatic Potential Energy of a Two Charges System:

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|}$$

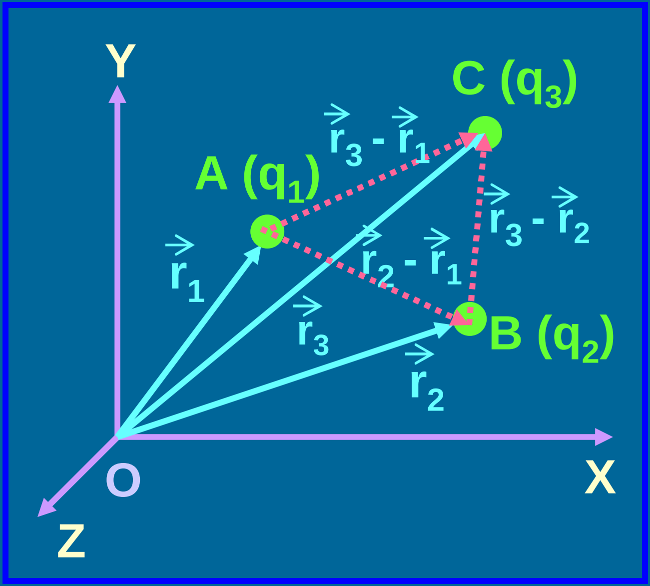
or

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$



ii) Electrostatic Potential Energy of a Three Charges System:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{|\vec{r}_3 - \vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{|\vec{r}_3 - \vec{r}_2|}$$



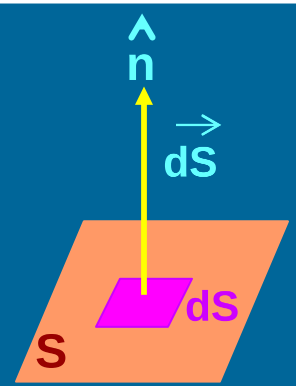
or

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{31}} + \frac{q_2 q_3}{r_{32}}\right]$$

iii) Electrostatic Potential Energy of an n - Charges System:

$$U = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n}\frac{q_i\, q_j}{|\vec{r}_j - \vec{r}_i|}\right]$$

Area Vector:



Small area of a surface can be represented by a vector.

$$\vec{dS} = dS\,\hat{n}$$

Electric Flux:

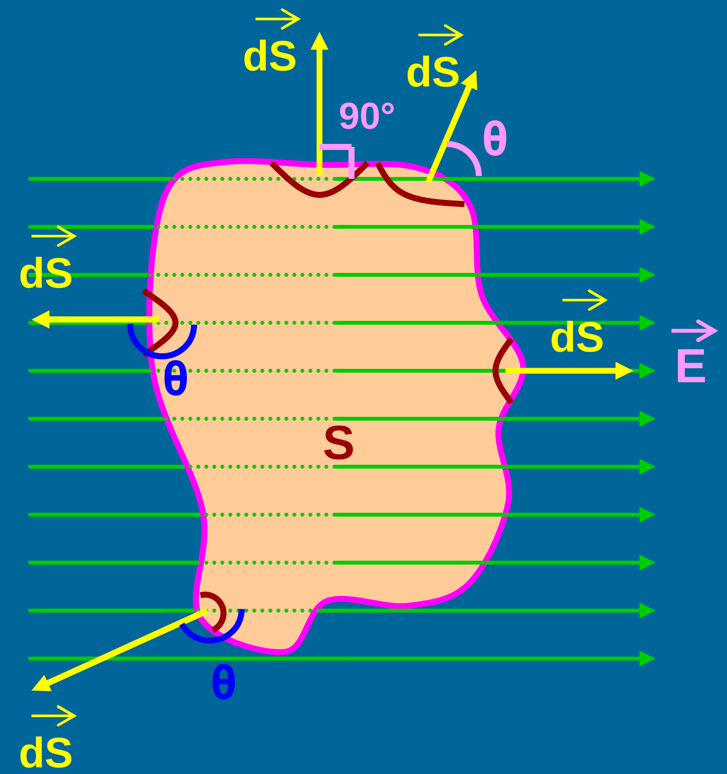
Electric flux linked with any surface is defined as the total number of electric lines of force that normally pass through that surface.

Electric flux dΦ through a small area element dS due to an electric field E at an angle θ with dS is

$$d\Phi = \vec{E} \cdot \vec{dS} = E\, dS \cos\theta$$

Total electric flux Φ over the whole surface S due to an electric field E is

$$\Phi = \int_S \vec{E} \cdot \vec{dS} = E\, S \cos\theta = \vec{E} \cdot \vec{S}$$



Electric flux is a scalar quantity. But it is a property of vector field.

SI unit of electric flux is N m^2 C^{-1} or J m C^{-1}.

Special Cases:

1. For $0° < \theta < 90°$, Φ is positive.
2. For $\theta = 90°$, Φ is zero.
3. For $90° < \theta < 180°$, Φ is negative.

Solid Angle:

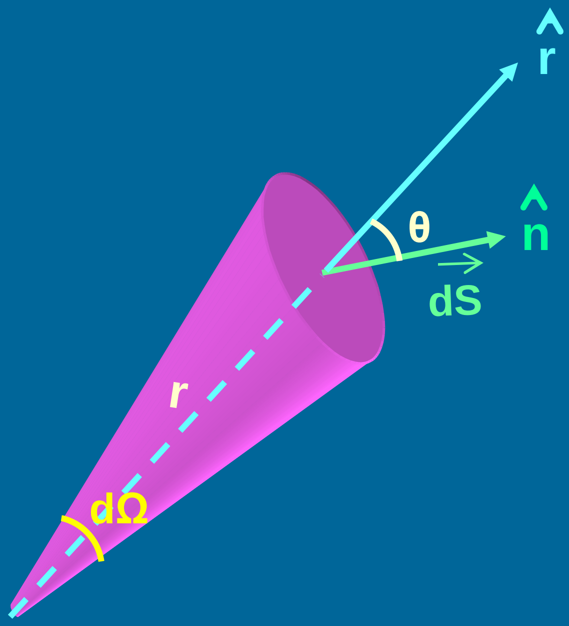
Solid angle is the three-dimensional equivalent of an ordinary two-dimensional plane angle.

SI unit of solid angle is steradian.

Solid angle subtended by area element dS at the centre O of a sphere of radius r is

$$d\Omega = \frac{dS \cos\theta}{r^2}$$

$$\Omega = \int_S d\Omega = \int_S \frac{dS \cos\theta}{r^2} = 4\pi \text{ steradian}$$



Gauss's Theorem:

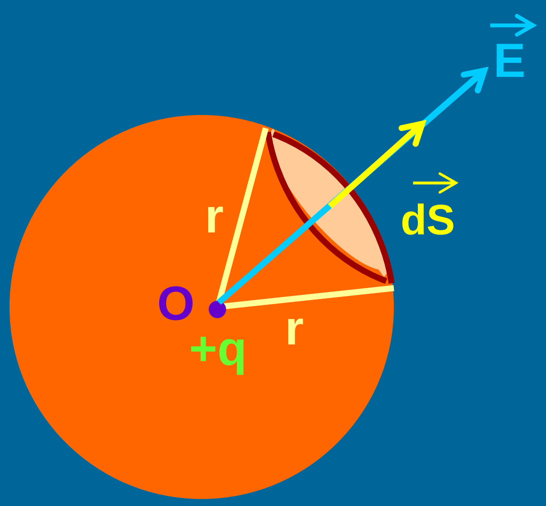
The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to 1 / ε_0 times the net charge enclosed within the surface.

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i$$

Proof of Gauss's Theorem for Spherically Symmetric Surfaces:

$$d\Phi = \vec{E} \cdot \vec{dS} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \cdot dS\, \hat{n}$$

$$d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q\, dS}{r^2} \hat{r} \cdot \hat{n}$$



Here, $\hat{r} \cdot \hat{n} = 1 \times 1 \cos 0° = 1$

$$\therefore \quad d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q\, dS}{r^2}$$

$$\Phi_E = \oint_S d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \oint_S dS = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Proof of Gauss's Theorem for a Closed Surface of any Shape:

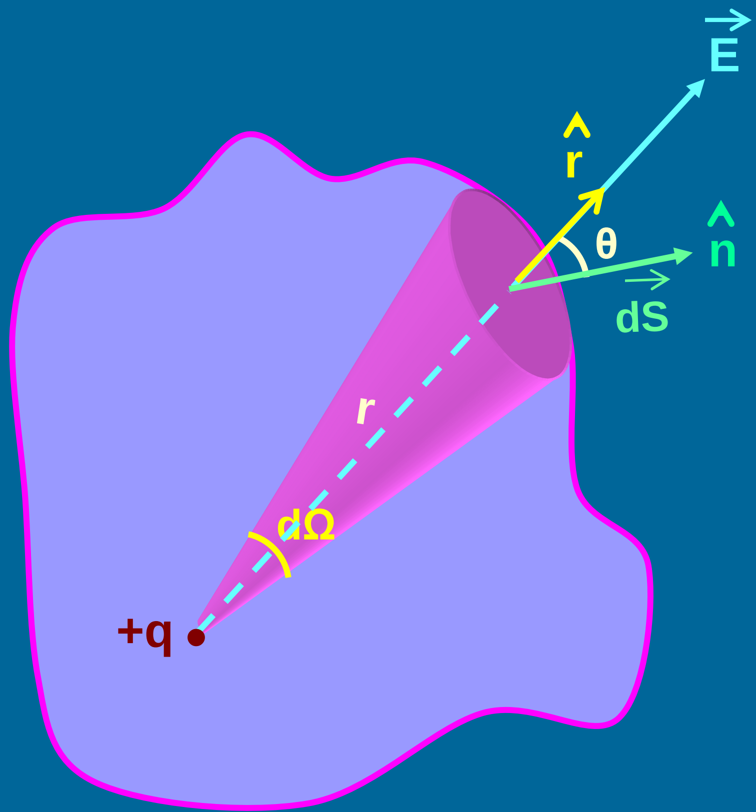
$$d\Phi = \vec{E} \cdot \vec{dS} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \cdot dS \, \hat{n}$$

$$d\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q \, dS}{r^2} \hat{r} \cdot \hat{n}$$

Here, $\hat{r} \cdot \hat{n} = 1 \times 1 \cos\theta = \cos\theta$

$$\therefore \quad d\Phi = \frac{q}{4\pi\varepsilon_0} \frac{dS \cos\theta}{r^2}$$

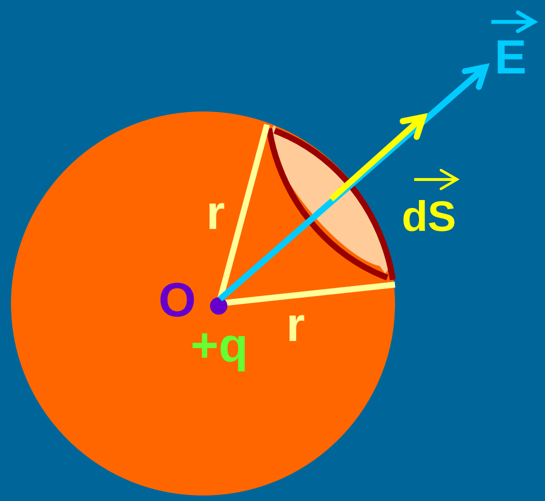
$$\Phi_E = \oint_S d\Phi = \frac{q}{4\pi\varepsilon_0} \oint_S d\Omega = \frac{q}{4\pi\varepsilon_0} 4\pi = \frac{q}{\varepsilon_0}$$



Deduction of Coulomb's Law from Gauss's Theorem:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$



Since $\vec{E}$ and $d\vec{S}$ are in the same direction,

$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{q}{4\pi\varepsilon_0 \; r^2}$$

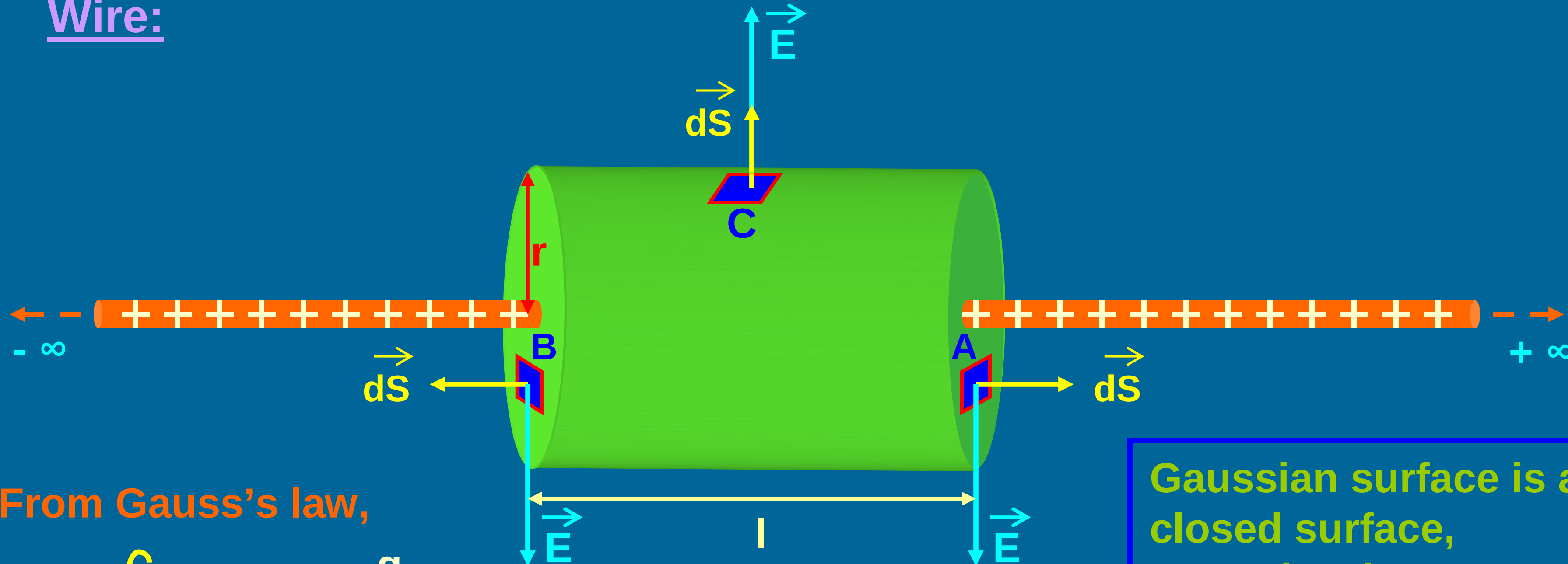
If a charge q_0 is placed at a point where E is calculated, then

$$F = \frac{q q_0}{4\pi\varepsilon_0 \; r^2}$$

which is Coulomb's Law.

Applications of Gauss's Theorem:

1. Electric Field Intensity due to an Infinitely Long Straight Charged Wire:



From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A \vec{E} \cdot \vec{dS} + \int_B \vec{E} \cdot \vec{dS} + \int_C \vec{E} \cdot \vec{dS}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A E\, dS \cos 90° + \int_B E\, dS \cos 90° + \int_C E\, dS \cos 0° = E \int_C dS \; = E \times 2\,\pi\, r\, l$$

Gaussian surface is a closed surface, around a charge distribution, such that the electric field intensity has a single fixed value at every point on the surface.

$$\frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$ (where λ is the liner charge density)

$$\therefore \quad E \times 2 \pi r l = \frac{\lambda l}{\varepsilon_0}$$

or $$E = \frac{1}{2 \pi \varepsilon_0} \frac{\lambda}{r}$$

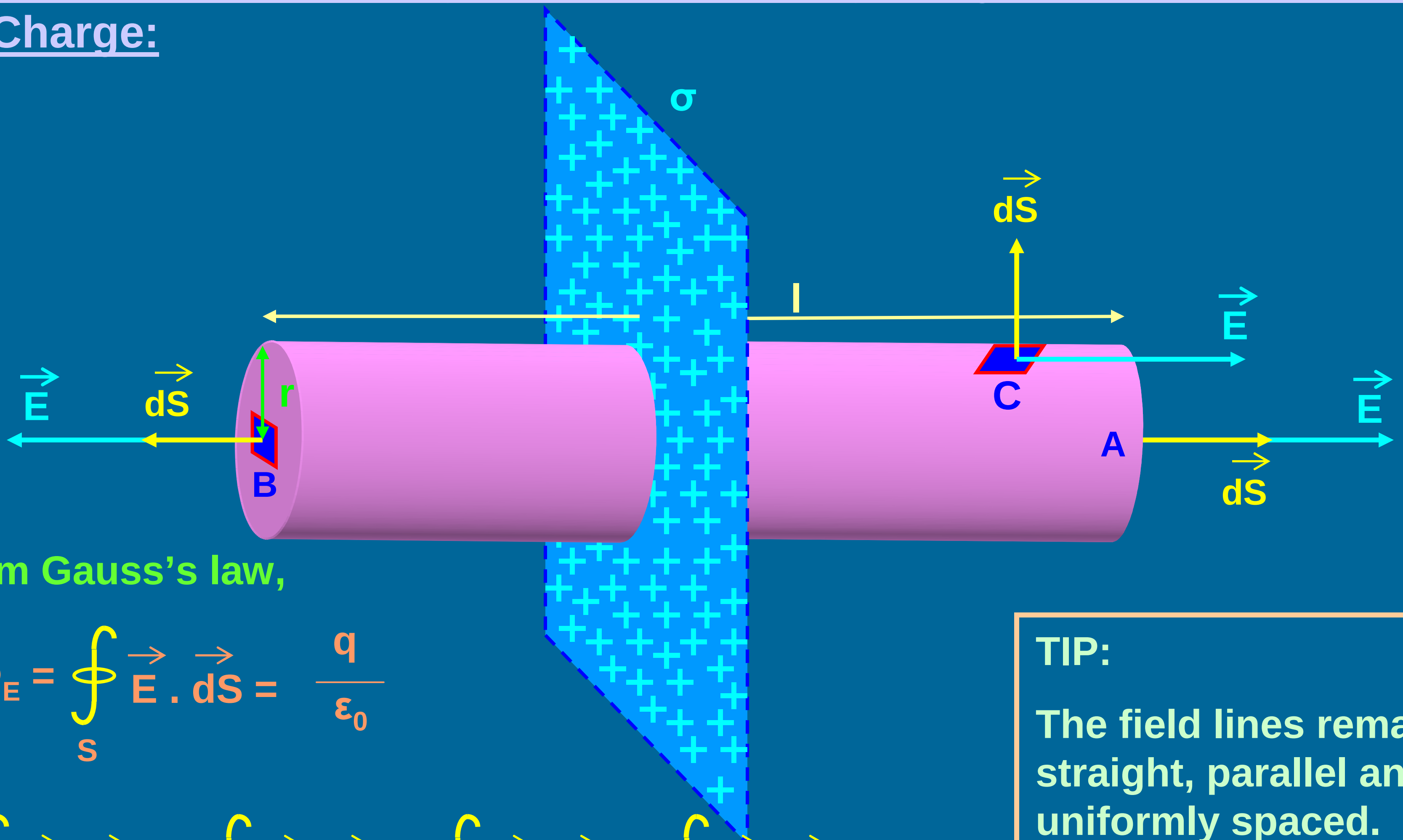
or $$\boxed{E = \frac{1}{4 \pi \varepsilon_0} \frac{2\lambda}{r}}$$

In vector form, $$\boxed{\vec{E}(\vec{r}) = \frac{1}{4 \pi \varepsilon_0} \frac{2\lambda}{r} \hat{r}}$$

The direction of the electric field intensity is radially outward from the positive line charge. For negative line charge, it will be radially inward.

Note:
The electric field intensity is independent of the size of the Gaussian surface constructed. It depends only on the distance of point of consideration. i.e. the Gaussian surface should contain the point of consideration.

2. Electric Field Intensity due to an Infinitely Long, Thin Plane Sheet of Charge:



From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A \vec{E} \cdot \vec{dS} + \int_B \vec{E} \cdot \vec{dS} + \int_C \vec{E} \cdot \vec{dS}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A E\, dS \cos 0° + \int_B E\, dS \cos 0° + \int_C E\, dS \cos 90° = 2E \int dS = 2E \times \pi r^2$$

TIP:

The field lines remain straight, parallel and uniformly spaced.

$$\frac{q}{\varepsilon_0} = \frac{\sigma \pi r^2}{\varepsilon_0}$$ (where σ is the surface charge density)

$$\therefore \quad 2 E \times \pi r^2 = \frac{\sigma \pi r^2}{\varepsilon_0}$$

or $$E = \frac{\sigma}{2 \varepsilon_0}$$

In vector form, $$\vec{E}(\vec{l}) = \frac{\sigma}{2 \varepsilon_0} \hat{l}$$

The direction of the electric field intensity is normal to the plane and away from the positive charge distribution. For negative charge distribution, it will be towards the plane.

Note:

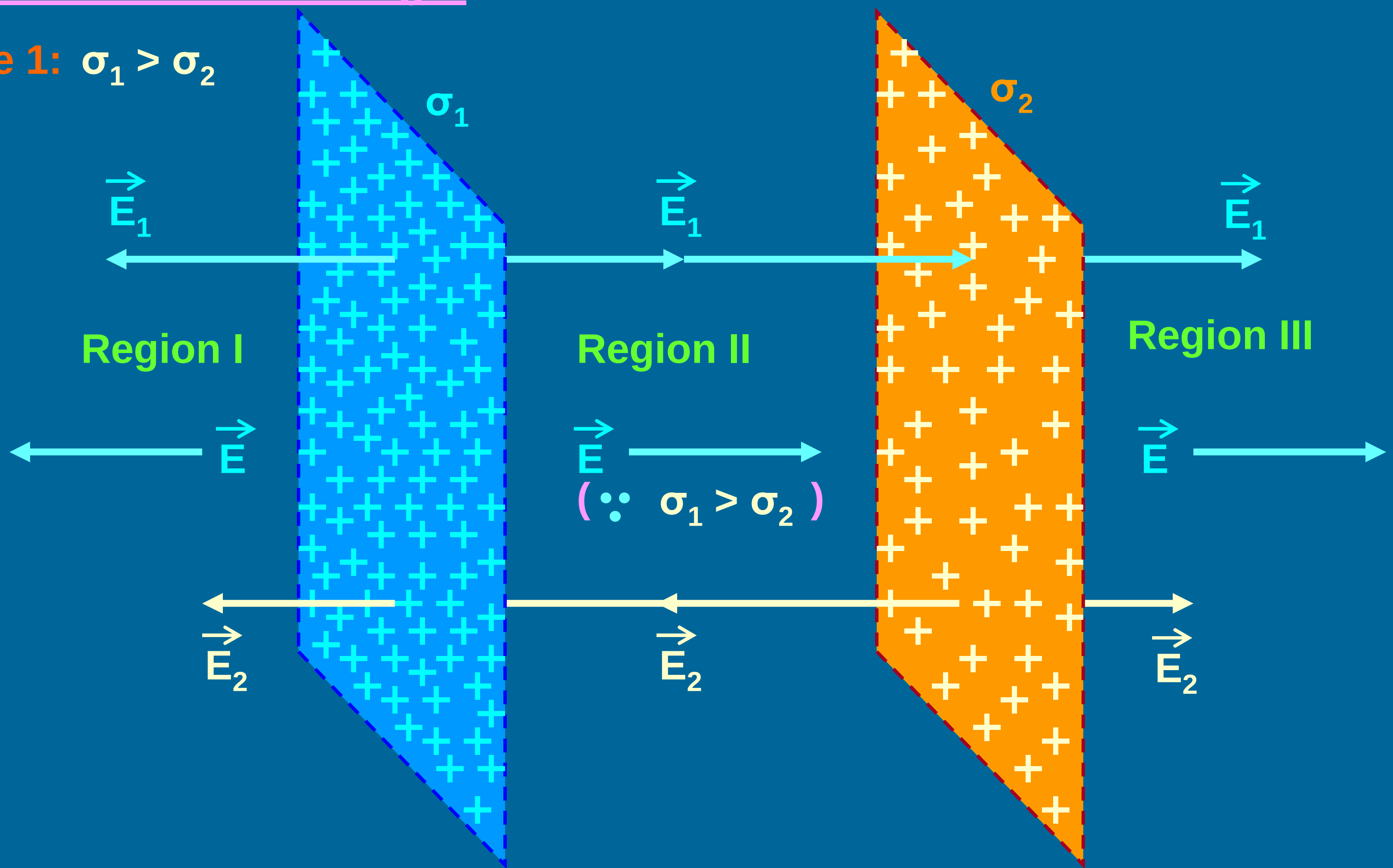
The electric field intensity is independent of the size of the Gaussian surface constructed. It neither depends on the distance of point of consideration nor the radius of the cylindrical surface.

If the plane sheet is thick, then the charge distribution will be available on both the sides. So, the charge enclosed within the Gaussian surface will be twice as before. Therefore, the field will be twice.

$$\therefore \quad E = \frac{\sigma}{\varepsilon_0}$$

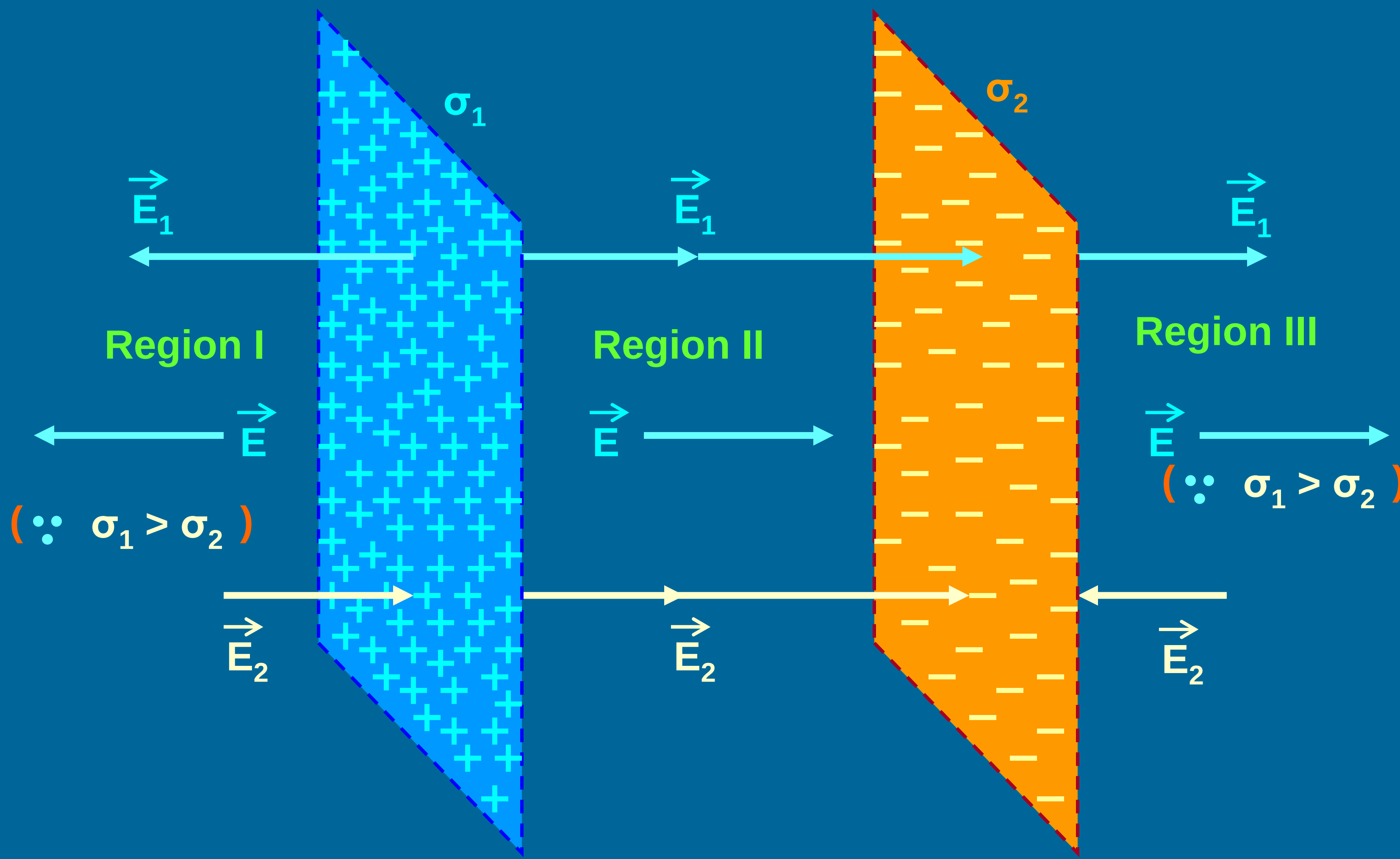
3. Electric Field Intensity due to Two Parallel, Infinitely Long, Thin Plane Sheet of Charge:

Case 1: $\sigma_1 > \sigma_2$



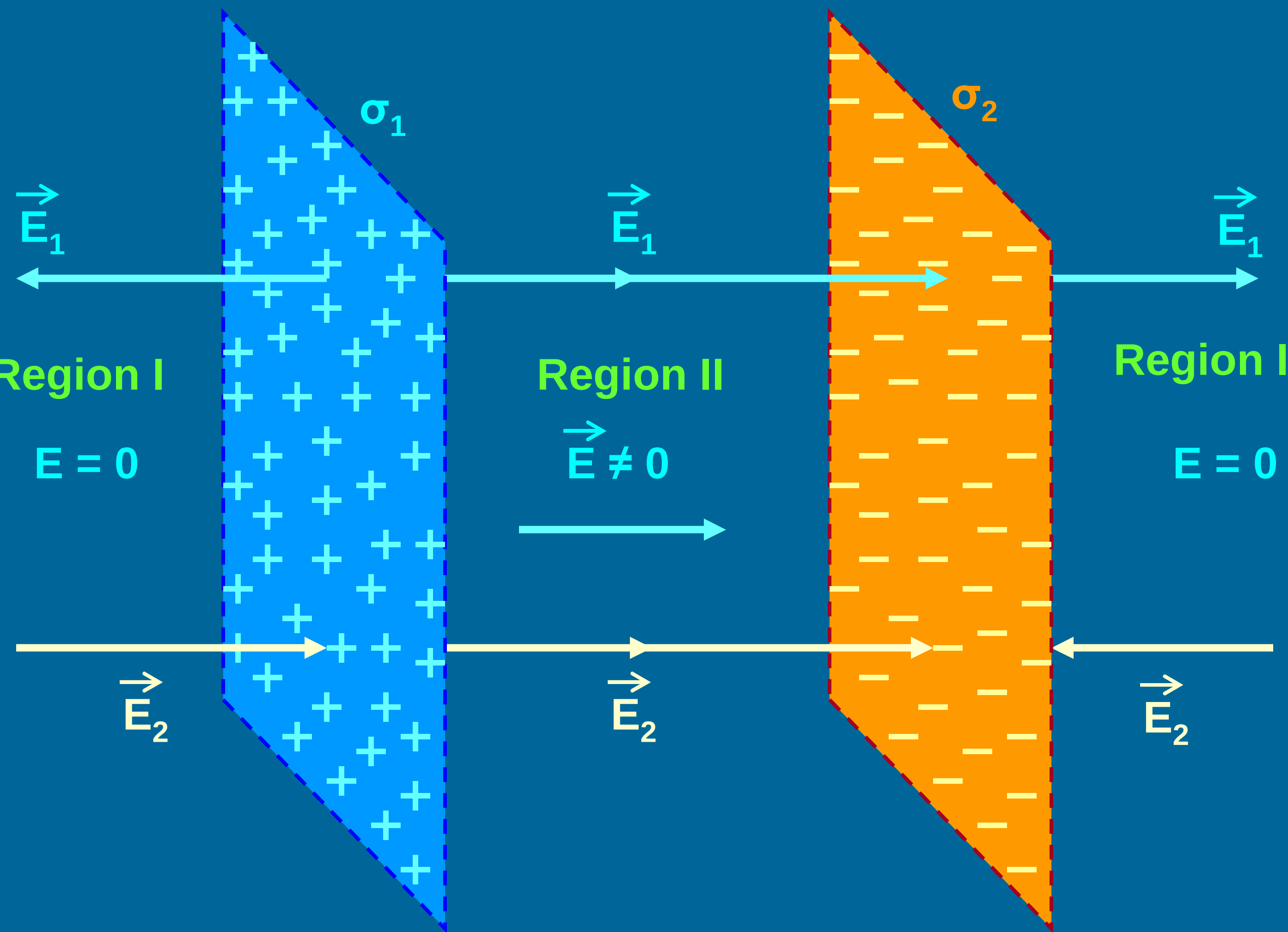
$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0}$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$

Case 2: $+\sigma_1$ & $-\sigma_2$



$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0}$	$E = E_1 + E_2$ $E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0}$	$E = E_1 - E_2$ $E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0}$

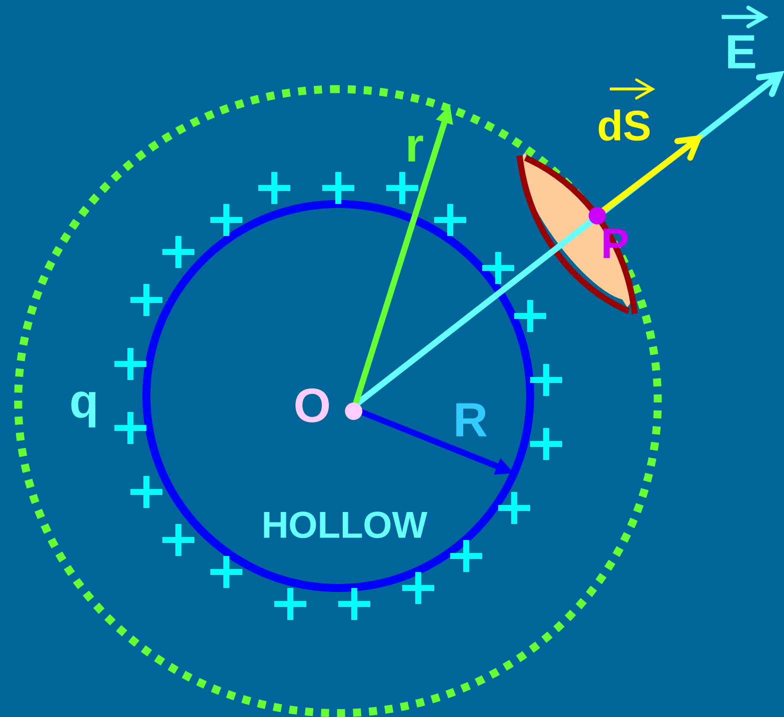
Case 3: $+\sigma$ & $-\sigma$



$E = E_1 - E_2$	$E = E_1 + E_2$	$E = E_1 - E_2$
$E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0} = 0$	$E = \dfrac{\sigma_1 + \sigma_2}{2\,\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$	$E = \dfrac{\sigma_1 - \sigma_2}{2\,\varepsilon_0} = 0$

4. Electric Field Intensity due to a Uniformed Charged This Spherical Shell:

i) At a point P outside the shell:



From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $d\vec{S}$ are in the same direction,

$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

or $$\Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

……… Gaussian Surface

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$ or $$E = \frac{q}{4\pi\varepsilon_0 \; r^2}$$

Since $q = \sigma \times 4\pi R^2$, $\therefore$ $$E = \frac{\sigma \; R^2}{\varepsilon_0 \; r^2}$$

Electric field due to a uniformly charged thin spherical shell at a point outside the shell is such as if the whole charge were concentrated at the centre of the shell.

ii) At a point A on the surface of the shell:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

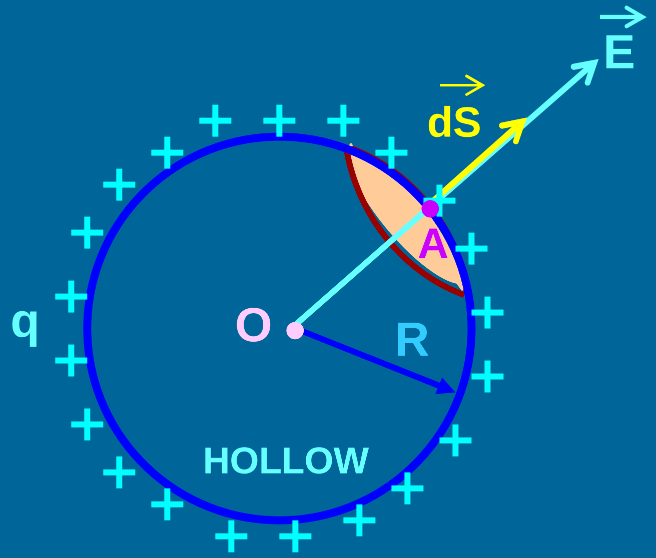
$$\therefore \quad \Phi_E = \oint_S E \; dS = \frac{q}{\varepsilon_0}$$

or $$\Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{q}{4\pi\varepsilon_0 R^2}$$

Since $q = \sigma \times 4\pi R^2$,

$$\therefore \quad E = \frac{\sigma}{\varepsilon_0}$$



Electric field due to a uniformly charged thin spherical shell at a point on the surface of the shell is maximum.

iii) At a point B inside the shell:

From Gauss's law,

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are in the same direction,

$$\therefore \quad \Phi_E = \oint_S E \, dS = \frac{q}{\varepsilon_0}$$

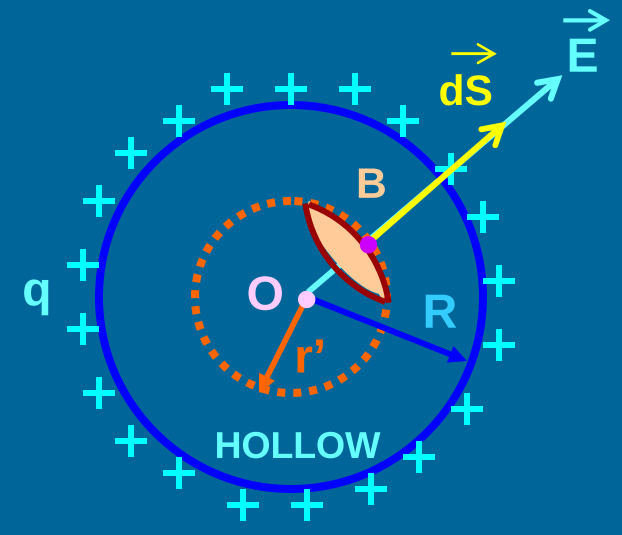
$$\text{or} \quad \Phi_E = E \oint_S dS = \frac{q}{\varepsilon_0}$$

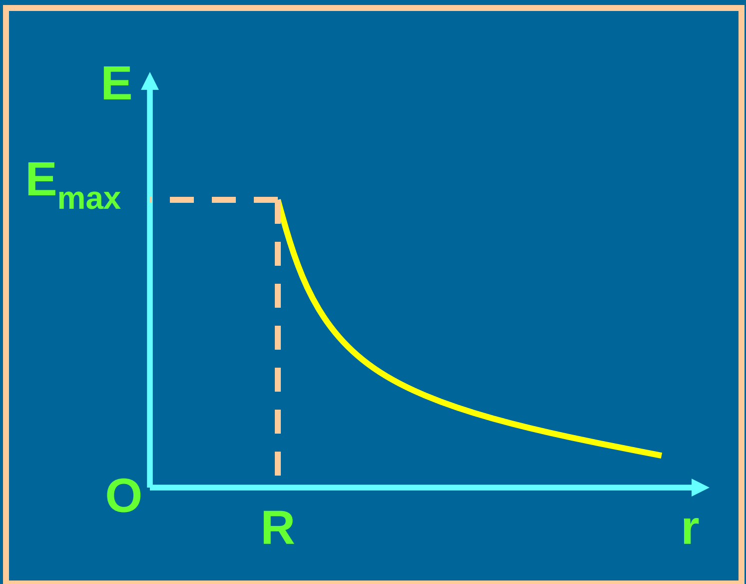
$$E \times 4\pi r'^2 = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{0}{4\pi\varepsilon_0 r'^2}$$

(since q = 0 inside the Gaussian surface)

$$\therefore \quad E = 0$$

This property E = 0 inside a cavity is used for electrostatic shielding.





ELECTROSTATICS - IV

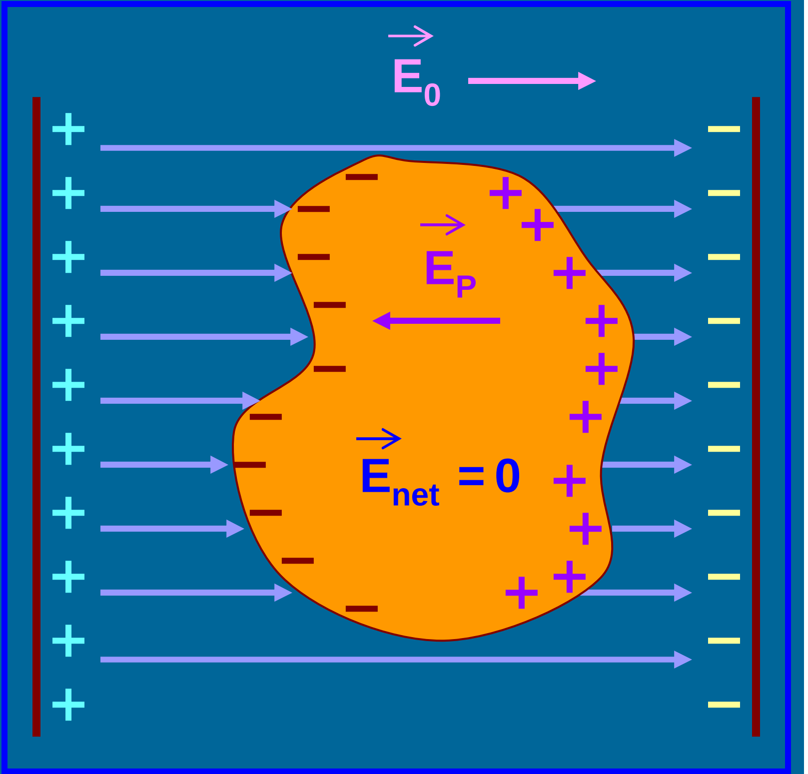
- Capacitance and Van de Graaff Generator

1. Behaviour of Conductors in Electrostatic Field
2. Electrical Capacitance
3. Principle of Capacitance
4. Capacitance of a Parallel Plate Capacitor
5. Series and Parallel Combination of Capacitors
6. Energy Stored in a Capacitor and Energy Density
7. Energy Stored in Series and Parallel Combination of Capacitors
8. Loss of Energy on Sharing Charges Between Two Capacitors
9. Polar and Non-polar Molecules
10. Polarization of a Dielectric
11. Polarizing Vector and Dielectric Strength
12. Parallel Plate Capacitor with a Dielectric Slab
13. Van de Graaff Generator

Behaviour of Conductors in the Electrostatic Field:

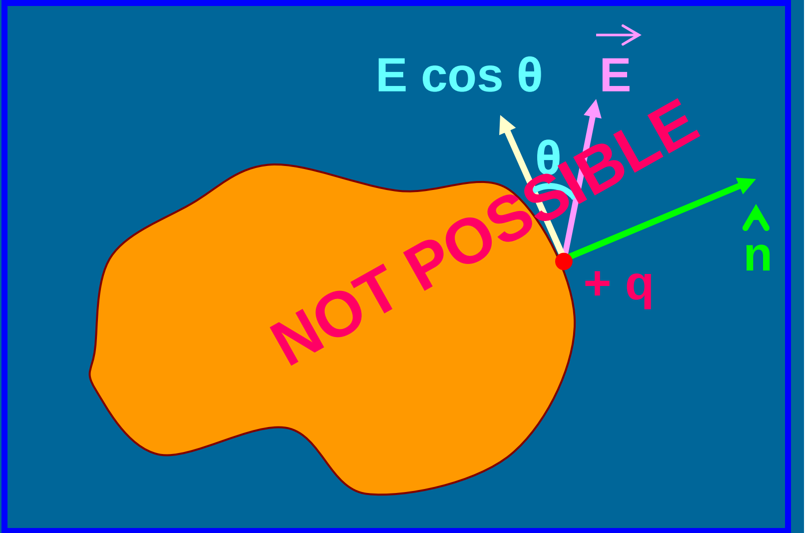
1. **Net electric field intensity in the interior of a conductor is zero.**

   When a conductor is placed in an electrostatic field, the charges (free electrons) drift towards the positive plate leaving the + ve core behind. At an equilibrium, the electric field due to the polarisation becomes equal to the applied field. So, the net electrostatic field inside the conductor is zero.



2. **Electric field just outside the charged conductor is perpendicular to the surface of the conductor.**

   Suppose the electric field is acting at an angle other than 90°, then there will be a component E cos θ acting along the tangent at that point to the surface which will tend to accelerate the charge on the surface leading to 'surface current'. But there is no surface current in electrostatics. So, θ = 90° and cos 90° = 0.

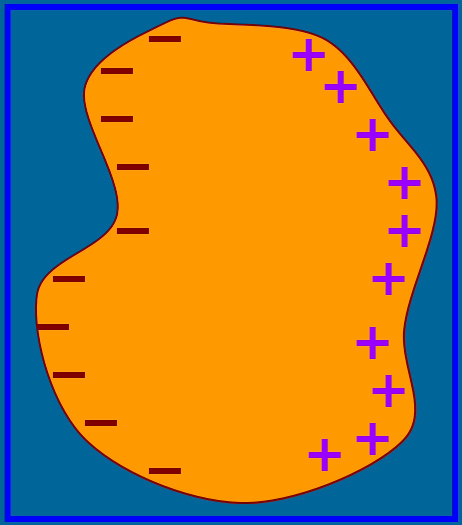


3. Net charge in the interior of a conductor is zero.

The charges are temporarily separated. The total charge of the system is zero.

$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

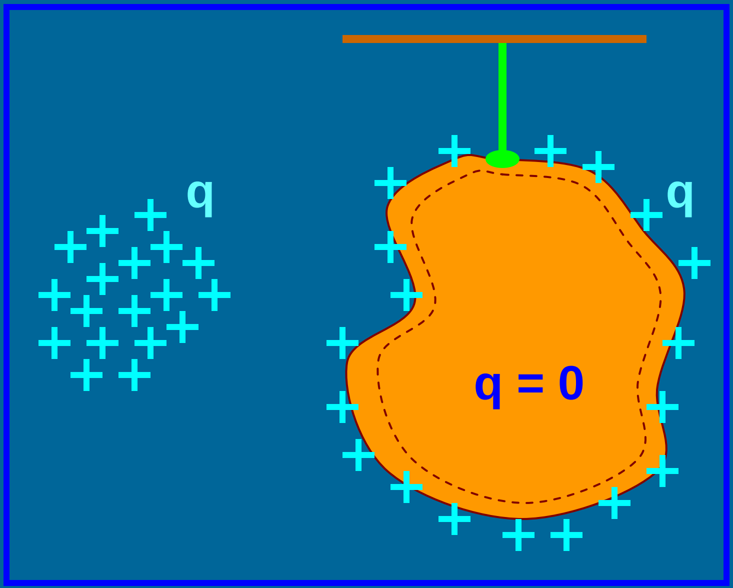
Since E = 0 in the interior of the conductor, therefore q = 0.



4. Charge always resides on the surface of a conductor.

Suppose a conductor is given some excess charge q. Construct a Gaussian surface just inside the conductor.

Since E = 0 in the interior of the conductor, therefore q = 0 inside the conductor.



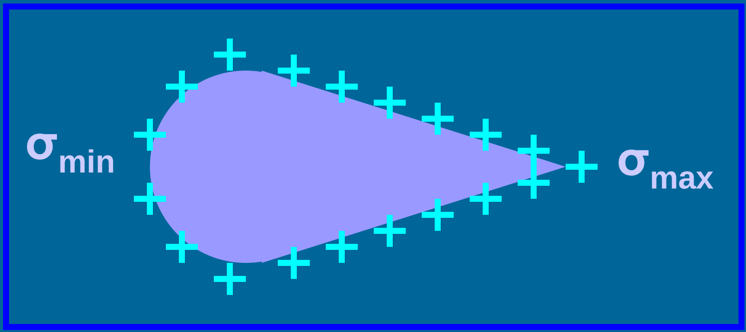
5. Electric potential is constant for the entire conductor.

$dV = - E \cdot dr$

Since E = 0 in the interior of the conductor, therefore dV = 0. i.e. V = constant

6. Surface charge distribution may be different at different points.

$$\sigma = \frac{q}{S}$$



Every conductor is an equipotential volume (three- dimensional) rather than just an equipotential surface (two- dimensional).

Electrical Capacitance:

The measure of the ability of a conductor to store charges is known as capacitance or capacity (old name).

$q \propto V$ or $q = C V$ or $C = \frac{q}{V}$

If $V = 1$ volt, then $C = q$

Capacitance of a conductor is defined as the charge required to raise its potential through one unit.

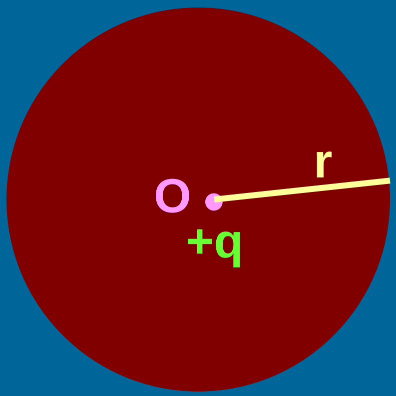
SI Unit of capacitance is 'farad' (F). Symbol of capacitance:

Capacitance is said to be 1 farad when 1 coulomb of charge raises the potential of conductor by 1 volt.

Since 1 coulomb is the big amount of charge, the capacitance will be usually in the range of milli farad, micro farad, nano farad or pico farad.

Capacitance of an Isolated Spherical Conductor:

Let a charge q be given to the sphere which is assumed to be concentrated at the centre.



Potential at any point on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

$$C = \frac{q}{V}$$

$$\therefore \quad C = 4\pi\varepsilon_0 r$$

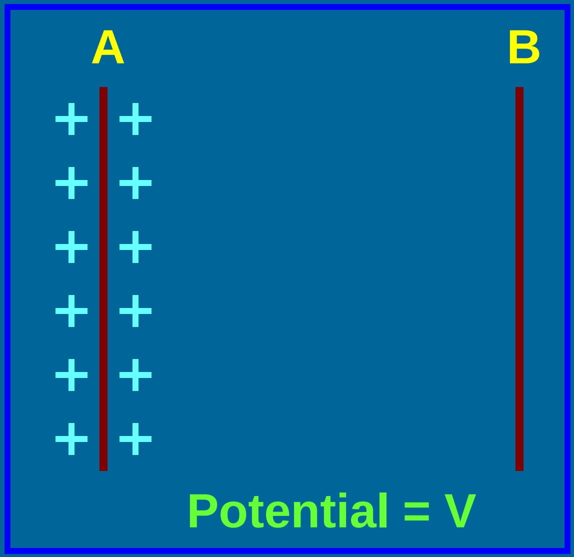
1. Capacitance of a spherical conductor is directly proportional to its radius.
2. The above equation is true for conducting spheres, hollow or solid.
3. IF the sphere is in a medium, then $C = 4\pi\varepsilon_0\varepsilon_r r$.
4. Capacitance of the earth is 711 μF.

Principle of Capacitance:

Step 1: Plate A is positively charged and B is neutral.

Step 2: When a neutral plate B is brought near A, charges are induced on B such that the side near A is negative and the other side is positive.

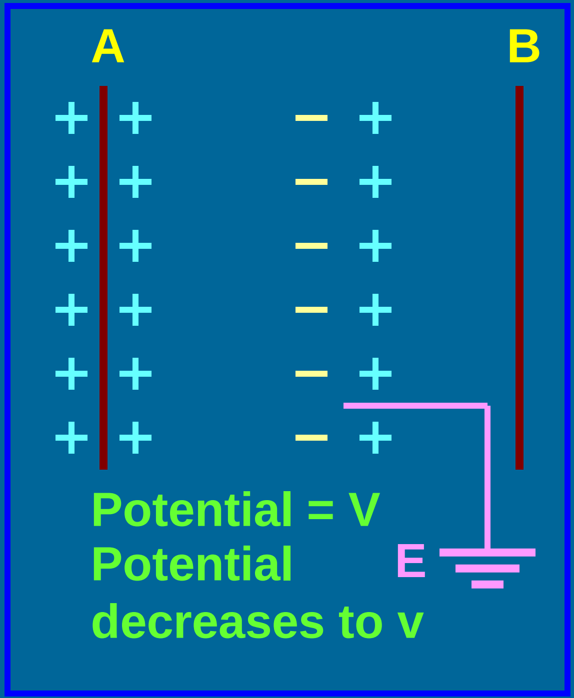
The potential of the system of A and B in step 1 and 2 remains the same because the potential due to positive and negative charges on B cancel out.



Step 3: When the farther side of B is earthed the positive charges on B get neutralised and B is left only with negative charges.

Now, the net potential of the system decreases due to the sum of positive potential on A and negative potential on B.

To increase the potential to the same value as was in step 2, an additional amount of charges can be given to plate A.



This means, the capacity of storing charges on A increases.

The system so formed is called a 'capacitor'.

Capacitance of Parallel Plate Capacitor:

Parallel plate capacitor is an arrangement of two parallel conducting plates of equal area separated by air medium or any other insulating medium such as paper, mica, glass, wood, ceramic, etc.

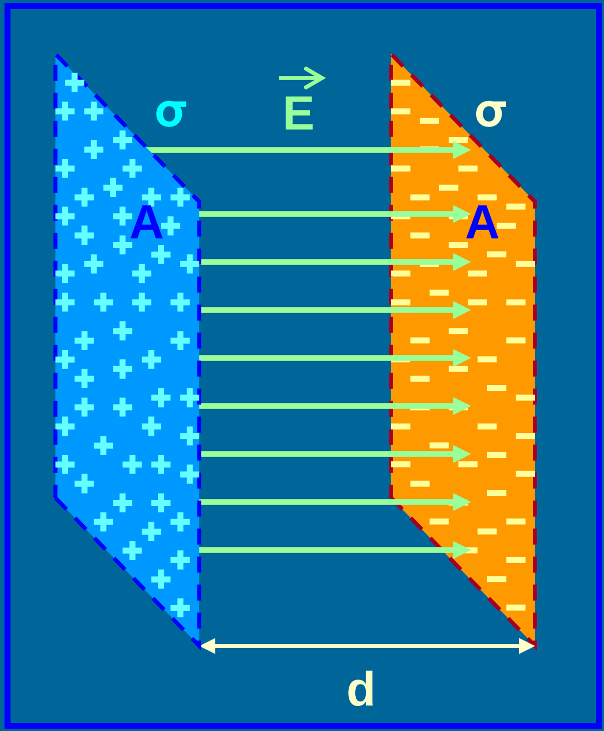
$$V = E\,d = \frac{\sigma}{\varepsilon_0}\,d$$

or

$$V = \frac{q\,d}{A\,\varepsilon_0}$$

But

$$C = \frac{q}{V} \quad \therefore \quad C = \frac{A\,\varepsilon_0}{d}$$



If the space between the plates is filled with dielectric medium of relative permittivity ε_r, then

$$C = \frac{A\,\varepsilon_0\,\varepsilon_r}{d}$$

Capacitance of a parallel plate capacitor is

(i) directly proportional to the area of the plates and

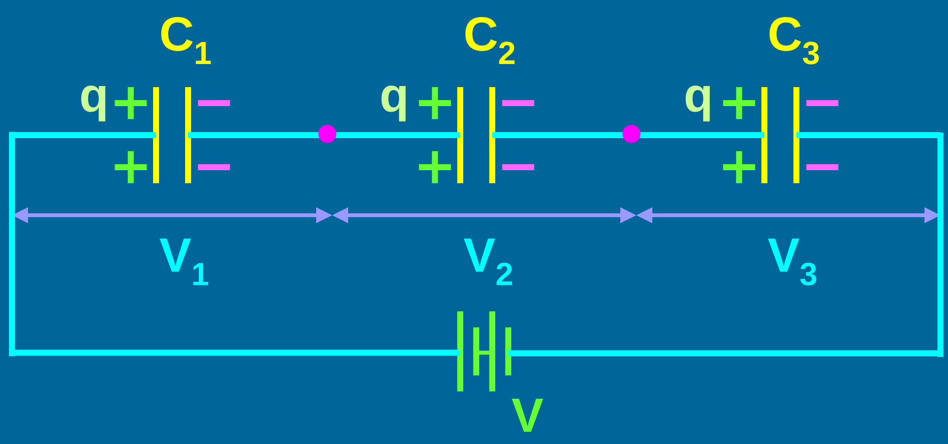
(ii) inversely proportional to the distance of separation between them.

Series Combination of Capacitors:

In series combination,

i) Charge is same in each capacitor

ii) Potential is distributed in inverse proportion to capacitances



i.e. $V = V_1 + V_2 + V_3$

But $V = \frac{q}{C}$, $V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$ and $V_3 = \frac{q}{C_3}$

$$\therefore \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

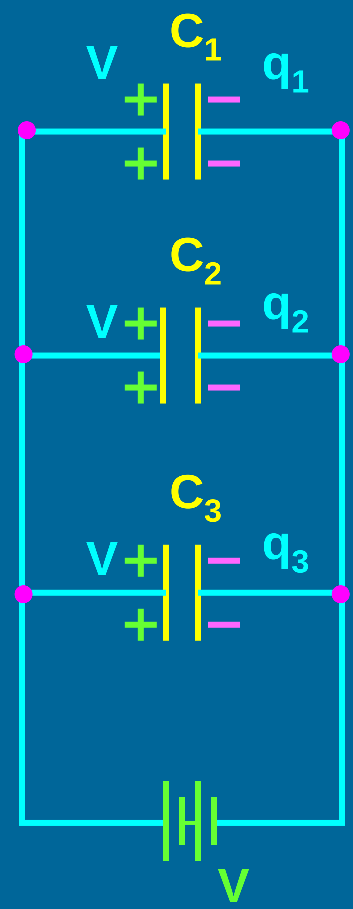
(where C is the equivalent capacitance or effective capacitance or net capacitance or total capacitance)

$$\frac{1}{C} = \sum_{i=1}^{n} \frac{1}{C_i}$$

The reciprocal of the effective capacitance is the sum of the reciprocals of the individual capacitances.

Note: The effective capacitance in series combination is less than the least of all the individual capacitances.

Parallel Combination of Capacitors:



In parallel combination,

i) Potential is same across each capacitor

ii) Charge is distributed in direct proportion to capacitances

i.e. $q = q_1 + q_2 + q_3$

But $q_1 = C_1 V$, $q_2 = C_2 V$, $q_3 = C_3 V$ and $q = C V$

$\therefore$ $C V = C_1 V + C_2 V + C_3 V$ (where C is the equivalent capacitance)

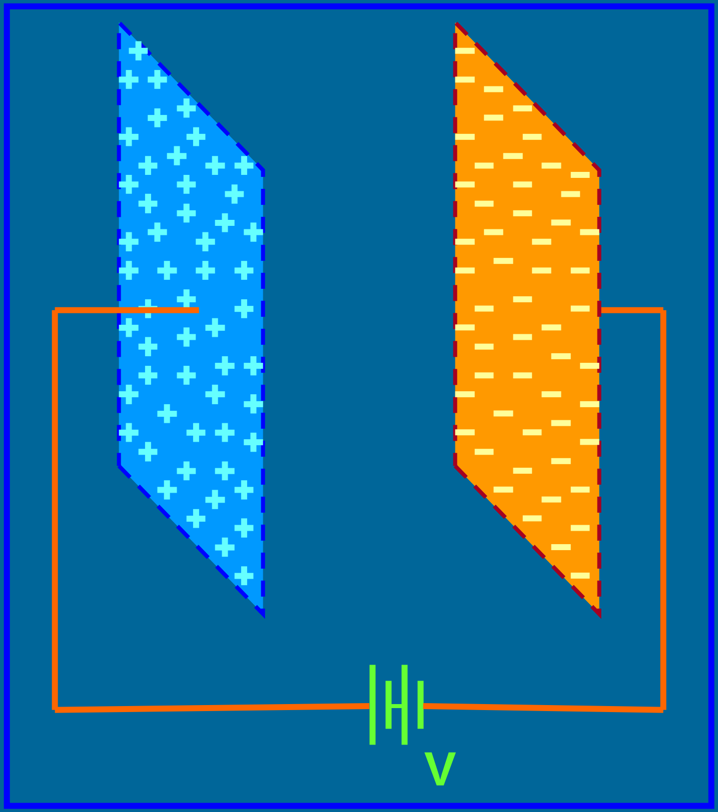
or $C = C_1 + C_2 + C_3$

$$C = \sum_{i=1}^{n} C_i$$

The effective capacitance is the sum of the individual capacitances.

Note: The effective capacitance in parallel combination is larger than the largest of all the individual capacitances.

Energy Stored in a Capacitor:



The process of charging a capacitor is equivalent to transferring charges from one plate to the other of the capacitor.

The moment charging starts, there is a potential difference between the plates. Therefore, to transfer charges against the potential difference some work is to be done. This work is stored as electrostatic potential energy in the capacitor.

If dq be the charge transferred against the potential difference V, then work done is

$$dU = dW = V\,dq$$

$$= \frac{q}{C}\,dq$$

The total work done (energy) to transfer charge q is

$$U = \int_0^q \frac{q}{C}\,dq \quad \text{or} \quad U = \frac{1}{2}\frac{q^2}{C} \quad \text{or} \quad U = \frac{1}{2} C V^2 \quad \text{or} \quad U = \frac{1}{2} q V$$

Energy Density:

$$U = \frac{1}{2} C V^2$$

But $C = \frac{A \varepsilon_0}{d}$ and $V = E d$

$$\therefore \; U = \frac{1}{2} \varepsilon_0 A d E^2 \quad \text{or} \quad \frac{U}{Ad} = \frac{1}{2} \varepsilon_0 E^2 \quad \text{or} \quad \overline{U} = \frac{1}{2} \varepsilon_0 E^2$$

SI unit of energy density is $J\ m^{-3}$.

Energy density is generalised as energy per unit volume of the field.

Energy Stored in a Series Combination of Capacitors:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots + \frac{1}{C_n}$$

$$U = \frac{1}{2} \frac{q^2}{C}$$

$$\therefore \; U = \frac{1}{2} q^2 \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots + \frac{1}{C_n} \right]$$

$$U = U_1 + U_2 + U_3 + \ldots\ldots\ldots + U_n$$

The total energy stored in the system is the sum of energy stored in the individual capacitors.

Energy Stored in a Parallel Combination of Capacitors:

$$C = C_1 + C_2 + C_3 + \ldots\ldots\ldots + C_n$$

$$U = \frac{1}{2} C V^2$$

$$\therefore \quad U = \frac{1}{2} V^2 \left(C_1 + C_2 + C_3 + \ldots\ldots\ldots + C_n \right)$$

$$U = U_1 + U_2 + U_3 + \ldots\ldots\ldots + U_n$$

The total energy stored in the system is the sum of energy stored in the individual capacitors.

Loss of Energy on Sharing of Charges between the Capacitors in Parallel:

Consider two capacitors of capacitances C_1, C_2, charges q_1, q_2 and potentials V_1,V_2.

Total charge after sharing = Total charge before sharing

$$\therefore \quad (C_1 + C_2) V = C_1 V_1 + C_2 V_2$$

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

The total energy before sharing is

$$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

The total energy after sharing is

$$U_f = \frac{1}{2} (C_1 + C_2) V^2$$

$$U_i - U_f = \frac{C_1 C_2 (V_1 - V_2)2}{2 (C_1 + C_2)}$$

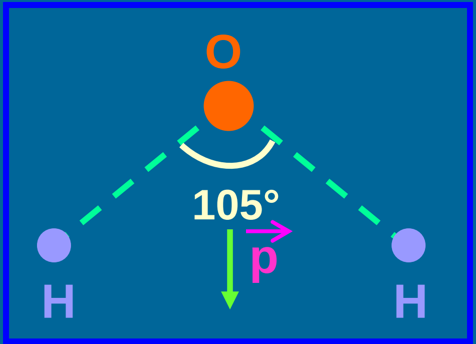
$U_i - U_f > 0$ or $U_i > U_f$

Therefore, there is some loss of energy when two charged capacitors are connected together.

The loss of energy appears as heat and the wire connecting the two capacitors may become hot.

Polar Molecules:

A molecule in which the centre of positive charges does not coincide with the centre of negative charges is called a polar molecule.

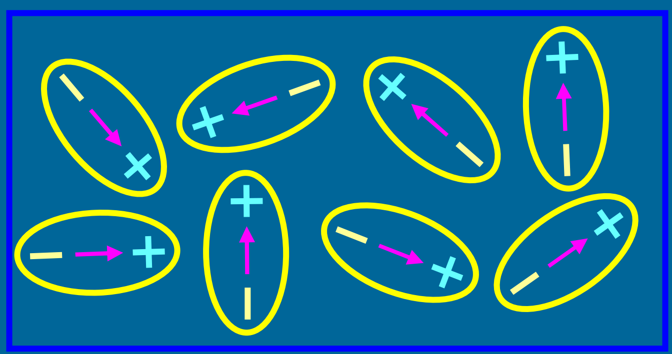


Polar molecule does not have symmetrical shape.

Eg. H Cl, H_2 O, N H_3, C O_2, alcohol, etc.

Effect of Electric Field on Polar Molecules:

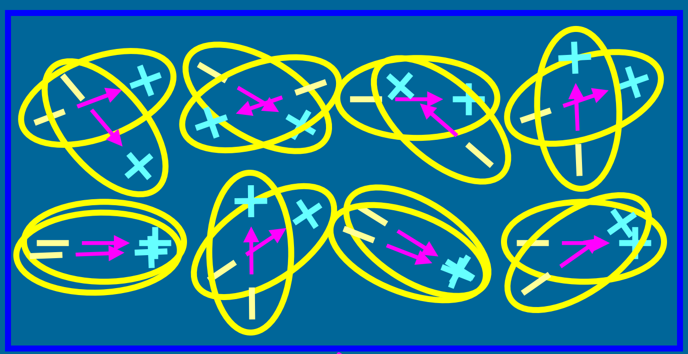
$\vec{E} = 0$



$\vec{p} = 0$

In the absence of external electric field, the permanent dipoles of the molecules orient in random directions and hence the net dipole moment is zero.

$\vec{E}$ →



$\vec{p}$ →

When electric field is applied, the dipoles orient themselves in a regular fashion and hence dipole moment is induced. Complete allignment is not possible due to thermal agitation.

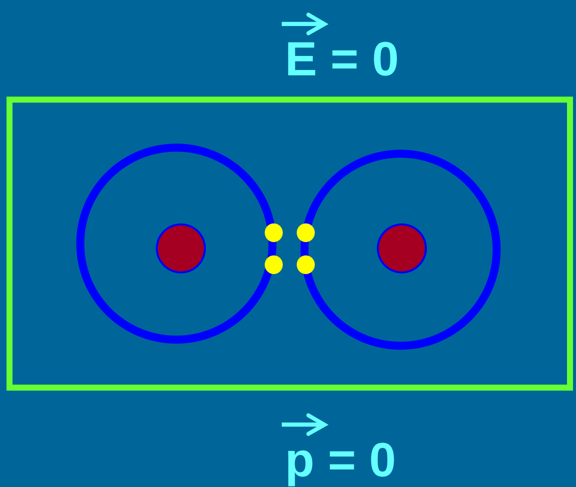
Non - polar Molecules:

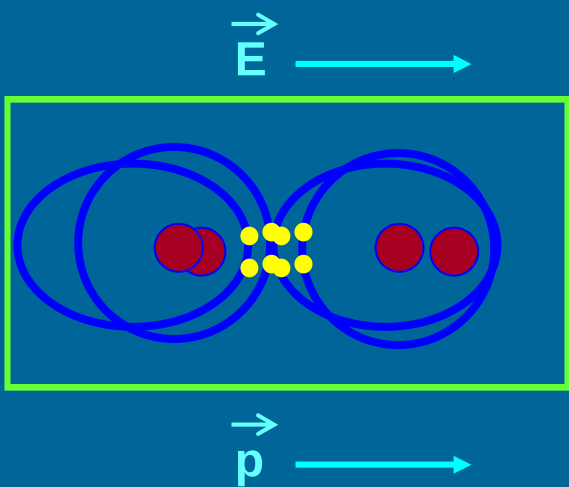
A molecule in which the centre of positive charges coincides with the centre of negative charges is called a non-polar molecule.

Non-polar molecule has symmetrical shape.

Eg. N_2 , $C\ H_4$, O_2, $C_6\ H_6$, etc.

Effect of Electric Field on Non-polar Molecules:





In the absence of external electric field, the effective positive and negative centres coincide and hence dipole is not formed.

When electric field is applied, the positive charges are pushed in the direction of electric field and the electrons are pulled in the direction opposite to the electric field. Due to separation of effective centres of positive and negative charges, dipole is formed.

Dielectrics:

Generally, a non-conducting medium or insulator is called a 'dielectric'.

Precisely, the non-conducting materials in which induced charges are produced on their faces on the application of electric fields are called dielectrics.

Eg. Air, H_2, glass, mica, paraffin wax, transformer oil, etc.

Polarization of Dielectrics:

When a non-polar dielectric slab is subjected to an electric field, dipoles are induced due to separation of effective positive and negative centres.

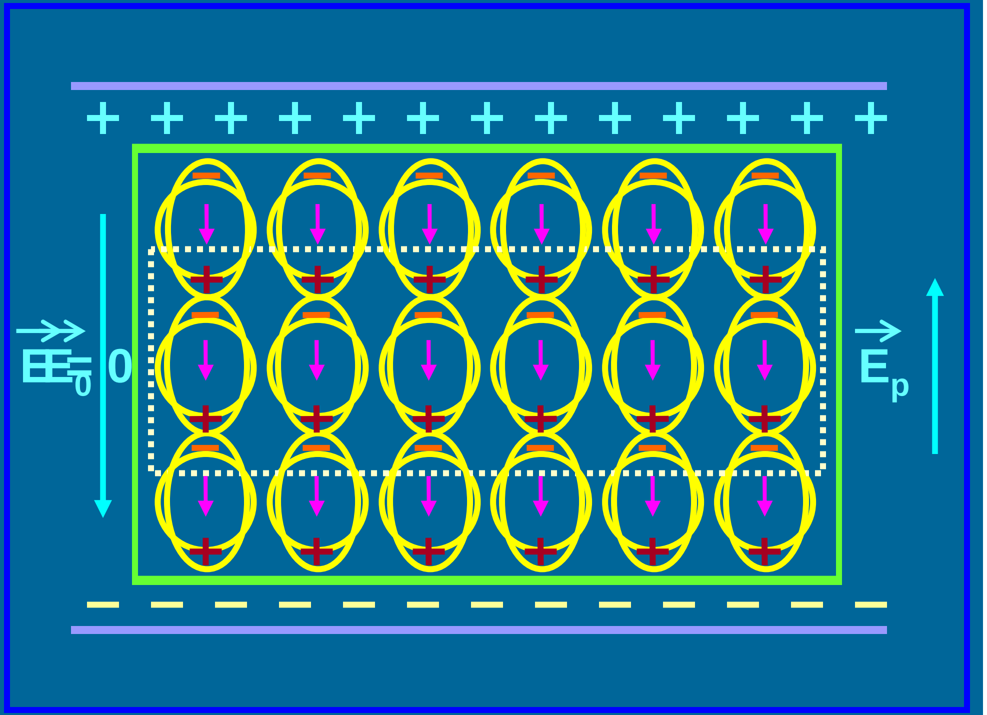
E_0 is the applied field and E_p is the induced field in the dielectric.

The net field is $E_N = E_0 - E_p$

i.e. the field is reduced when a dielectric slab is introduced.

The dielectric constant is given by

$$K = \frac{E_0}{E_0 - E_p}$$



Polarization Vector:

The polarization vector measures the degree of polarization of the dielectric. It is defined as the dipole moment of the unit volume of the polarized dielectric.

If n is the number of atoms or molecules per unit volume of the dielectric, then polarization vector is

$$\vec{P} = n\,\vec{p}$$

SI unit of polarization vector is $C\ m^{-2}$.

Dielectric Strength:

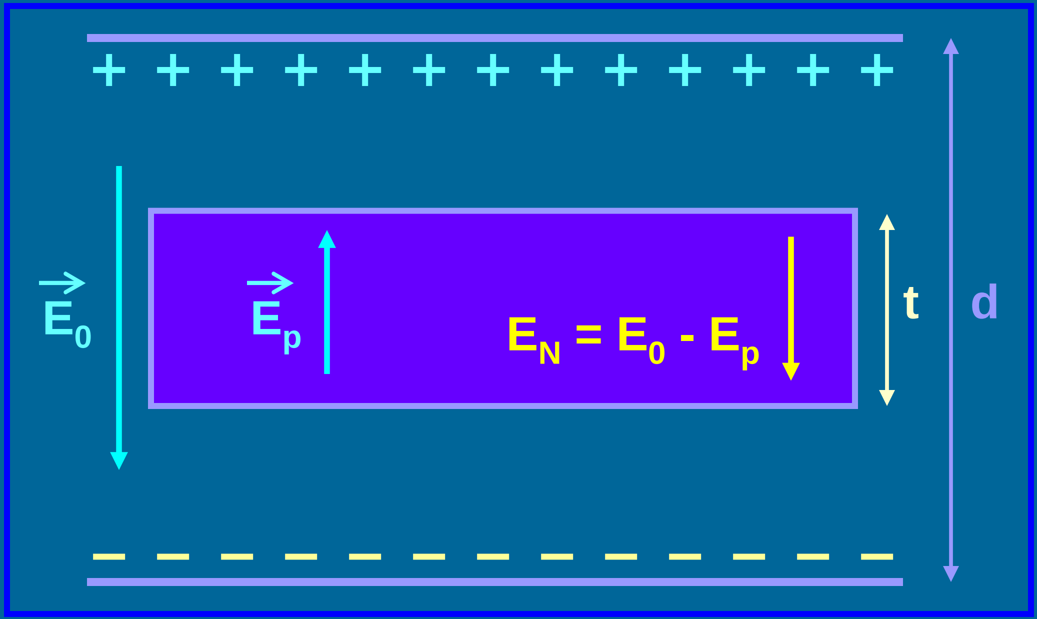
Dielectric strength is the maximum value of the electric field intensity that can be applied to the dielectric without its electric break down.

Its SI unit is $V\ m^{-1}$.

Its practical unit is $kV\ mm^{-1}$.

Dielectric	Dielectric strength (kV / mm)
Vacuum	∞
Air	0.8 – 1
Porcelain	4 – 8
Pyrex	14
Paper	14 – 16
Rubber	21
Mica	160 – 200

Capacitance of Parallel Plate Capacitor with Dielectric Slab:



$$V = E_0 (d - t) + E_N t$$

$$K = \frac{E_0}{E_N} \quad \text{or} \quad E_N = \frac{E_0}{K}$$

$$\therefore \quad V = E_0 (d - t) + \frac{E_0}{K} t$$

$$V = E_0 \left[(d - t) + \frac{t}{K} \right]$$

But $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{qA}{\varepsilon_0}$

and $C = \dfrac{q}{V}$

$$\therefore \quad C = \frac{A\,\varepsilon_0}{\left[(d - t) + \frac{t}{K} \right]}$$

or

$$C = \frac{A\,\varepsilon_0}{d\left[1 - \frac{t}{d}\left(1 - \frac{t}{K}\right) \right]}$$

or

$$C = \frac{C_0}{\left[1 - \frac{t}{d}\left(1 - \frac{t}{K}\right) \right]}$$

$C > C_0$. i.e. Capacitance increases with introduction of dielectric slab.

If the dielectric slab occupies the whole space between the plates, i.e. t = d, then

$$C = K C_0$$

Dielectric Constant

$$K = \frac{C}{C_0}$$

WITH DIELECTRIC SLAB

Physcial Quantity	With Battery disconnected	With Battery connected
Charge	Remains the same	Increases ($K C_0 V_0$)
Capacitance	Increases ($K C_0$)	Increases ($K C_0$)
Electric Field	Decreases $E_N = E_0 - E_p$	Remains the same
Potential Difference	Decreases	Remains the same
Energy stored	Remains the same	Increases ($K U_0$)

Van de Graaff Generator:

S – Large Copper sphere

C_1, C_2 – Combs with sharp points

P_1, P_2 – Pulleys to run belt

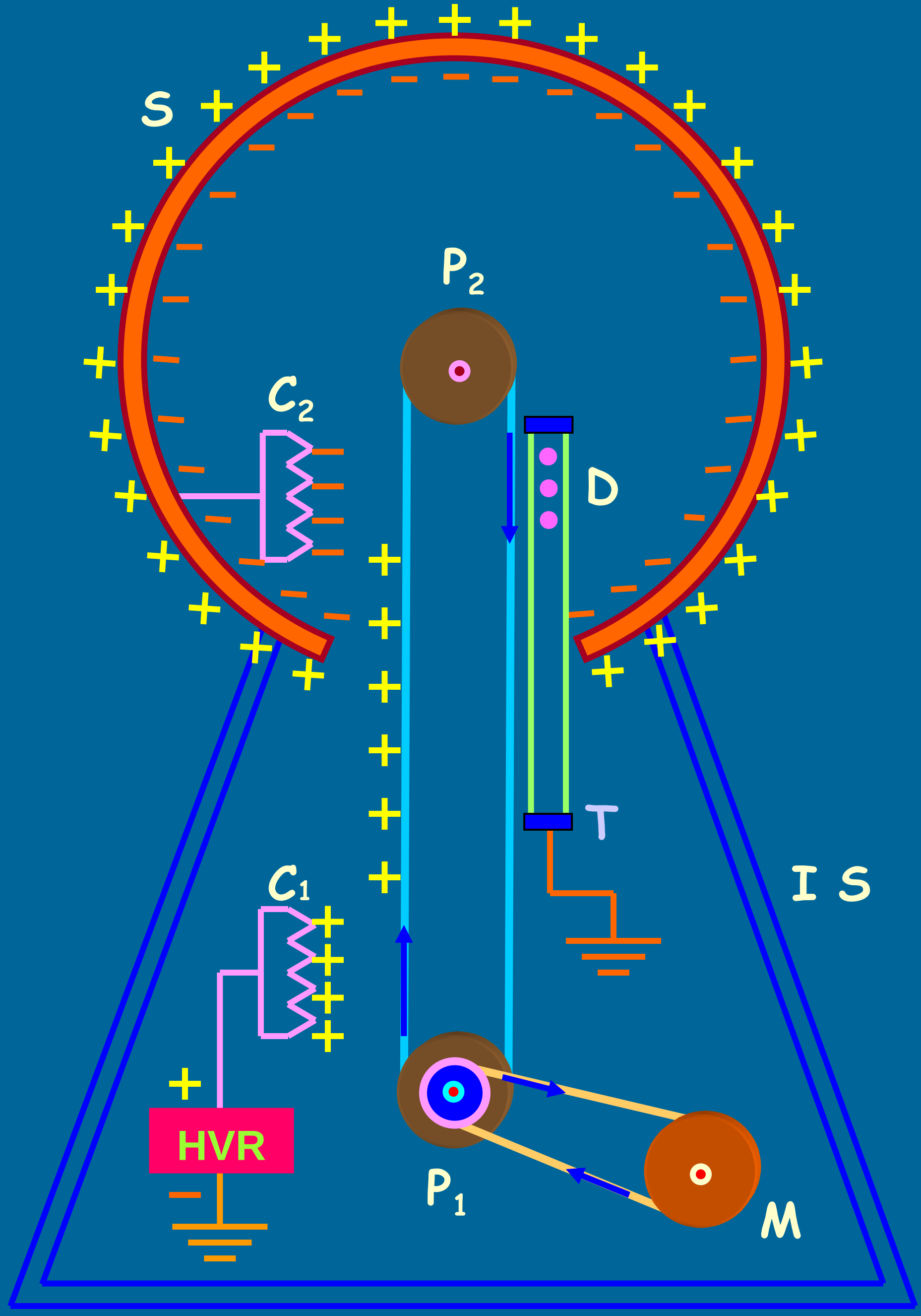
HVR – High Voltage Rectifier

M – Motor

IS – Insulating Stand

D – Gas Discharge Tube

T – Target



Principle:

Consider two charged conducting spherical shells such that one is smaller and the other is larger. When the smaller one is kept inside the larger one and connected together, charge from the smaller one is transferred to larger shell irrespective of the higher potential of the larger shell. i.e. The charge resides on the outer surface of the outer shell and the potential of the outer shell increases considerably.

Sharp pointed surfaces of a conductor have large surface charge densities and hence the electric field created by them is very high compared to the dielectric strength of the dielectric (air).

Therefore air surrounding these conductors get ionized and the like charges are repelled by the charged pointed conductors causing discharging action known as Corona Discharge or Action of Points. The sprayed charges moving with high speed cause electric wind.

Opposite charges are induced on the teeth of collecting comb (conductor) and again opposite charges are induced on the outer surface of the collecting sphere (Dome).

Construction:

Van de Graaff Generator consists of a large (about a few metres in radius) copper spherical shell (S) supported on an insulating stand (IS) which is of several metres high above the ground.

A belt made of insulating fabric (silk, rubber, etc.) is made to run over the pulleys (P_1, P_2) operated by an electric motor (M) such that it ascends on the side of the combs.

Comb (C_1) near the lower pulley is connected to High Voltage Rectifier (HVR) whose other end is earthed. Comb (C_2) near the upper pulley is connected to the sphere S through a conducting rod.

A tube (T) with the charged particles to be accelerated at its top and the target at the bottom is placed as shown in the figure. The bottom end of the tube is earthed for maintaining lower potential.

To avoid the leakage of charges from the sphere, the generator is enclosed in the steel tank filled with air or nitrogen at very high pressure (15 atmospheres).

Working:

Let the positive terminal of the High Voltage Rectifier (HVR) is connected to the comb (C_1). Due to action of points, electric wind is caused and the positive charges are sprayed on to the belt (silk or rubber). The belt made ascending by electric motor (EM) and pulley (P_1) carries these charges in the upward direction.

The comb (C_2) is induced with the negative charges which are carried by conduction to inner surface of the collecting sphere (dome) S through a metallic wire which in turn induces positive charges on the outer surface of the dome.

The comb (C_2) being negatively charged causes electric wind by spraying negative charges due to action of points which neutralize the positive charges on the belt. Therefore the belt does not carry any charge back while descending. (Thus the principle of conservation of charge is obeyed.)

Contd..

The process continues for a longer time to store more and more charges on the sphere and the potential of the sphere increases considerably. When the charge on the sphere is very high, the leakage of charges due to ionization of surrounding air also increases.

Maximum potential occurs when the rate of charge carried in by the belt is equal to the rate at which charge leaks from the shell due to ionization of air.

Now, if the positively charged particles which are to be accelerated are kept at the top of the tube T, they get accelerated due to difference in potential (the lower end of the tube is connected to the earth and hence at the lower potential) and are made to hit the target for causing nuclear reactions, etc.

Uses:

Van de Graaff Generator is used to produce very high potential difference (of the order of several million volts) for accelerating charged particles.

- The beam of accelerated charged particles are used to trigger nuclear reactions.

- The beam is used to break atoms for various experiments in Physics.

- In medicine, such beams are used to treat cancer.

It is used for research purposes.

CURRENT ELECTRICITY - I

1. Electric Current
2. Conventional Current
3. Drift Velocity of electrons and current
4. Current Density
5. Ohm's Law
6. Resistance, Resistivity, Conductance & Conductivity
7. Temperature dependence of resistance
8. Colour Codes for Carbon Resistors
9. Series and Parallel combination of resistors
10. EMF and Potential Difference of a cell
11. Internal Resistance of a cell
12. Series and Parallel combination of cells

Electric Current:

The electric current is defined as the charge flowing through any section of the conductor in one second.

I = q / t (if the rate of flow of charge is steady)

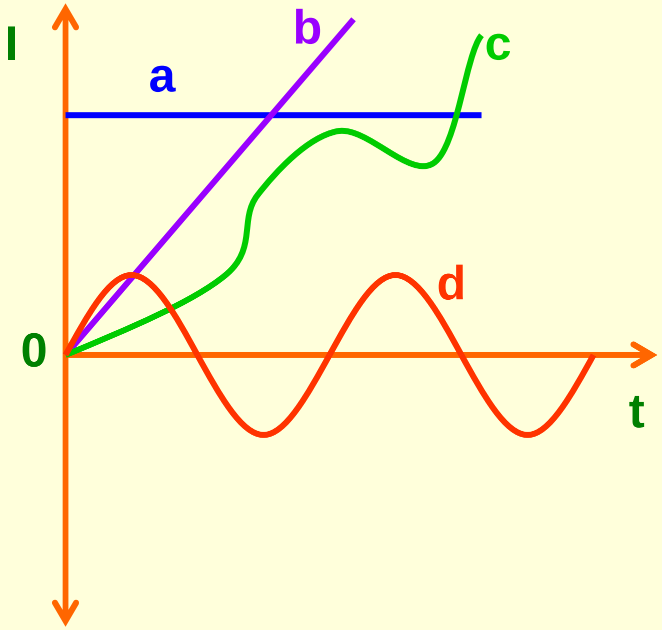
I = dq / dt (if the rate of flow of charge varies with time)

Different types of current:

a) Steady current which does not vary with time

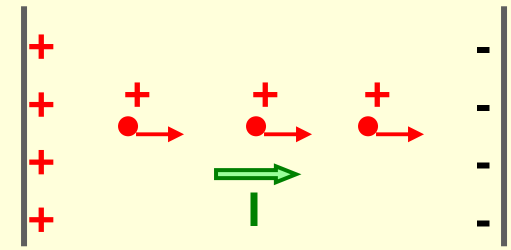
b) & c) Varying current whose magnitude varies with time

d) Alternating current whose magnitude varies continuously and direction changes periodically

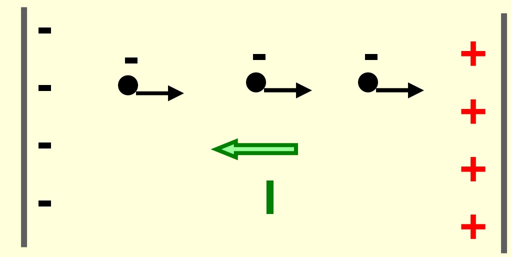


Conventional Current:

Conventional current is the current whose direction is along the direction of the motion of positive charge under the action of electric field.

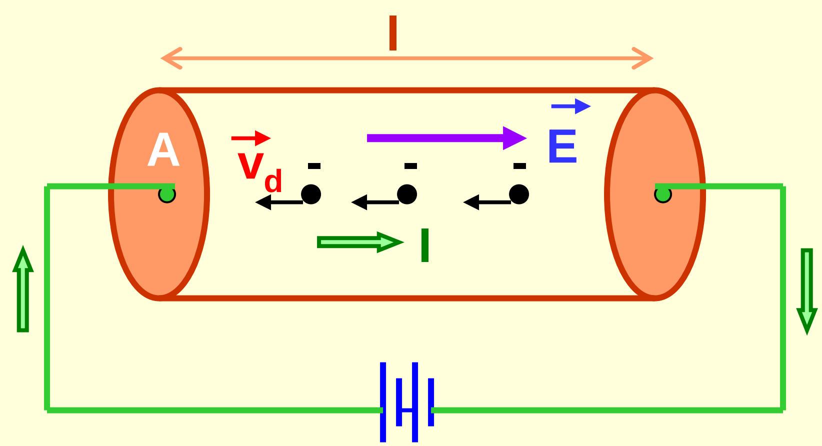


Conventional current due to motion of electrons is in the direction opposite to that of motion of electrons.



Drift Velocity and Current:

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied electric field.



$\vec{v}_d = \vec{a}\,\tau$ $\quad$ $\vec{v}_d = -(e\vec{E} / m)\,\tau$ $\quad$ $I = neA\,\vec{v}_d$

Current is directly proportional to drift velocity.

$\vec{v}_d$ - drift velocity, $\vec{a}$ – acceleration, τ – relaxation time, $\vec{E}$ – electric field,
e – electronic charge, m – mass of electron, n – number density of electrons,
l – length of the conductor and A – Area of cross-section

Current density:

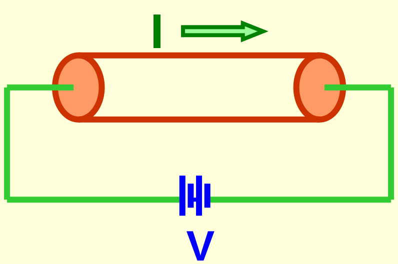
Current density at a point, within a conductor, is the current through a unit area of the conductor, around that point, provided the area is perpendicular to the direction of flow of current at that point.

$J = I / A = nev_d$

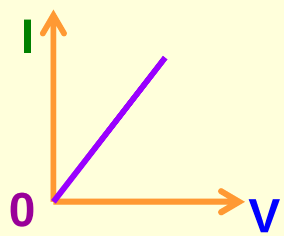
In vector form, $I = \vec{J} \cdot \vec{A}$

Ohm's Law:

The electric current flowing through a conductor is directly proportional to the potential difference across the two ends of the conductor when physical conditions such as temperature, mechanical strain, etc. remain the same.



$I \propto V$ or $V \propto I$ or $V = RI$



Resistance:

The resistance of conductor is the opposition offered by the conductor to the flow of electric current through it.

$$R = V / I$$

Resistance in terms of physical features of the conductor:

$$I = neA\,|\vec{v}_d|$$

$$I = neA\,(e\,|\vec{E}|/m)\,\tau$$

$$I = \frac{ne^2A\tau}{m}\frac{V}{l}$$

$$\frac{V}{I} = \frac{ml}{ne^2A\tau}$$

$$R = \frac{m}{ne^2\tau}\frac{l}{A}$$

$$R = \rho\frac{l}{A}$$

where $\rho = \dfrac{m}{ne^2\tau}$ is resistivity or specific resistance

Resistance is directly proportional to length and inversely proportional to cross-sectional area of the conductor and depends on nature of material.

Resistivity depends upon nature of material and not on the geometrical dimensions of the conductor.

Relations between v_d, ρ, I, E, J and V:

$\rho = E / J = E / nev_d$ (since, $J = I / A = nev_d$)

$v_d = E /(ne\rho)$

$v_d = V /(ne\rho l)$ (since, $E = V / l$)

When temperature increases, v_d decreases and ρ increases.

When I increases, v_d decreases.

Conductance and conductivity:

Conductance is the reciprocal of resistance. Its S.I unit is mho.

Conductivity is the reciprocal of resistivity. Its S.I unit is mho / m.

Temperature dependence of Resistances:

$$R = \frac{m}{ne^2\tau}\frac{l}{A}$$

When temperature increases, the no. of collisions increases due to more internal energy and relaxation time decreases. Therefore, Resistance increases.

Temperature coefficient of Resistance:

$$\alpha = \frac{R_t - R_0}{R_0\, t} \quad \text{or} \quad \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

R_0 – Resistance at 0°C

R_t – Resistance at t°C

R_1 – Resistance at t_1°C

R_2 – Resistance at t_2°C

If $R_2 < R_1$, then α is – ve.

Colour code for carbon resistors:

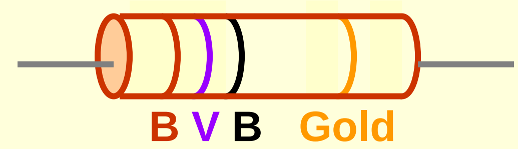
The first two rings from the end give the first two significant figures of resistance in ohm.

The third ring indicates the decimal multiplier.

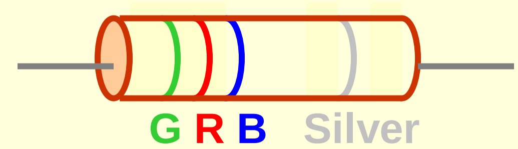
The last ring indicates the tolerance in per cent about the indicated value.

Eg. $AB \times 10^{C} \pm D\ \%$ ohm

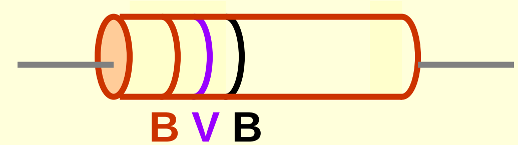
Letter	Colour	Number	Colour	Tolerance
B	Black	0	Gold	5%
B	Brown	1	Silver	10%
R	Red	2	No colour	20%
O	Orange	3		
Y	Yellow	4		
G	Green	5		
B	Blue	6		
V	Violet	7		
G	Grey	8		
W	White	9		



$17 \times 10^{0} = 17 \pm 5\%\ \Omega$



$52 \times 10^{6} \pm 10\%\ \Omega$



$52 \times 10^{0} = 52 \pm 20\%\ \Omega$

B B ROY of Great Britain has Very Good Wife

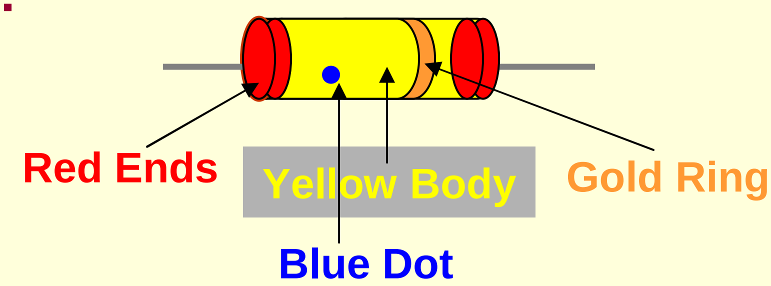
Another Colour code for carbon resistors:

i) The colour of the body gives the first significant figure.

ii) The colour of the ends gives the second significant figure.

iii) The colour of the dot gives the decimal multipier.

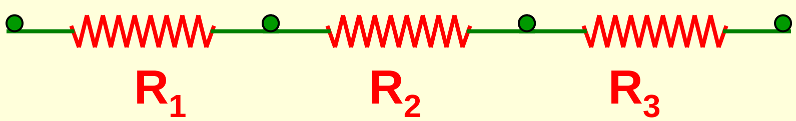
iv) The colour of the ring gives the tolerance.



YRB Gold

$42 \times 10^6 \pm 5\% \ \Omega$

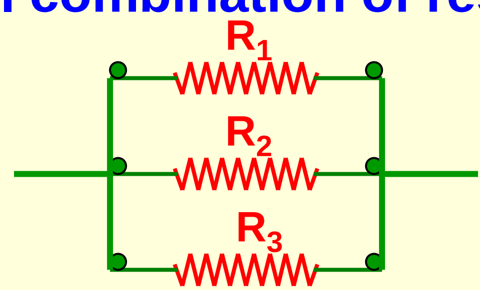
Series combination of resistors:



$R = R_1 + R_2 + R_3$

R is greater than the greatest of all.

Parallel combination of resistors:



$1/R = 1/R_1 + 1/R_2 + 1/R_3$

R is smaller than the smallest of all.

Sources of emf:

The electro motive force is the maximum potential difference between the two electrodes of the cell when no current is drawn from the cell.

Comparison of EMF and P.D:

	EMF	Potential Difference
1	EMF is the maximum potential difference between the two electrodes of the cell when no current is drawn from the cell i.e. when the circuit is open.	P.D is the difference of potentials between any two points in a closed circuit.
2	It is independent of the resistance of the circuit.	It is proportional to the resistance between the given points.
3	The term 'emf' is used only for the source of emf.	It is measured between any two points of the circuit.
4	It is greater than the potential difference between any two points in a circuit.	However, p.d. is greater than emf when the cell is being charged.

Internal Resistance of a cell:

The opposition offered by the electrolyte of the cell to the flow of electric current through it is called the internal resistance of the cell.

Factors affecting Internal Resistance of a cell:

i) Larger the separation between the electrodes of the cell, more the length of the electrolyte through which current has to flow and consequently a higher value of internal resistance.

ii) Greater the conductivity of the electrolyte, lesser is the internal resistance of the cell. i.e. internal resistance depends on the nature of the electrolyte.

iii) The internal resistance of a cell is inversely proportional to the common area of the electrodes dipping in the electrolyte.

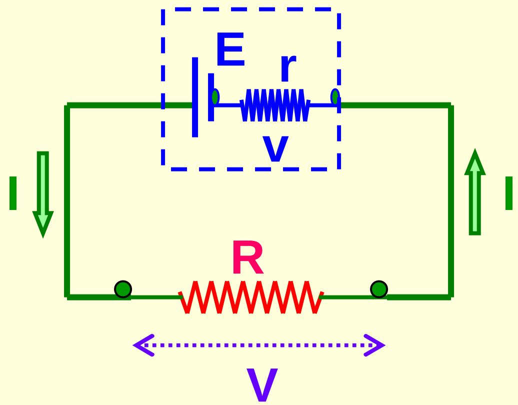
iv) The internal resistance of a cell depends on the nature of the electrodes.

$$E = V + v$$

$$= IR + Ir$$

$$= I\,(R + r)$$

$$I = E / (R + r)$$



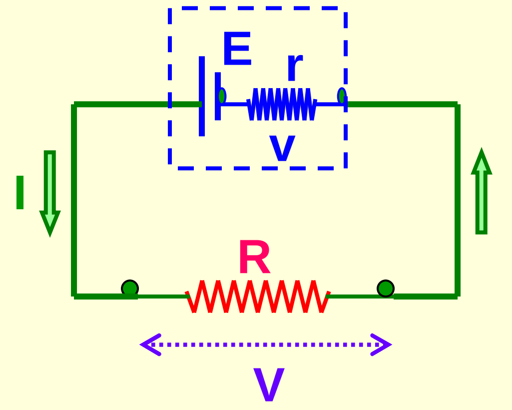
This relation is called circuit equation.

Internal Resistance of a cell in terms of E,V and R:

$E = V + v$

$\quad = V + Ir$

$Ir = E - V$



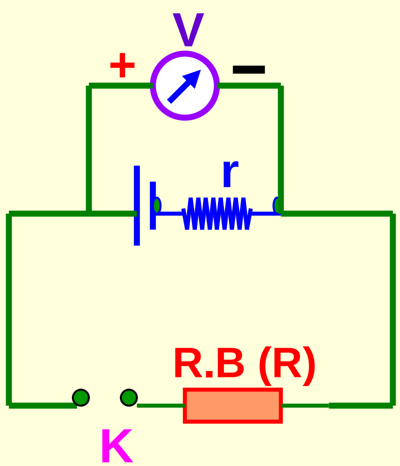
Dividing by IR = V,

$$\frac{Ir}{IR} = \frac{E - V}{V}$$

$\Longrightarrow$

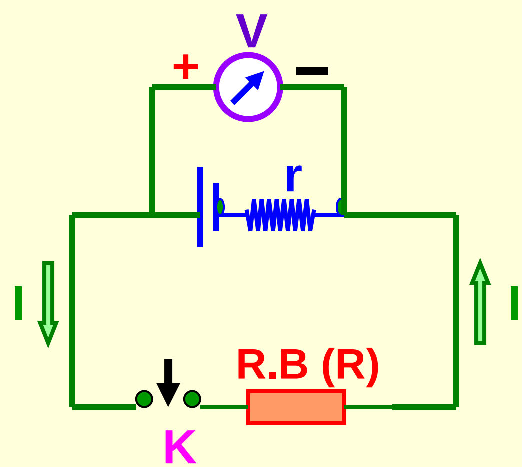
$$r = \left(\frac{E}{V} - 1\right) R$$

Determination of Internal Resistance of a cell by voltmeter method:



Open circuit (No current is drawn)

EMF (E) is measured



Closed circuit (Current is drawn)

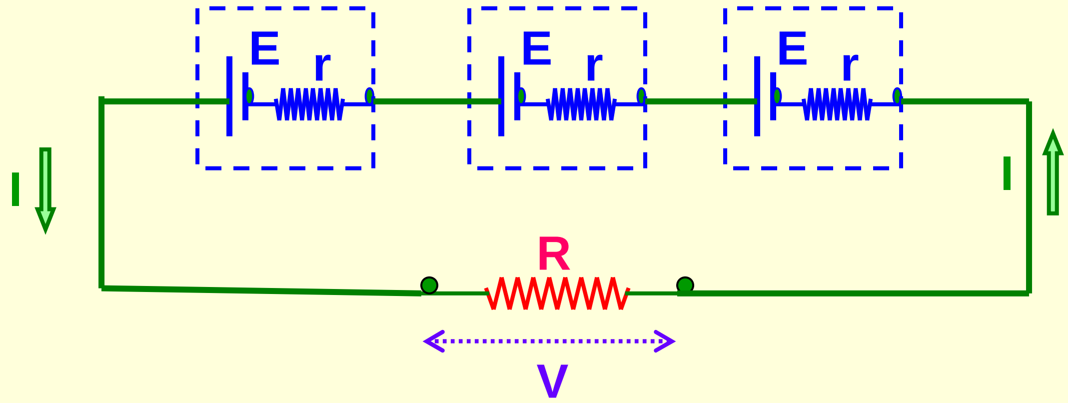
Potential Difference (V) is measured

Cells in Series combination:

Cells are connected in series when they are joined end to end so that the same quantity of electricity must flow through each cell.

NOTE:

1. The emf of the battery is the sum of the individual emfs
2. The current in each cell is the same and is identical with the current in the entire arrangement.
3. The total internal resistance of the battery is the sum of the individual internal resistances.



Total emf of the battery = nE (for n no. of identical cells)

Total Internal resistance of the battery = nr

Total resistance of the circuit = nr + R

Current $I = \dfrac{nE}{nr + R}$

(i) If R << nr, then I = E / r (ii) If nr << R, then I = n (E / R)

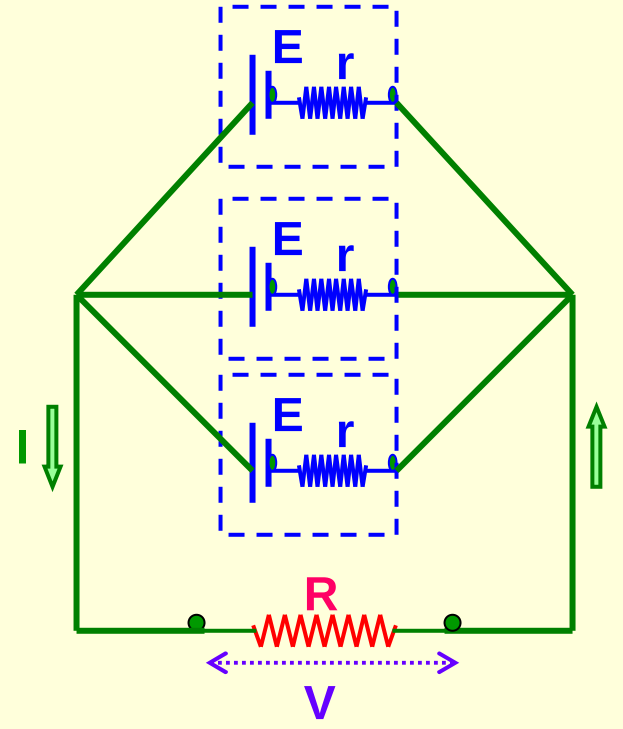
Conclusion: When internal resistance is negligible in comparison to the external resistance, then the cells are connected in series to get maximum current.

Cells in Parallel combination:

Cells are said to be connected in parallel when they are joined positive to positive and negative to negative such that current is divided between the cells.

NOTE:

1. The emf of the battery is the same as that of a single cell.
2. The current in the external circuit is divided equally among the cells.
3. The reciprocal of the total internal resistance is the sum of the reciprocals of the individual internal resistances.



Total emf of the battery = E

Total Internal resistance of the battery = r / n

Total resistance of the circuit = (r / n) + R

$$\text{Current } I = \frac{nE}{nR + r}$$

(i) If R << r/n, then I = n(E / r) (ii) If r/n << R, then I = E / R

Conclusion: When external resistance is negligible in comparison to the internal resistance, then the cells are connected in parallel to get maximum current.

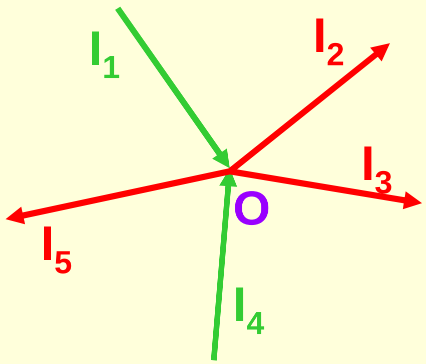
CURRENT ELECTRICITY - II

1. Kirchhoff's Laws of electricity
2. Wheatstone Bridge
3. Metre Bridge
4. Potentiometer
    i) Principle
    ii) Comparison of emf of primary cells

KIRCHHOFF'S LAWS:

I Law or Current Law or Junction Rule:

The algebraic sum of electric currents at a junction in any electrical network is always zero.



$$I_1 - I_2 - I_3 + I_4 - I_5 = 0$$

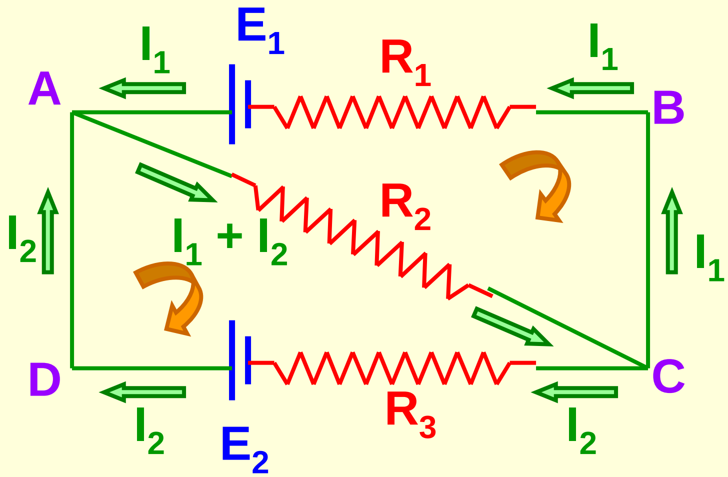
Sign Conventions:

1. The incoming currents towards the junction are taken positive.
2. The outgoing currents away from the junction are taken negative.

Note: The charges cannot accumulate at a junction. The number of charges that arrive at a junction in a given time must leave in the same time in accordance with conservation of charges.

II Law or Voltage Law or Loop Rule:

The algebraic sum of all the potential drops and emf's along any closed path in an electrical network is always zero.



Loop ABCA:

$- E_1 + I_1.R_1 + (I_1 + I_2).R_2 = 0$

Loop ACDA:

$- (I_1 + I_2).R_2 - I_2.R_3 + E_2 = 0$

Sign Conventions:

1. The emf is taken negative when we traverse from positive to negative terminal of the cell through the electrolyte.
2. The emf is taken positive when we traverse from negative to positive terminal of the cell through the electrolyte.

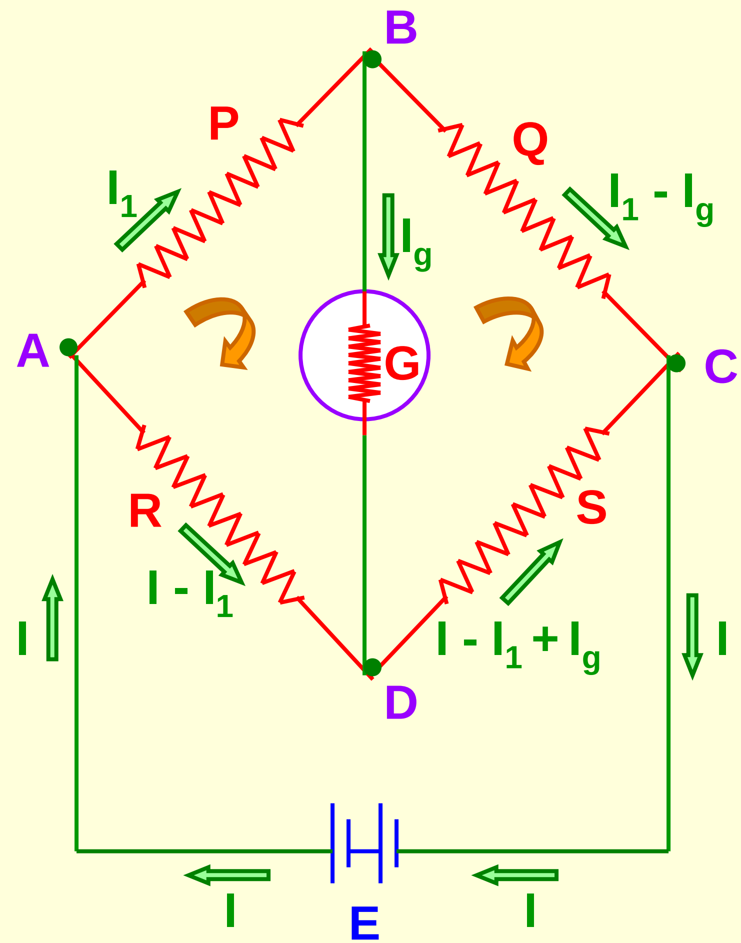
The potential falls along the direction of current in a current path and it rises along the direction opposite to the current path.

3. The potential fall is taken negative.
4. The potential rise is taken positive.

Note: The path can be traversed in clockwise or anticlockwise direction of the loop.

Wheatstone Bridge:

Currents through the arms are assumed by applying Kirchhoff's Junction Rule.



Applying Kirchhoff's Loop Rule for:

Loop ABDA:

$-I_1.P - I_g.G + (I - I_1).R = 0$

Loop BCDB:

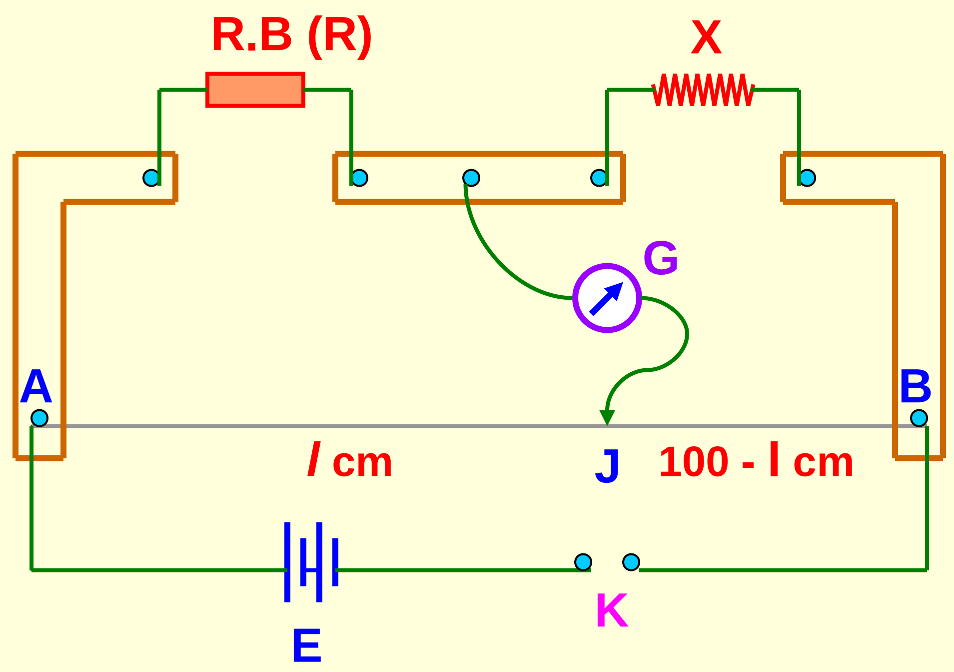
$-(I_1 - I_g).Q + (I - I_1 + I_g).S + I_g.G = 0$

When $I_g = 0$, the bridge is said to balanced.

By manipulating the above equations, we get

$$\frac{P}{Q} = \frac{R}{S}$$

Metre Bridge:



Metre Bridge is based on the principle of Wheatstone Bridge.

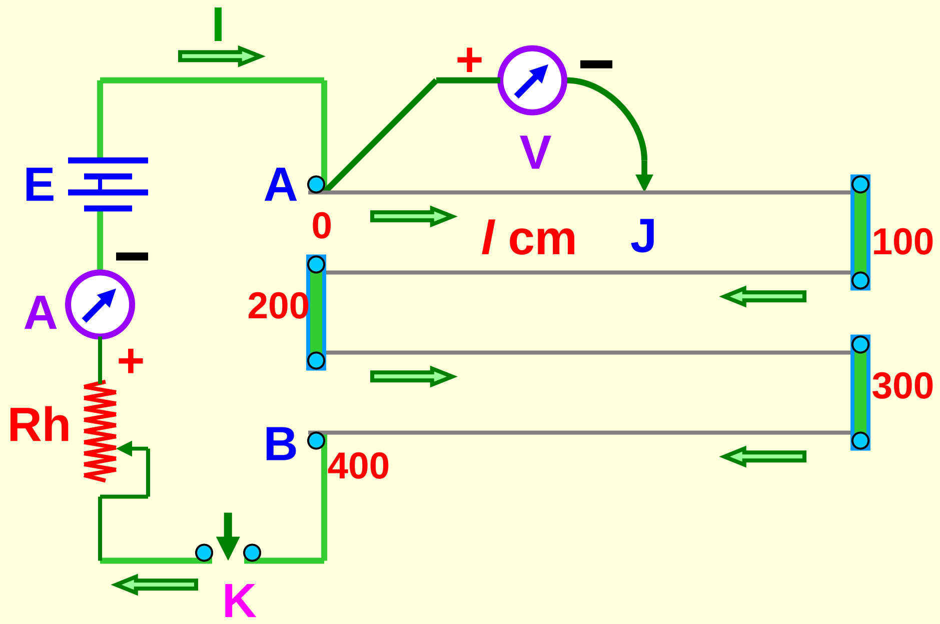
When the galvanometer current is made zero by adjusting the jockey position on the metre-bridge wire for the given values of known and unknown resistances,

$$\frac{R}{X} = \frac{R_{AJ}}{R_{JB}} \Rightarrow \frac{R}{X} = \frac{AJ}{JB} \Rightarrow \frac{R}{X} = \frac{l}{100 - l}$$

(Since, Resistance α length)

Therefore, $X = R\,(100 - l)\,/\,l$

Potentiometer:



Principle:

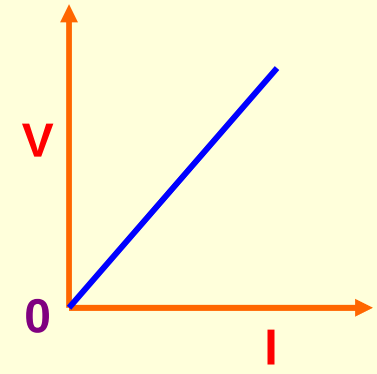
V = I R

= I ρl/A

If the constant current flows through the potentiometer wire of uniform cross sectional area (A) and uniform composition of material (ρ), then

V = Kl   or   V α l

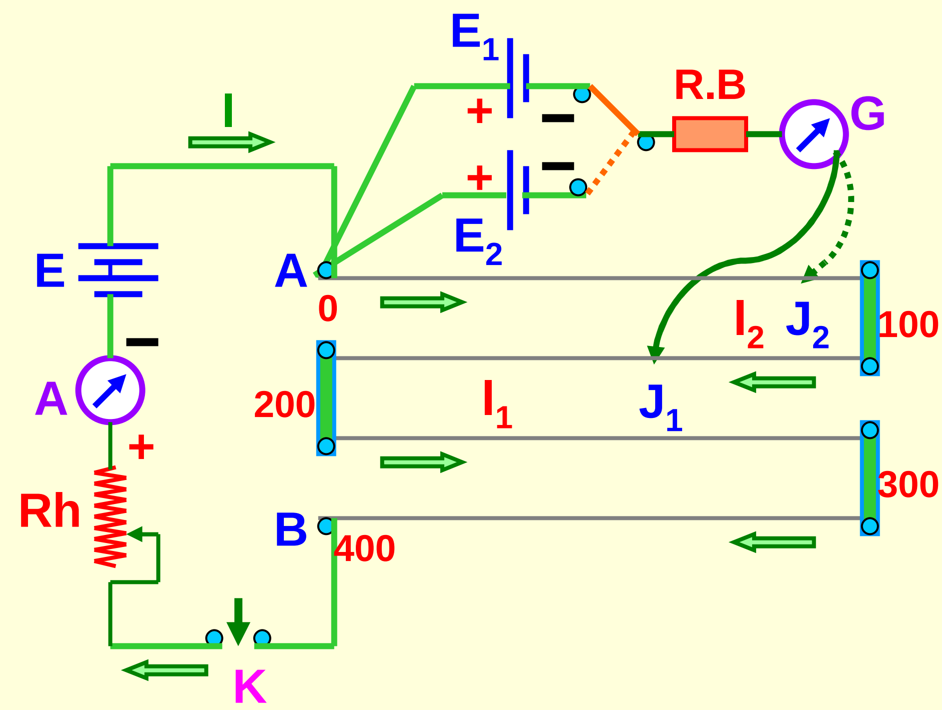
V /l is a constant.



The potential difference across any length of a wire of uniform cross-section and uniform composition is proportional to its length when a constant current flows through it.

Comparison of emf's using Potentiometer:

The balance point is obtained for the cell when the potential at a point on the potentiometer wire is equal and opposite to the emf of the cell.



$$E_1 = V_{AJ_1} = I\,\rho l_1 / A$$

$$E_2 = V_{AJ_2} = I\,\rho l_2 / A$$

$$E_1 / E_2 = l_1 / l_2$$

Note:

The balance point will not be obtained on the potentiometer wire if the fall of potential along the potentiometer wire is less than the emf of the cell to be measured.

The working of the potentiometer is based on null deflection method. So the resistance of the wire becomes infinite. Thus potentiometer can be regarded as an ideal voltmeter.

MAGNETIC EFFECT OF CURRENT - I

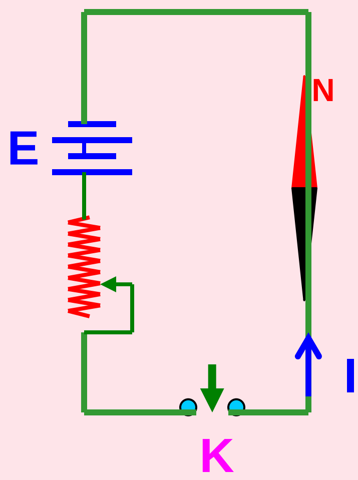
1. Magnetic Effect of Current – Oersted's Experiment
2. Ampere's Swimming Rule
3. Maxwell's Cork Screw Rule
4. Right Hand Thumb Rule
5. Biot – Savart's Law
6. Magnetic Field due to Infinitely Long Straight Current – carrying Conductor
7. Magnetic Field due to a Circular Loop carrying current
8. Magnetic Field due to a Solenoid

Magnetic Effect of Current:

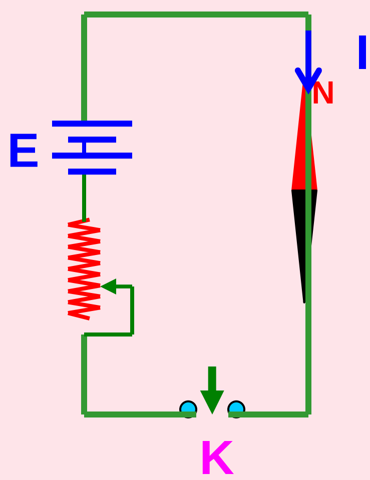
An electric current (i.e. flow of electric charge) produces magnetic effect in the space around the conductor called strength of Magnetic field or simply Magnetic field.

Oersted's Experiment:

When current was allowed to flow through a wire placed parallel to the axis of a magnetic needle kept directly below the wire, the needle was found to deflect from its normal position.



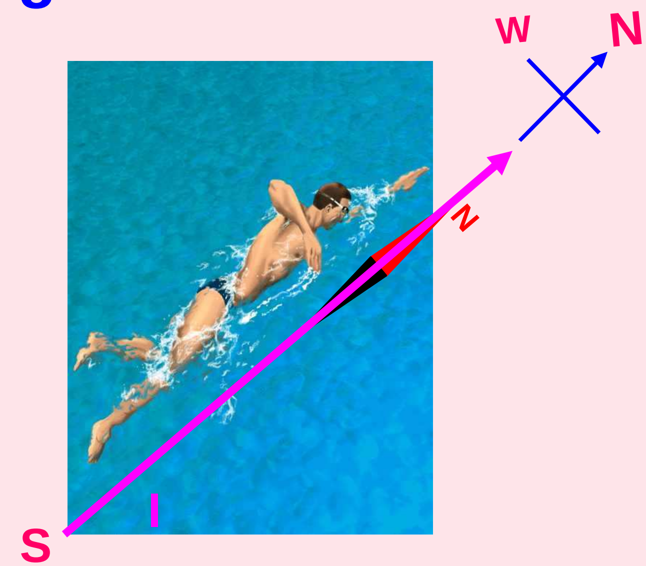
When current was reversed through the wire, the needle was found to deflect in the opposite direction to the earlier case.



Rules to determine the direction of magnetic field:

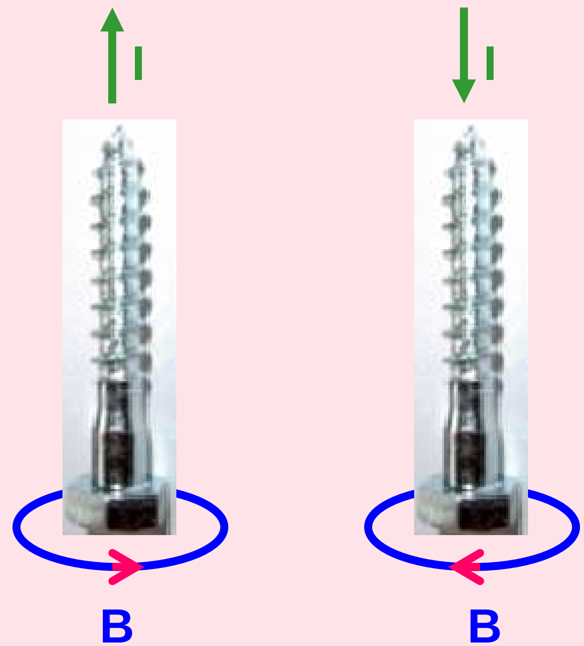
Ampere's Swimming Rule:

Imagining a man who swims in the direction of current from south to north facing a magnetic needle kept under him such that current enters his feet then the North pole of the needle will deflect towards his left hand, i.e. towards West.



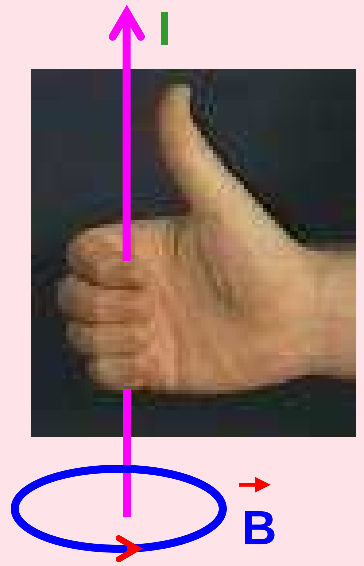
Maxwell's Cork Screw Rule or Right Hand Screw Rule:

If the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.



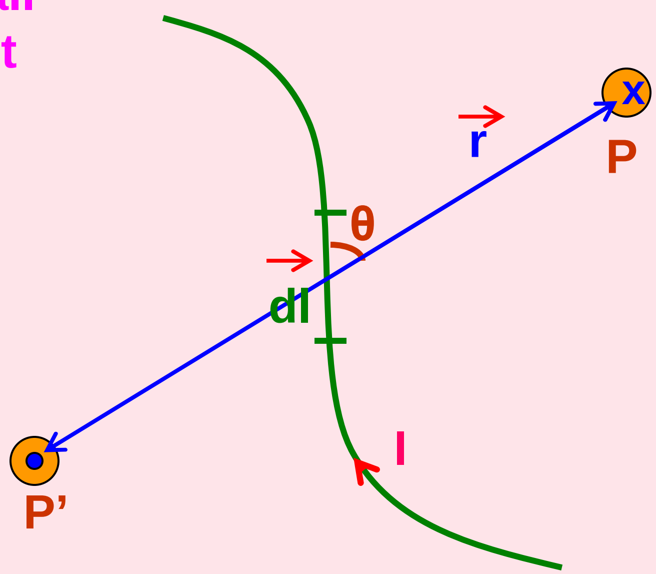
Right Hand Thumb Rule or Curl Rule:

If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.



Biot – Savart's Law:

The strength of magnetic field dB due to a small current element dl carrying a current I at a point P distant r from the element is directly proportional to I, dl, sin θ and inversely proportional to the square of the distance (r^2) where θ is the angle between dl and r.



i) dB α I

ii) dB α dl

iii) dB α sin θ

iv) dB α 1 / r^2

$$dB \;\alpha\; \frac{I\,dl\,\sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\,\frac{I\,dl\,\sin\theta}{r^2}$$

Biot – Savart's Law in vector form:

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \, \vec{dl} \times \hat{r}}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \, \vec{dl} \times \vec{r}}{r^3}$$

Value of $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1} or Wb m^{-1} A^{-1}

Direction of $\vec{dB}$ is same as that of direction of $\vec{dl} \times \vec{r}$ which can be determined by Right Hand Screw Rule.

It is emerging $\odot$ at P' and entering $\otimes$ at P into the plane of the diagram.

Current element is a vector quantity whose magnitude is the vector product of current and length of small element having the direction of the flow of current. ($I \, \vec{dl}$)

Magnetic Field due to a Straight Wire carrying current:

According to Biot – Savart's law

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

$\sin\theta = a / r = \cos\Phi$

or $r = a / \cos\Phi$

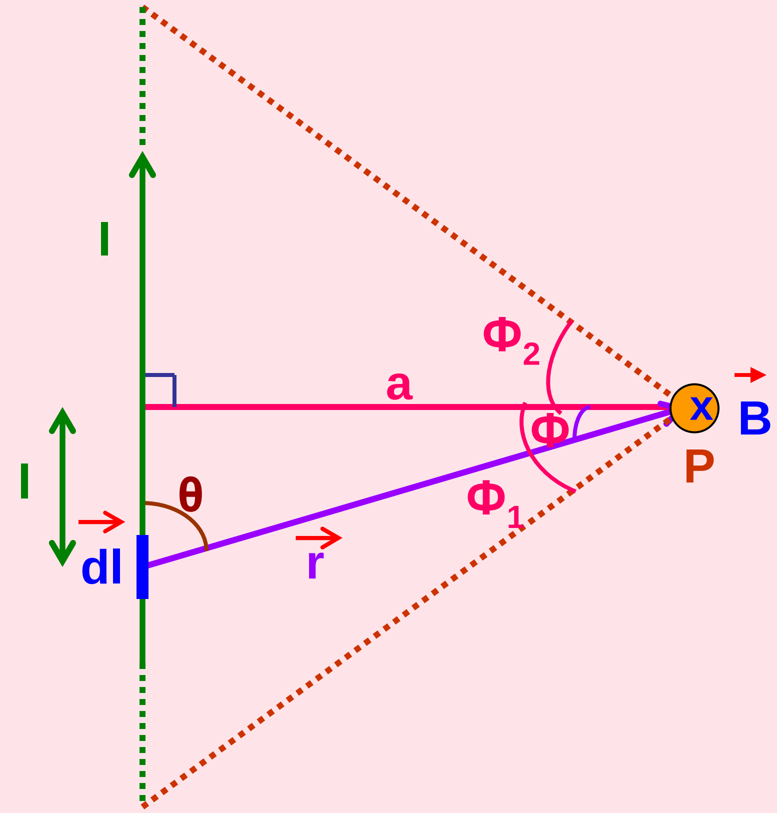
$\tan\Phi = l / a$

or $l = a \tan\Phi$

$dl = a \sec^2\Phi \, d\Phi$

Substituting for r and dl in dB,

$$dB = \frac{\mu_0}{4\pi} \frac{I \cos\Phi \, d\Phi}{a}$$



Magnetic field due to whole conductor is obtained by integrating with limits - Φ_1 to Φ_2. (Φ_1 is taken negative since it is anticlockwise)

$$B = \int dB = \int_{-\Phi_1}^{\Phi_2} \frac{\mu_0}{4\pi} \frac{I \cos\Phi \, d\Phi}{a}$$

$$B = \frac{\mu_0 I (\sin\Phi_1 + \sin\Phi_2)}{4\pi a}$$

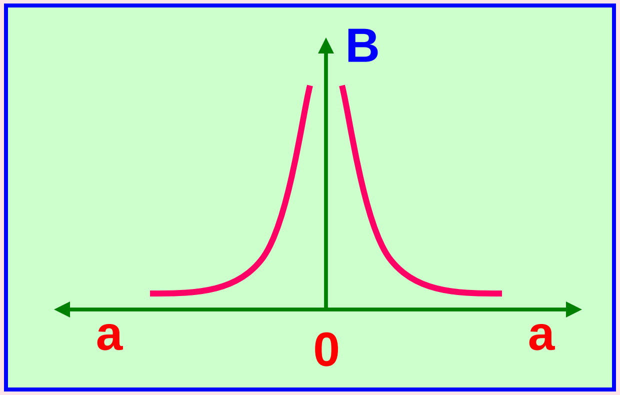
If the straight wire is infinitely long,

then $\Phi_1 = \Phi_2 = \pi / 2$

$$B = \frac{\mu_0 \, 2I}{4\pi a}$$

or

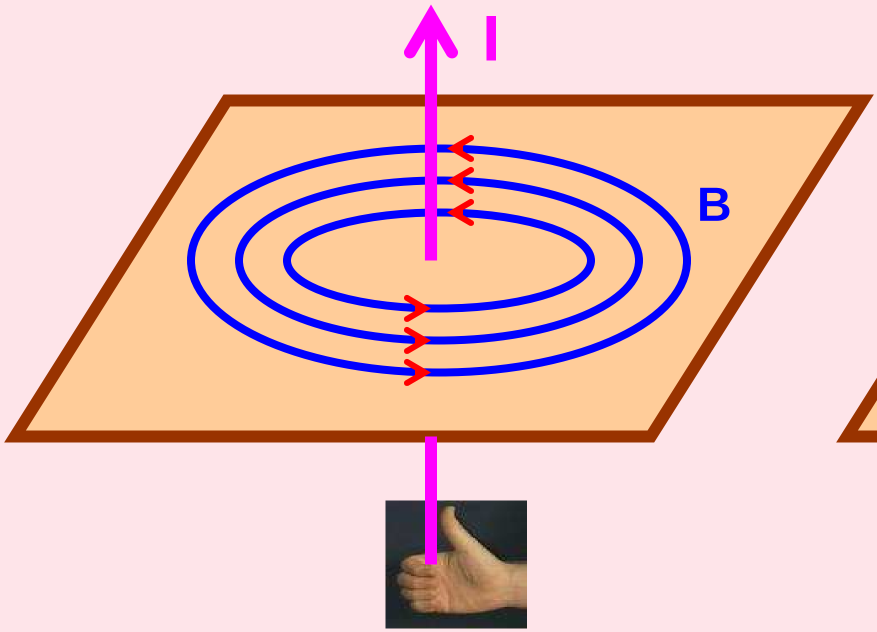
$$B = \frac{\mu_0 \, I}{2\pi a}$$

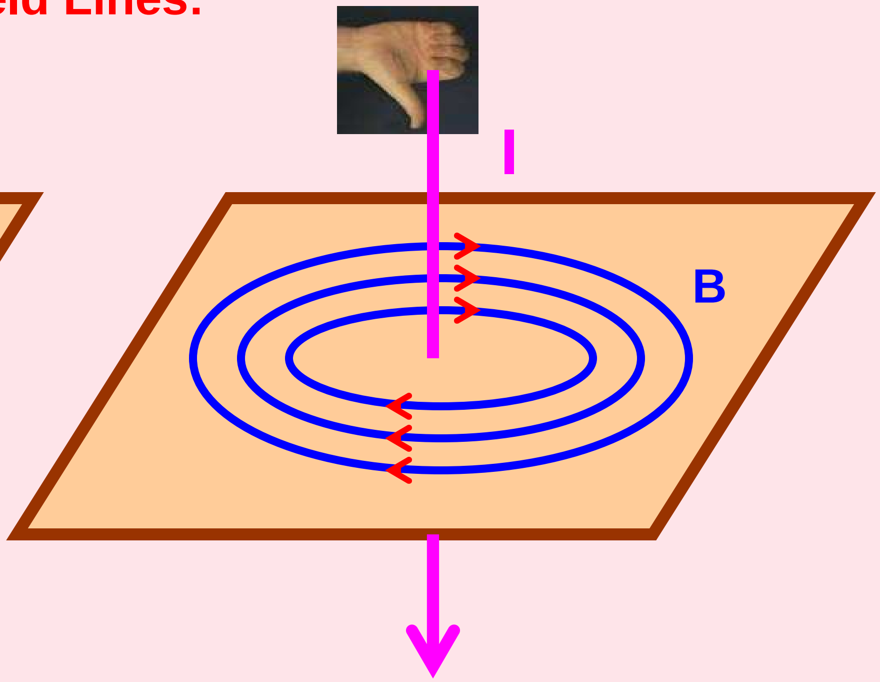


Direction of $\vec{B}$ is same as that of direction of $\vec{dl} \times \vec{r}$ which can be determined by Right Hand Screw Rule.

It is perpendicular to the plane of the diagram and entering into the plane at P.

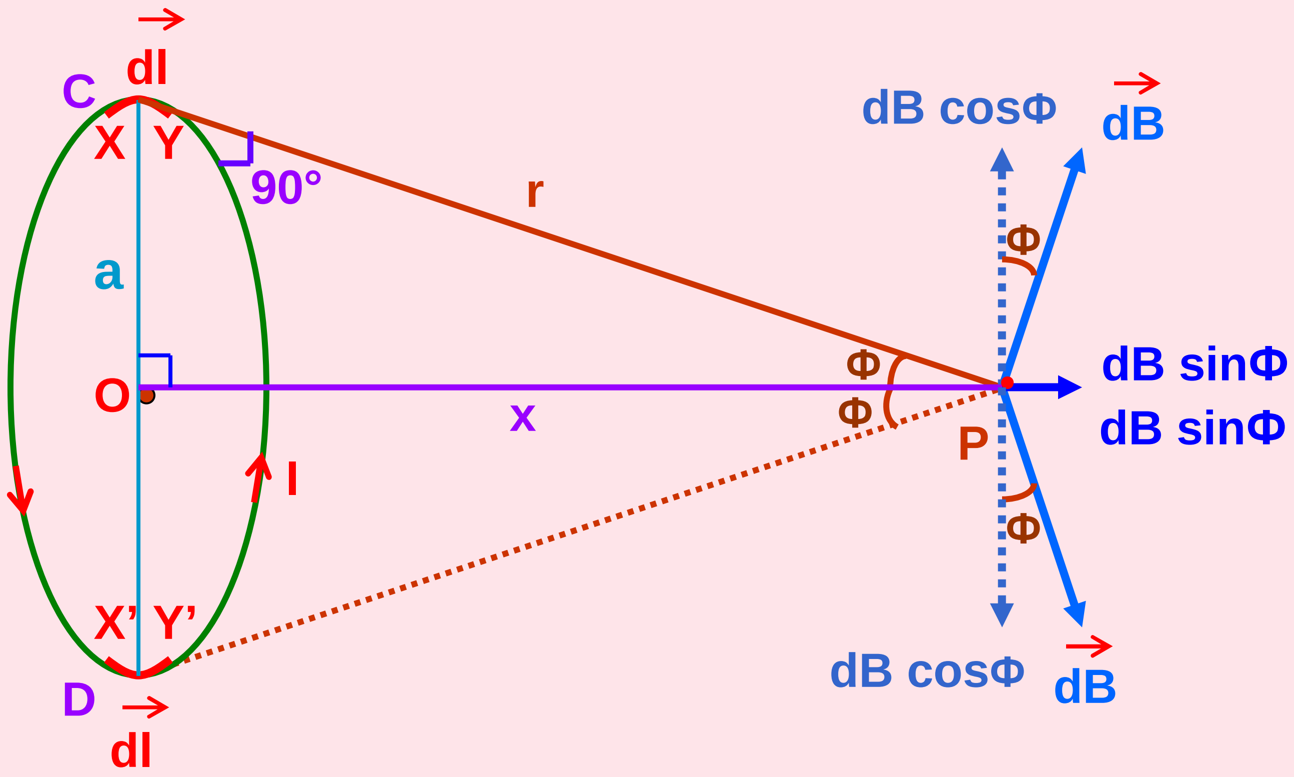
Magnetic Field Lines:





Magnetic Field due to a Circular Loop carrying current:

1) At a point on the axial line:



The plane of the coil is considered perpendicular to the plane of the diagram such that the direction of magnetic field can be visualized on the plane of the diagram.

At C and D current elements XY and X'Y' are considered such that current at C emerges out and at D enters into the plane of the diagram.

$$dB = \frac{\mu_0 \; I \, dl \sin\theta}{4\pi \quad r^2} \quad \text{or} \quad dB = \frac{\mu_0 \; I \, dl}{4\pi \; r^2}$$

The angle θ between $\vec{dl}$ and $\vec{r}$ is 90° because the radius of the loop is very small and since sin 90°= 1

The semi-vertical angle made by $\vec{r}$ to the loop is Φ and the angle between $\vec{r}$ and $\vec{dB}$ is 90° . Therefore, the angle between vertical axis and $\vec{dB}$ is also Φ.

$\vec{dB}$ is resolved into components dB cosΦ and dB sinΦ .

Due to diametrically opposite current elements, cosΦ components are always opposite to each other and hence they cancel out each other.

SinΦ components due to all current elements $\vec{dl}$ get added up along the same direction (in the direction away from the loop).

$$B = \int dB \sin\Phi = \int \frac{\mu_0 \; I \, dl \sin\Phi}{4\pi \; r^2} \quad \text{or} \quad B = \frac{\mu_0 \, I \, (2\pi a) \, a}{4\pi \, (a^2 + x^2) \, (a^2 + x^2)^{1/2}}$$

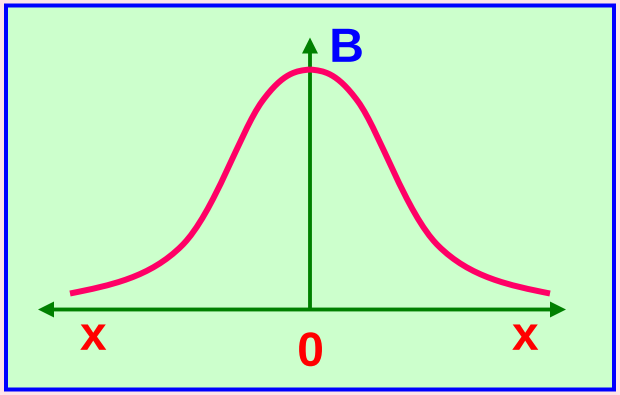
$$B = \frac{\mu_0 \, I \, a^2}{2(a^2 + x^2)^{3/2}}$$

(μ_0 , I, a, sinΦ are constants, $\int dl = 2\pi a$ and r & sinΦ are replaced with measurable and constant values.)

Special Cases:

i) At the centre O, x = 0.

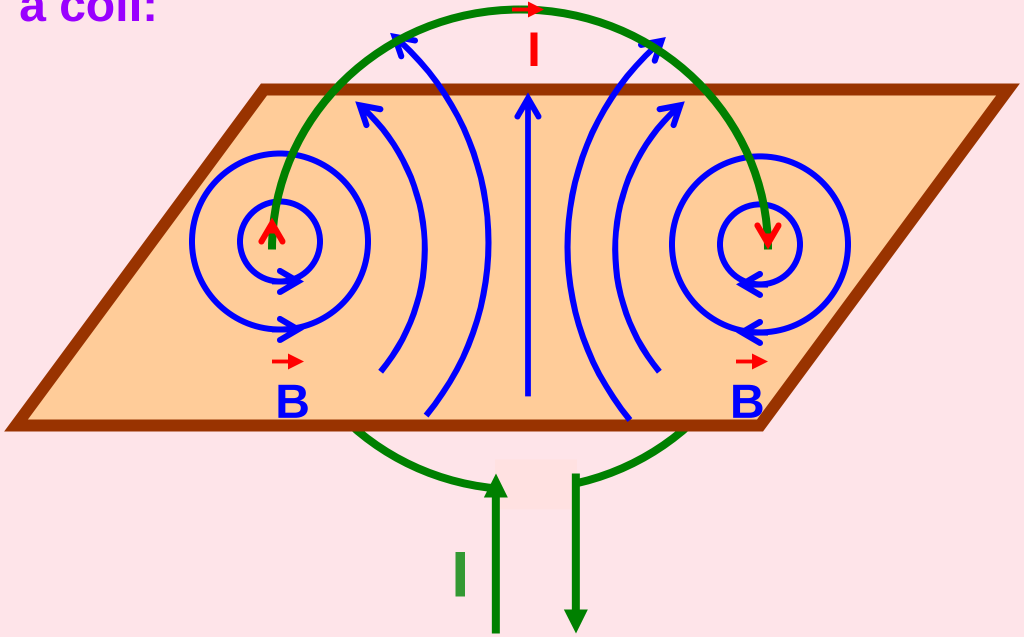
$$\therefore \quad B = \frac{\mu_0 I}{2a}$$

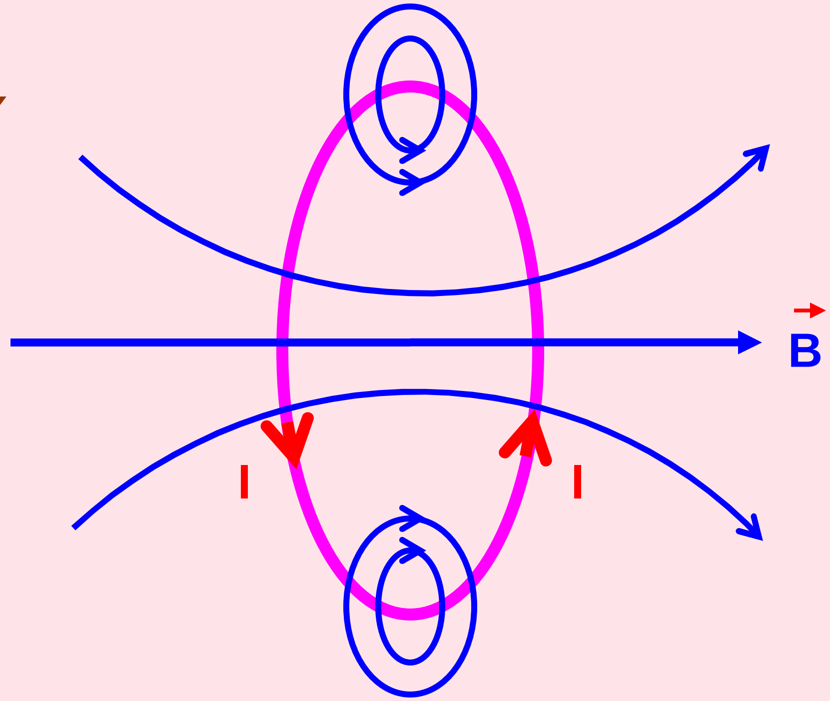


ii) If the observation point is far away from the coil, then a << x. So, a^2 can be neglected in comparison with x^2.

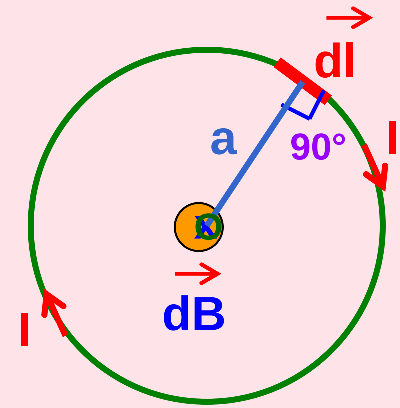
$$\therefore \quad B = \frac{\mu_0 I a^2}{2 x^3}$$

Different views of direction of current and magnetic field due to circular loop of a coil:





2) B at the centre of the loop:



The plane of the coil is lying on the plane of the diagram and the direction of current is clockwise such that the direction of magnetic field is perpendicular and into the plane.

$$dB = \frac{\mu_0 \; I \; dl \sin\theta}{4\pi \; a^2}$$

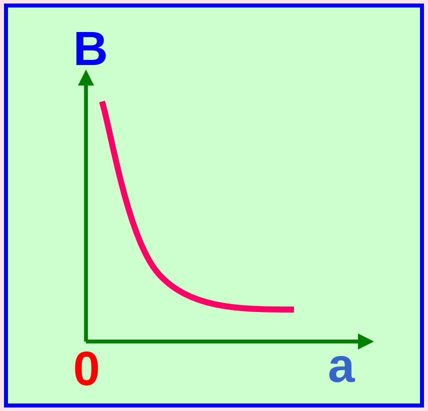
$$dB = \frac{\mu_0 \; I \; dl}{4\pi \; a^2}$$

The angle θ between dl and a is 90° because the radius of the loop is very small and since sin 90°= 1

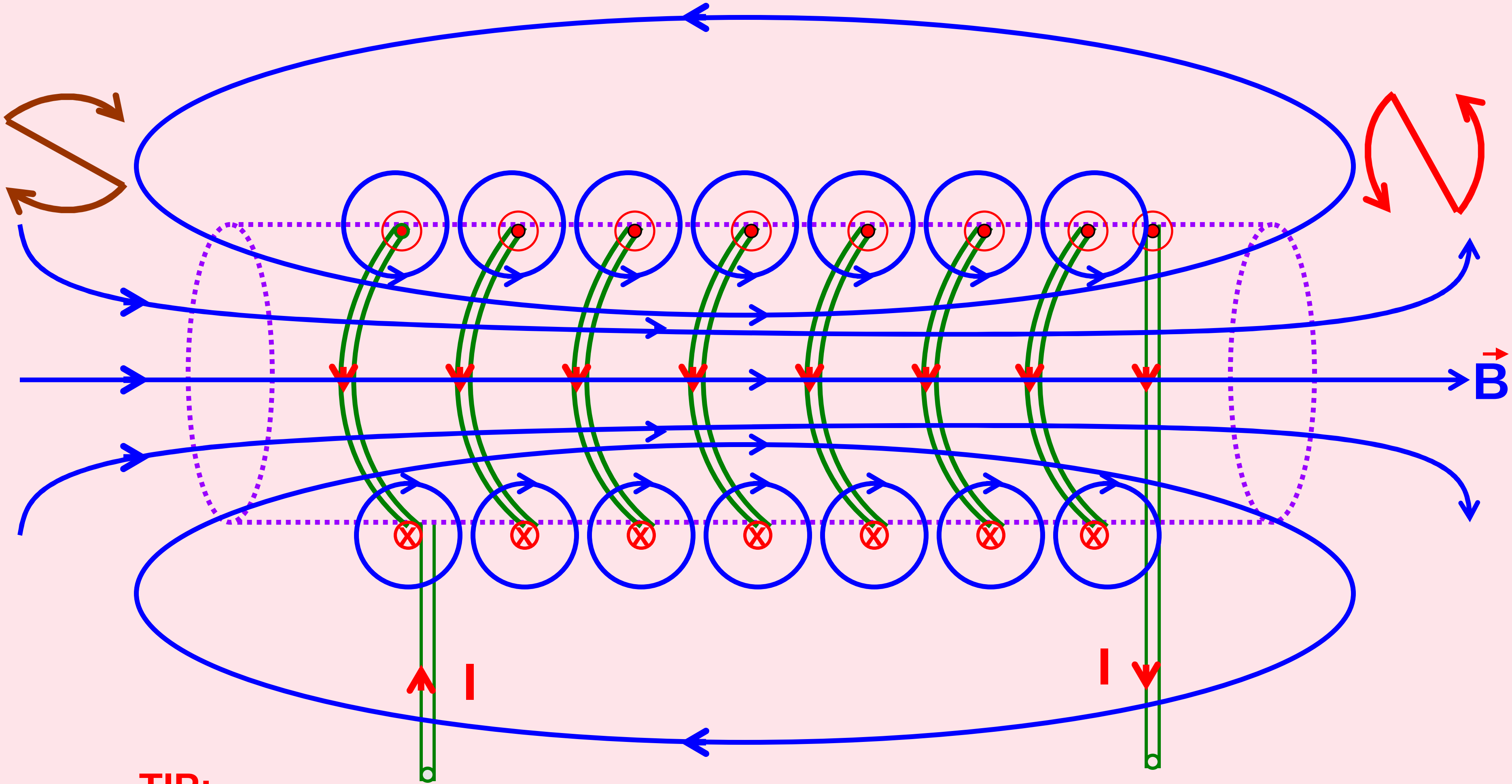
$$B = \int dB = \int \frac{\mu_0 \; I \; dl}{4\pi \; a^2}$$

$$B = \frac{\mu_0 \, I}{2a}$$

(μ_0 , I, a are constants and $\int dl = 2\pi a$)



Magnetic Field due to a Solenoid:



TIP:

When we look at any end of the coil carrying current, if the current is in anti-clockwise direction then that end of coil behaves like North Pole and if the current is in clockwise direction then that end of the coil behaves like South Pole.

MAGNETIC EFFECT OF CURRENT - II

1. Lorentz Magnetic Force
2. Fleming's Left Hand Rule
3. Force on a moving charge in uniform Electric and Magnetic fields
4. Force on a current carrying conductor in a uniform Magnetic Field
5. Force between two infinitely long parallel current-carrying conductors
6. Definition of ampere
7. Representation of fields due to parallel currents
8. Torque experienced by a current-carrying coil in a uniform Magnetic Field
9. Moving Coil Galvanometer
10. Conversion of Galvanometer into Ammeter and Voltmeter
11. Differences between Ammeter and Voltmeter

Lorentz Magnetic Force:

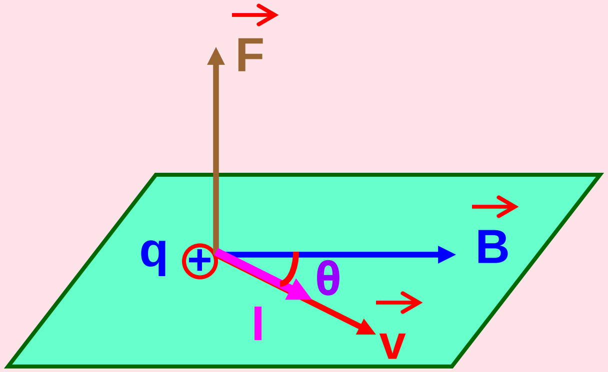
A current carrying conductor placed in a magnetic field experiences a force which means that a moving charge in a magnetic field experiences force.

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

or

$$\vec{F}_m = (q\,v\,B\,\sin\theta)\,\hat{n}$$

where θ is the angle between $\vec{v}$ and $\vec{B}$



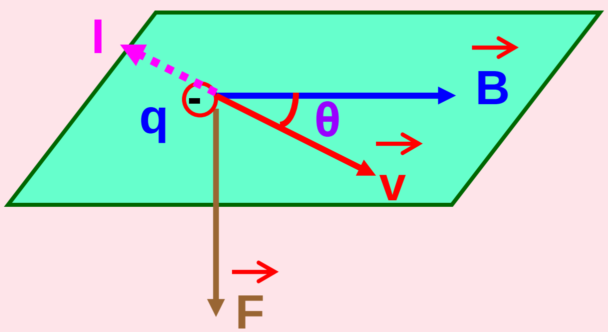
Special Cases:

i) If the charge is at rest, i.e. v = 0, then $F_m = 0$. So, a stationary charge in a magnetic field does not experience any force.

ii) If θ = 0° or 180° i.e. if the charge moves parallel or anti-parallel to the direction of the magnetic field, then $F_m = 0$.

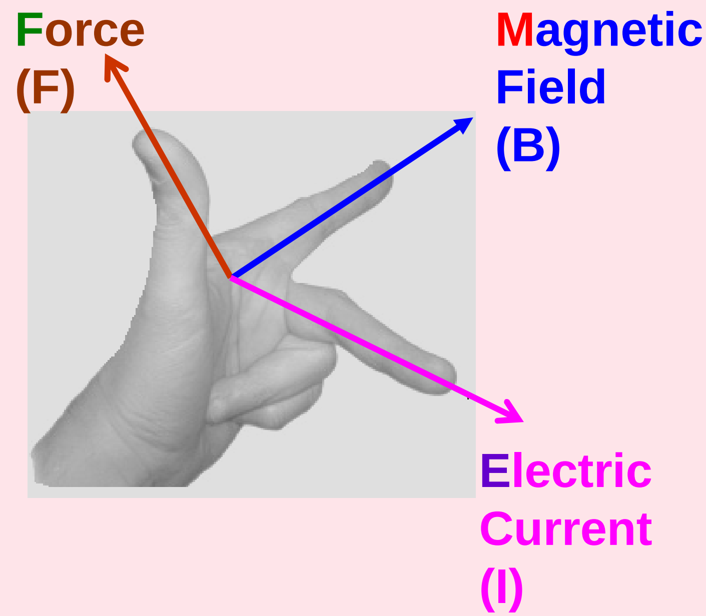
iii) If θ = 90° i.e. if the charge moves perpendicular to the magnetic field, then the force is maximum.

$$F_{m\,(max)} = q\,v\,B$$



Fleming's Left Hand Rule:

If the central finger, fore finger and thumb of left hand are stretched mutually perpendicular to each other and the central finger points to current, fore finger points to magnetic field, then thumb points in the direction of motion (force) on the current carrying conductor.



TIP:

Remember the phrase 'e m f' to represent electric current, magnetic field and force in anticlockwise direction of the fingers of left hand.

Force on a moving charge in uniform Electric and Magnetic Fields:

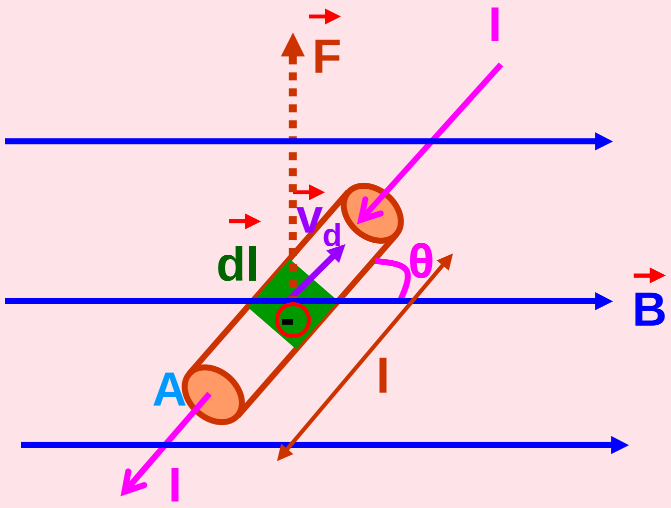
When a charge q moves with velocity $\vec{v}$ in region in which both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, then the Lorentz force is

$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$ or $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Force on a current-carrying conductor in a uniform Magnetic Field:

Force experienced by each electron in the conductor is

$\vec{f} = -e(\vec{v}_d \times \vec{B})$



If n be the number density of electrons, A be the area of cross section of the conductor, then no. of electrons in the element dl is n A dl.

Force experienced by the electrons in dl is

$\vec{dF} = n\,A\,dl\,[-e(\vec{v}_d \times \vec{B})] = -\,n\,e\,A\,v_d\,(\vec{dl} \times \vec{B})$

$= I(\vec{dl} \times \vec{B})$

where $I = neAv_d$ and -ve sign represents that the direction of dl is opposite to that of v_d)

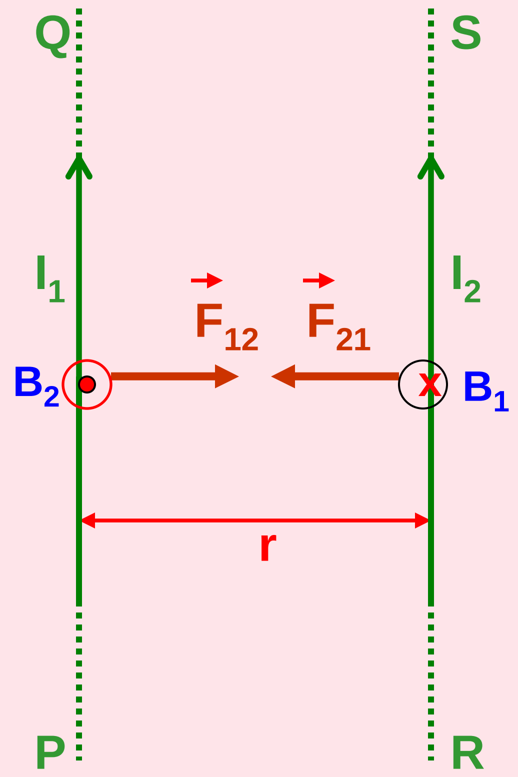
$\vec{F} = \int \vec{dF} = \int I(\vec{dl} \times \vec{B})$

$\vec{F} = I(\vec{l} \times \vec{B})$ or $F = I\,l\,B\sin\theta$

Forces between two parallel infinitely long current-carrying conductors:

Magnetic Field on RS due to current in PQ is

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$ (in magnitude)



Force acting on RS due to current I_2 through it is

$$F_{21} = \frac{\mu_0 I_1}{2\pi r} I_2 \, l \sin 90^\circ \quad \text{or} \quad F_{21} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

B_1 acts perpendicular and into the plane of the diagram by Right Hand Thumb Rule. So, the angle between l and B_1 is 90° .

l is length of the conductor.

Magnetic Field on PQ due to current in RS is

$$B_2 = \frac{\mu_0 I_2}{2\pi r}$$ (in magnitude)

Force acting on PQ due to current I_1 through it is

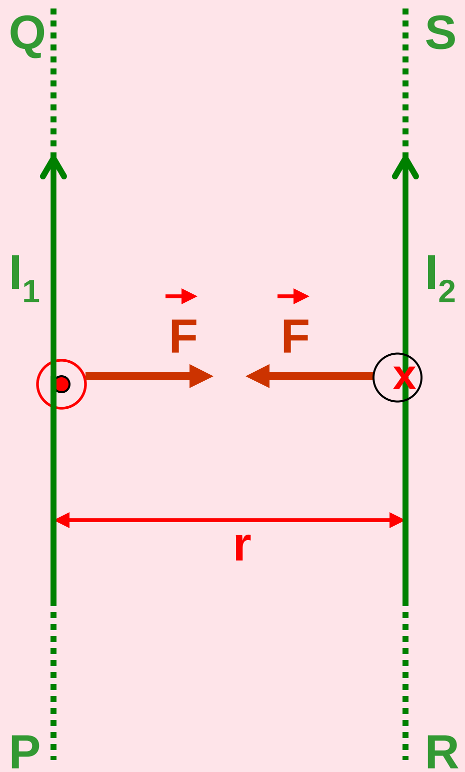
$$F_{12} = \frac{\mu_0 I_2}{2\pi r} I_1 \, l \sin 90^\circ \quad \text{or} \quad F_{12} = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

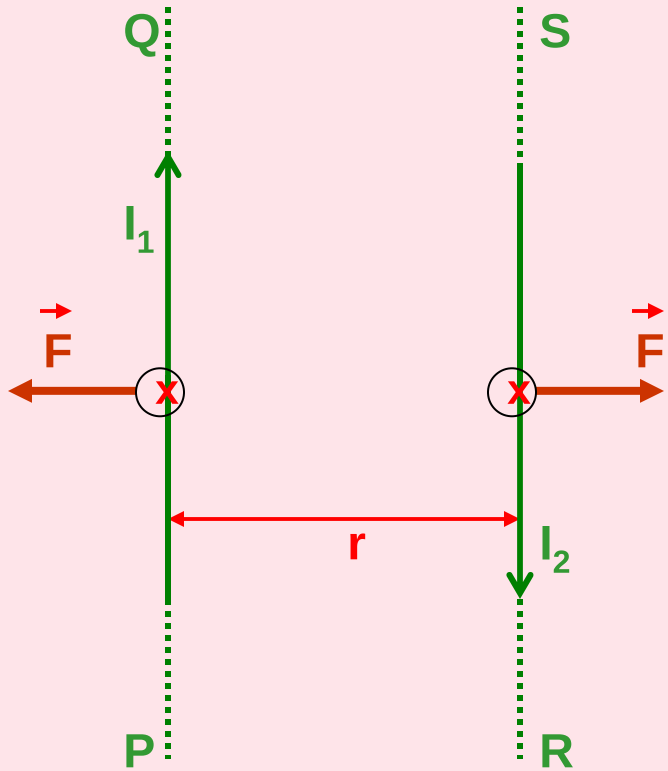
(The angle between l and B_2 is 90° and B_2 Is emerging out)

$$F_{12} = F_{21} = F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

Force per unit length of the conductor is

$$F / l = \frac{\mu_0 I_1 I_2}{2\pi r} \text{ N / m}$$





By Fleming's Left Hand Rule, the conductors experience force towards each other and hence attract each other.

By Fleming's Left Hand Rule, the conductors experience force away from each other and hence repel each other.

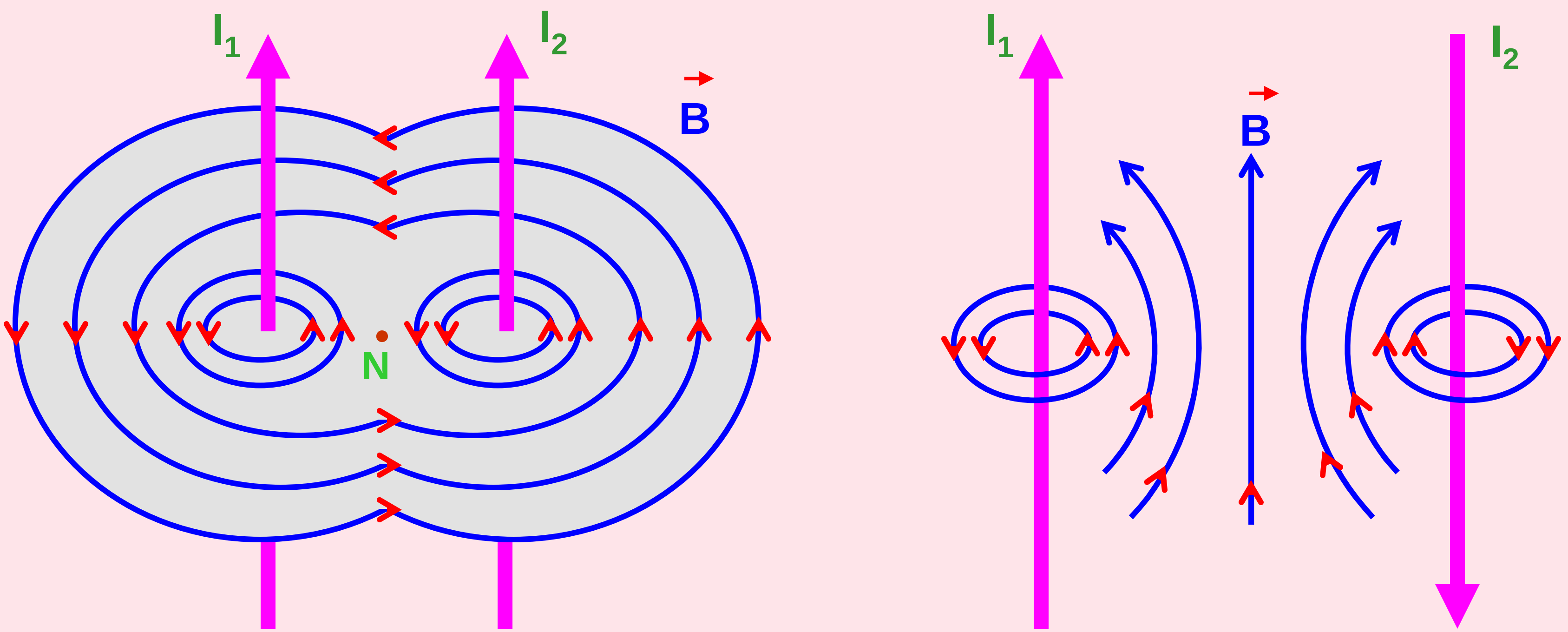
Definition of Ampere:

Force per unit length of the conductor is $F / l = \frac{\mu_0 I_1 I_2}{2\pi r}$ N / m

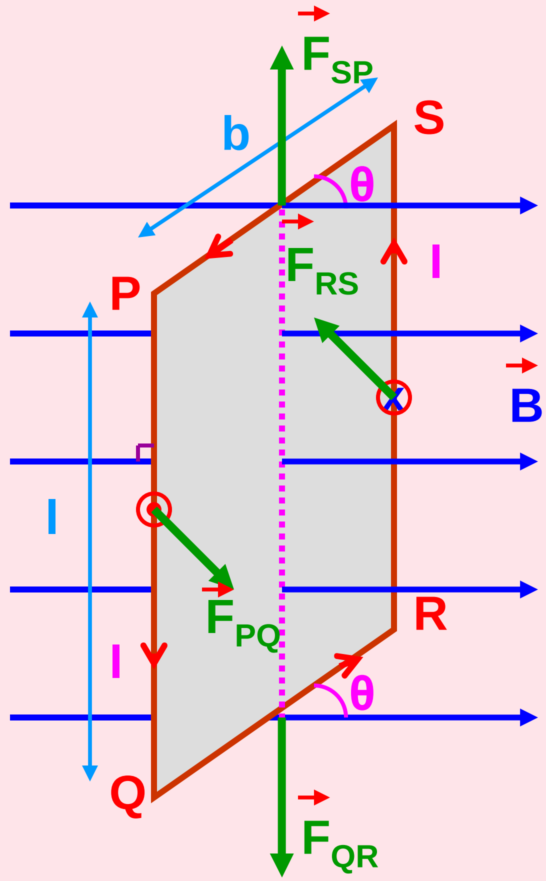
When $I_1 = I_2 = 1$ Ampere and $r = 1$ m, then $F = 2 \times 10^{-7}$ N/m.

One ampere is that current which, if passed in each of two parallel conductors of infinite length and placed 1 m apart in vacuum causes each conductor to experience a force of 2×10^{-7} Newton per metre of length of the conductor.

Representation of Field due to Parallel Currents:



Torque experienced by a Current Loop (Rectangular) in a uniform Magnetic Field:



Let θ be the angle between the plane of the loop and the direction of the magnetic field. The axis of the coil is perpendicular to the magnetic field.

$\vec{F}_{SP} = I\,(\vec{b} \times \vec{B})$

$|\vec{F}_{SP}| = I\,b\,B\,\sin\theta$

$\vec{F}_{QR} = I\,(\vec{b} \times \vec{B})$

$|\vec{F}_{QR}| = I\,b\,B\,\sin\theta$

Forces $\vec{F}_{SP}$ and $\vec{F}_{QR}$ are equal in magnitude but opposite in direction and they cancel out each other. Moreover they act along the same line of action (axis) and hence do not produce torque.

$\vec{F}_{PQ} = I\,(\vec{l} \times \vec{B})$

$|\vec{F}_{PQ}| = I\,l\,B\,\sin 90° = I\,l\,B$

$\vec{F}_{RS} = I\,(\vec{l} \times \vec{B})$

$|\vec{F}_{Rs}| = I\,l\,B\,\sin 90° = I\,l\,B$

Forces $\vec{F}_{PQ}$ and $\vec{F}_{RS}$ being equal in magnitude but opposite in direction cancel out each other and do not produce any translational motion. But they act along different lines of action and hence produce torque about the axis of the coil.

Torque experienced by the coil is

$\tau = F_{PQ} \times PN$ (in magnitude)

$\tau = I\, l\, B\, (b \cos\theta)$

$\tau = I\, lb\, B \cos\theta$

$\tau = I\, A\, B \cos\theta$ (A = lb)

$\tau = N\, I\, A\, B \cos\theta$ (where N is the no. of turns)

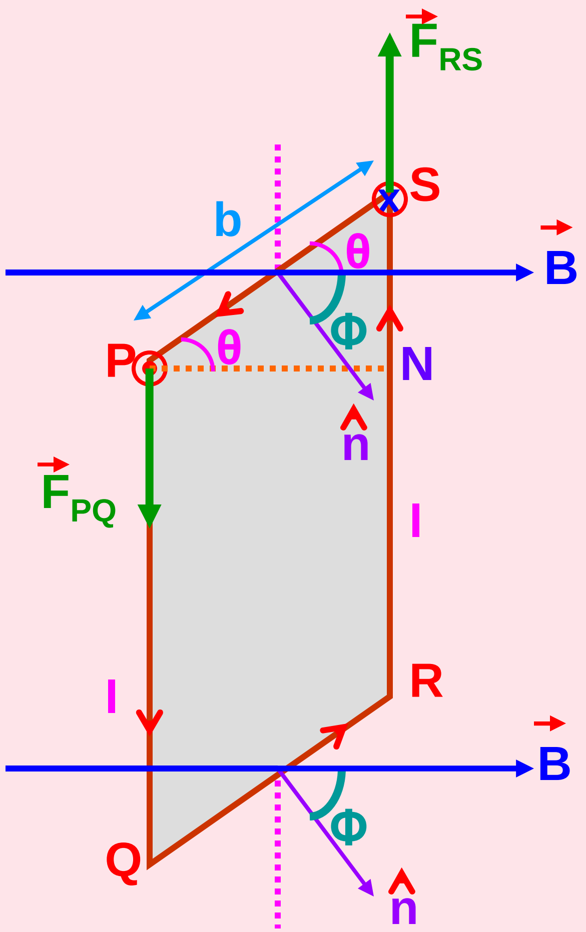
If Φ is the angle between the normal to the coil and the direction of the magnetic field, then

$\Phi + \theta = 90°$ i.e. $\theta = 90° - \Phi$

So,

$\tau = I\, A\, B \cos(90° - \Phi)$

$\tau = N\, I\, A\, B \sin\Phi$



NOTE:

One must be very careful in using the formula in terms of cos or sin since it depends on the angle taken whether with the plane of the coil or the normal of the coil.

Torque in Vector form:

$\tau = N\,I\,A\,B \sin\Phi$

$\vec{\tau} = (N\,I\,A\,B \sin\Phi)\,\hat{n}$ (where $\hat{n}$ is unit vector normal to the plane of the loop)

$\vec{\tau} = N\,I\,(\vec{A} \times \vec{B})$ or $\vec{\tau} = N\,(\vec{M} \times \vec{B})$

(since $\vec{M} = I\,\vec{A}$ is the Magnetic Dipole Moment)

Note:

1) The coil will rotate in the anticlockwise direction (from the top view, according to the figure) about the axis of the coil shown by the dotted line.

2) The torque acts in the upward direction along the dotted line (according to Maxwell's Screw Rule).

3) If $\Phi = 0°$, then $\tau = 0$.

4) If $\Phi = 90°$, then τ is maximum. i.e. $\tau_{max} = N\,I\,A\,B$

5) Units: B in Tesla, I in Ampere, A in m^2 and τ in Nm.

6) The above formulae for torque can be used for any loop irrespective of its shape.

Moving Coil or Suspended Coil or D' Arsonval Type Galvanometer:

Torque experienced by the coil is

τ = N I A B sin Φ

Restoring torque in the coil is

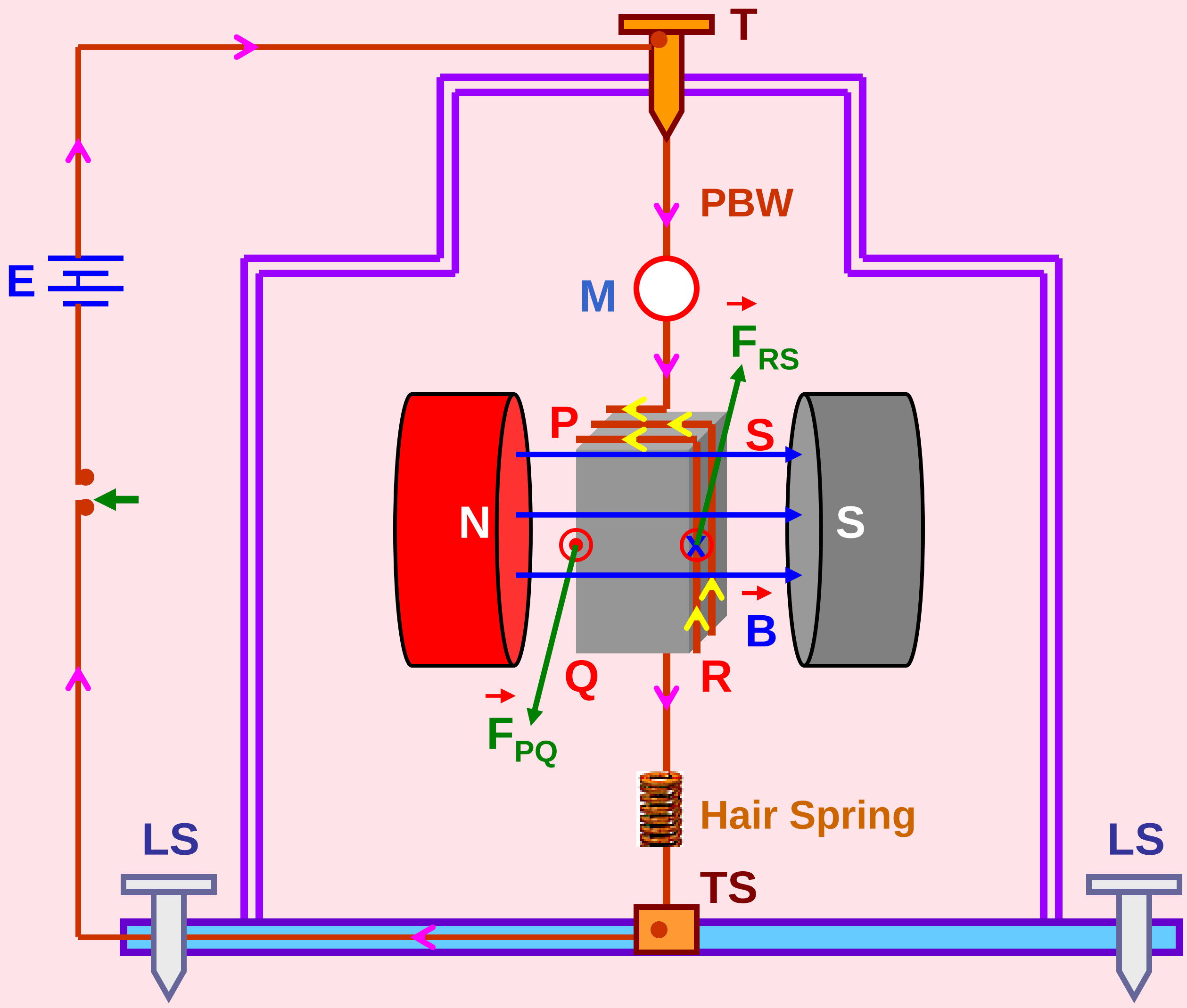
τ = k α (where k is restoring torque per unit angular twist, α is the angular twist in the wire)

At equilibrium,

N I A B sin Φ = k α

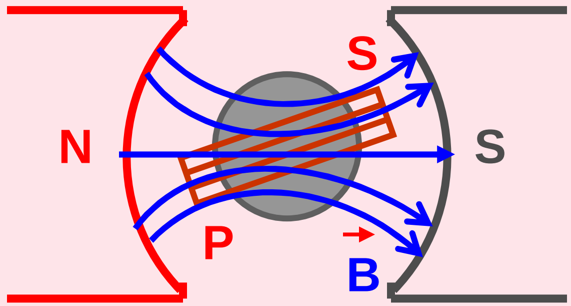
$$\therefore \quad I = \frac{k}{N\,A\,B\,\sin\Phi}\,\alpha$$

The factor sin Φ can be eliminated by choosing Radial Magnetic Field.



T – Torsion Head, TS – Terminal screw, M – Mirror, N,S – Poles pieces of a magnet, LS – Levelling Screws, PQRS – Rectangular coil, PBW – Phosphor Bronze Wire

Radial Magnetic Field:

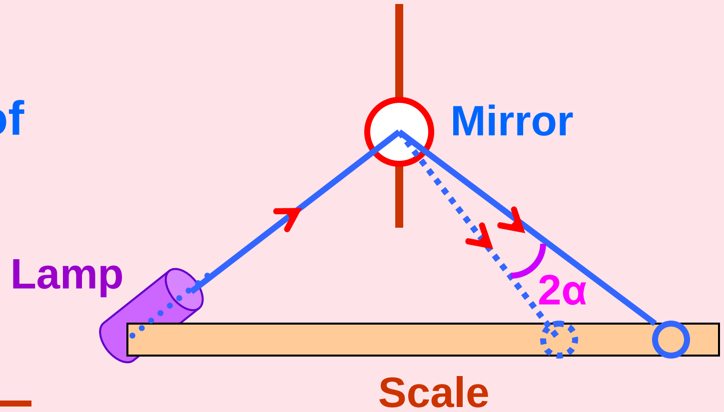


The (top view PS of) plane of the coil PQRS lies along the magnetic lines of force in whichever position the coil comes to rest in equilibrium.

So, the angle between the plane of the coil and the magnetic field is 0°.

or the angle between the normal to the plane of the coil and the magnetic field is 90°.

i.e. $\sin \Phi = \sin 90° = 1$



$$\therefore \quad I = \frac{k}{NAB}\alpha \quad \text{or} \quad I = G\alpha \quad \text{where } G = \frac{k}{NAB}$$

is called Galvanometer constant

Current Sensitivity of Galvanometer:

It is the defection of galvanometer per unit current.

$$\frac{\alpha}{I} = \frac{NAB}{k}$$

Voltage Sensitivity of Galvanometer:

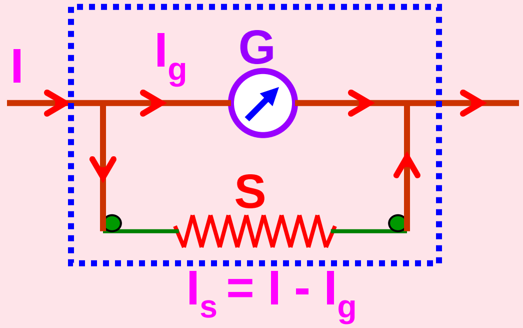
It is the defection of galvanometer per unit voltage.

$$\frac{\alpha}{V} = \frac{NAB}{kR}$$

Conversion of Galvanometer to Ammeter:

Galvanometer can be converted into ammeter by shunting it with a very small resistance.

Potential difference across the galvanometer and shunt resistance are equal.

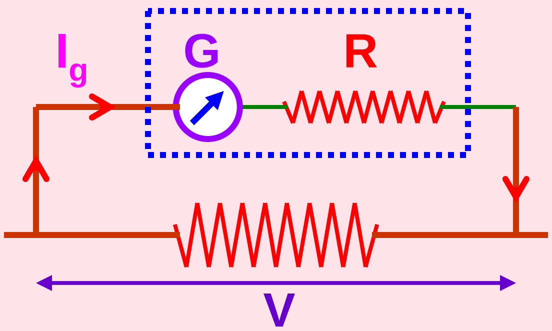


$\therefore$ $(I - I_g)\,S = I_g\,G$ or $S = \dfrac{I_g\,G}{I - I_g}$

Conversion of Galvanometer to Voltmeter:

Galvanometer can be converted into voltmeter by connecting it with a very high resistance.

Potential difference across the given load resistance is the sum of p.d across galvanometer and p.d. across the high resistance.



$\therefore$ $V = I_g\,(G + R)$ or $R = \dfrac{V}{I_g} - G$

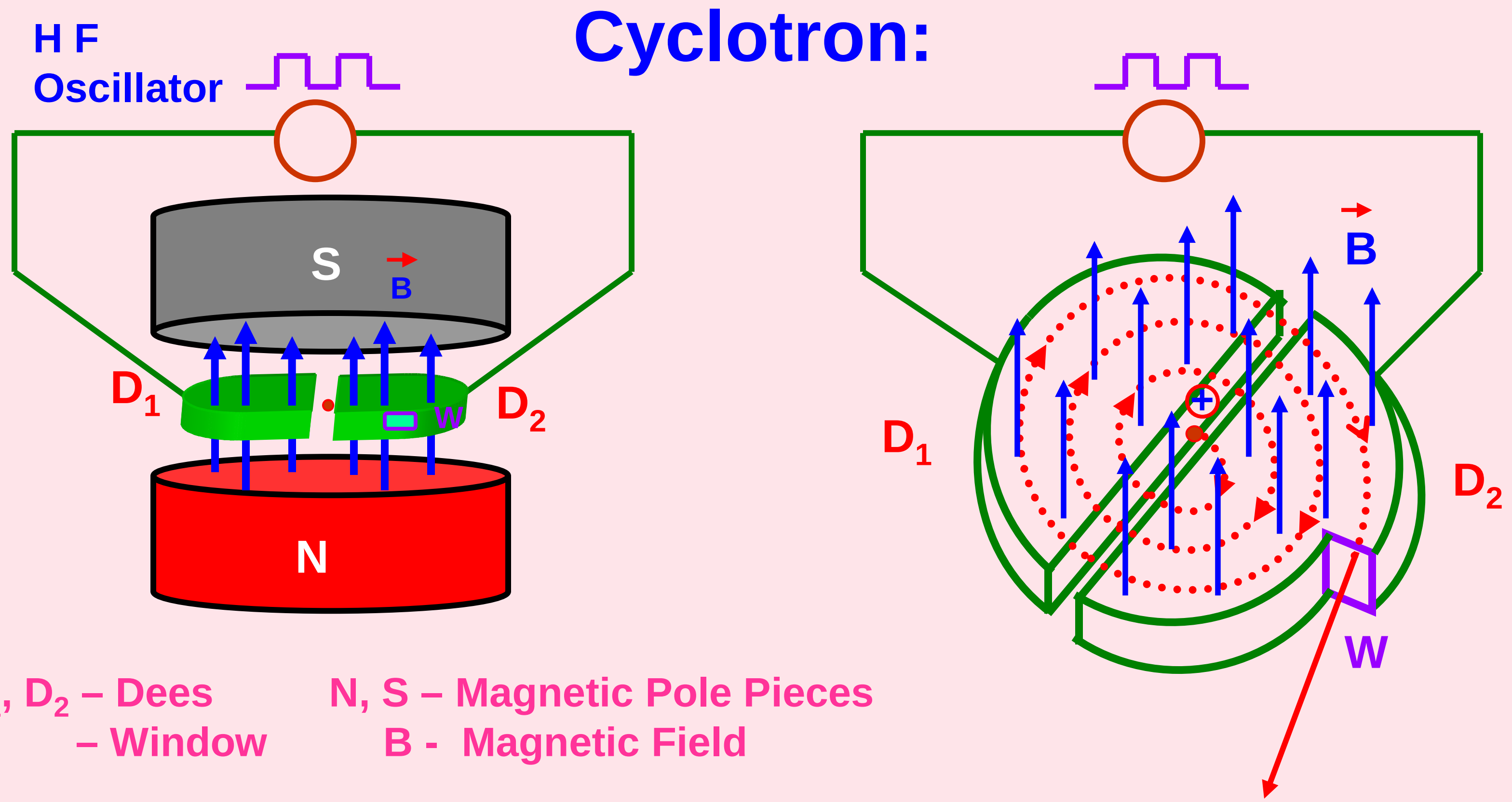
Difference between Ammeter and Voltmeter:

S.No.	Ammeter	Voltmeter
1	It is a low resistance instrument.	It is a high resistance instrument.
2	Resistance is GS / (G + S)	Resistance is G + R
3	Shunt Resistance is $(GI_g) / (I - I_g)$ and is very small.	Series Resistance is $(V / I_g) - G$ and is very high.
4	It is always connected in series.	It is always connected in parallel.
5	Resistance of an ideal ammeter is zero.	Resistance of an ideal voltmeter is infinity.
6	Its resistance is less than that of the galvanometer.	Its resistance is greater than that of the voltmeter.
7	It is not possible to decrease the range of the given ammeter.	It is possible to decrease the range of the given voltmeter.

MAGNETIC EFFECT OF CURRENT - III

1. Cyclotron
2. Ampere's Circuital Law
3. Magnetic Field due to a Straight Solenoid
4. Magnetic Field due to a Toroidal Solenoid

Cyclotron:



D_1, D_2 – Dees N, S – Magnetic Pole Pieces

W – Window B - Magnetic Field

Working: Imagining D_1 is positive and D_2 is negative, the + vely charged particle kept at the centre and in the gap between the dees get accelerated towards D_2. Due to perpendicular magnetic field and according to Fleming's Left Hand Rule the charge gets deflected and describes semi-circular path.

When it is about to leave D_2, D_2 becomes + ve and D_1 becomes – ve. Therefore the particle is again accelerated into D_1 where it continues to describe the semi-circular path. The process continues till the charge traverses through the whole space in the dees and finally it comes out with very high speed through the window.

Theory:

The magnetic force experienced by the charge provides centripetal force required to describe circular path.

∴ $mv^2 / r = qvB \sin 90°$

$$v = \frac{B\,q\,r}{m}$$

(where m – mass of the charged particle, q – charge, v – velocity on the path of radius – r, B is magnetic field and 90° is the angle b/n v and B)

If t is the time taken by the charge to describe the semi-circular path inside the dee, then

$$t = \frac{\pi\ r}{v} \quad \text{or} \quad t = \frac{\pi\,m}{B\,q}$$

Time taken inside the dee depends only on the magnetic field and m/q ratio and not on the speed of the charge or the radius of the path.

If T is the time period of the high frequency oscillator, then for resonance,

$$T = 2\,t \qquad \text{or} \qquad T = \frac{2\pi m}{B\,q}$$

If f is the frequency of the high frequency oscillator (Cyclotron Frequency), then

$$f = \frac{B\,q}{2\pi m}$$

Maximum Energy of the Particle:

Kinetic Energy of the charged particle is

$$K.E. = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{B q r}{m}\right)^2 = \frac{1}{2} \frac{B^2 q^2 r^2}{m}$$

Maximum Kinetic Energy of the charged particle is when r = R (radius of the D's).

$$K.E._{max} = \frac{1}{2} \frac{B^2 q^2 R^2}{m}$$

The expressions for Time period and Cyclotron frequency only when m remains constant. (Other quantities are already constant.)

But m varies with v according to Einstein's Relativistic Principle as per

$$m = \frac{m_0}{[1 - (v^2 / c^2)]^{1/2}}$$

If frequency is varied in synchronisation with the variation of mass of the charged particle (by maintaining B as constant) to have resonance, then the cyclotron is called synchro – cyclotron.

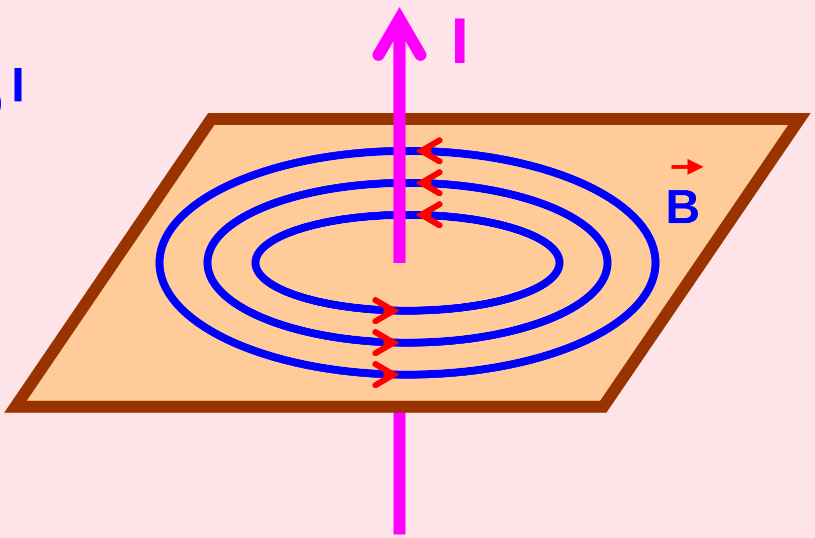
If magnetic field is varied in synchronisation with the variation of mass of the charged particle (by maintaining f as constant) to have resonance, then the cyclotron is called isochronous – cyclotron.

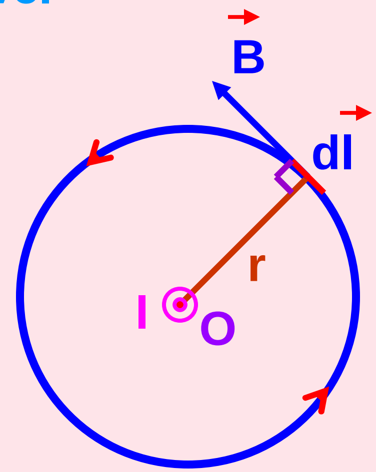
NOTE: Cyclotron can not be used for accelerating neutral particles. Electrons can not be accelerated because they gain speed very quickly due to their lighter mass and go out of phase with alternating e.m.f. and get lost within the dees.

Ampere's Circuital Law:

The line integral $\oint \vec{B} \cdot \vec{dl}$ for a closed curve is equal to μ_0 times the net current I threading through the area bounded by the curve.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$





Current is emerging out and the magnetic field is anticlockwise.

Proof:

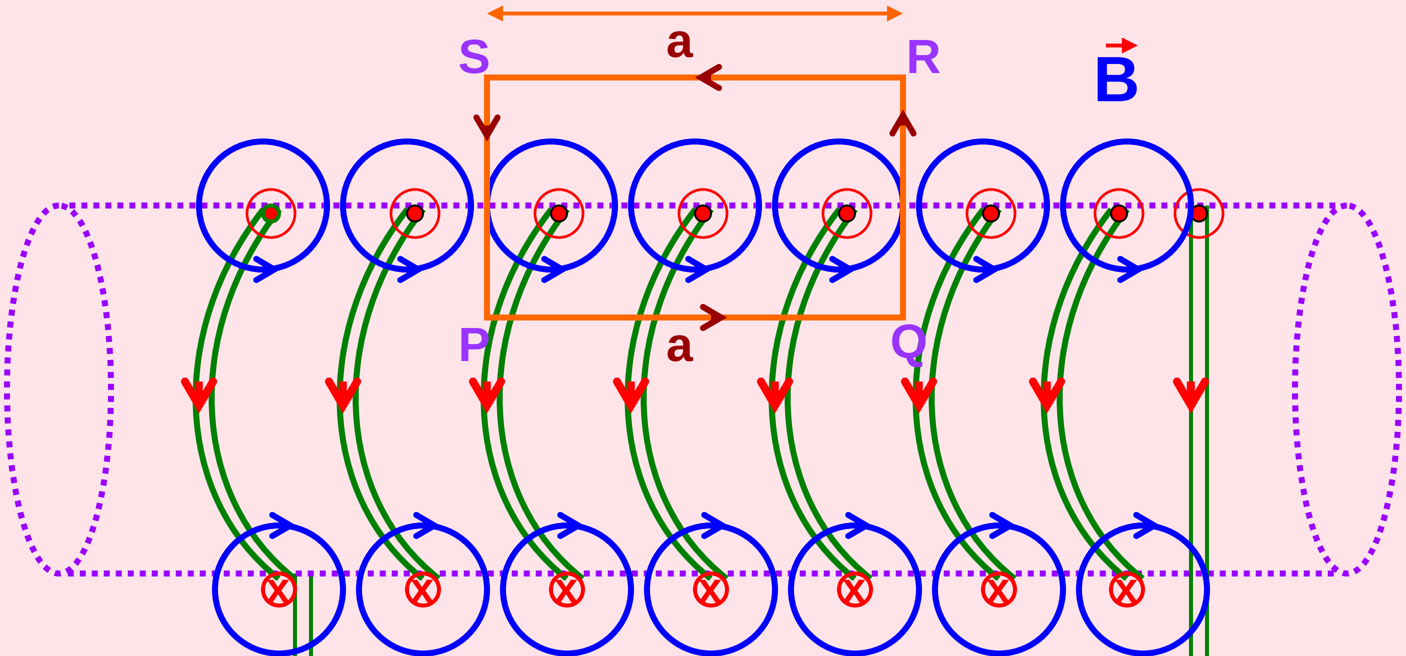
$$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl \cos 0°$$

$$= \oint B \cdot dl = B \oint dl$$

$$= B (2\pi r) = (\mu_0 I / 2\pi r) \times 2\pi r$$

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$$

Magnetic Field at the centre of a Straight Solenoid:



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_0$$

(where I_0 is the net current threading through the solenoid)

$$\oint \vec{B} \cdot \vec{dl} = \oint_{PQ} \vec{B} \cdot \vec{dl} + \oint_{QR} \vec{B} \cdot \vec{dl} + \oint_{RS} \vec{B} \cdot \vec{dl} + \oint_{SP} \vec{B} \cdot \vec{dl}$$

$$= \oint B \, . \, dl \cos 0° + \oint B \, . \, dl \cos 90° + \oint 0 \, . \, dl \cos 0° + \oint B \, . \, dl \cos 90°$$

$= B \oint dl = B.a$ and $\mu_0 I_0 = \mu_0 n a I$ $\therefore$ $B = \mu_0 n I$

(where **n** is no. of turns per unit length, **a** is the length of the path and **I** is the current passing through the lead of the solenoid)

Magnetic Field due to Toroidal Solenoid (Toroid):

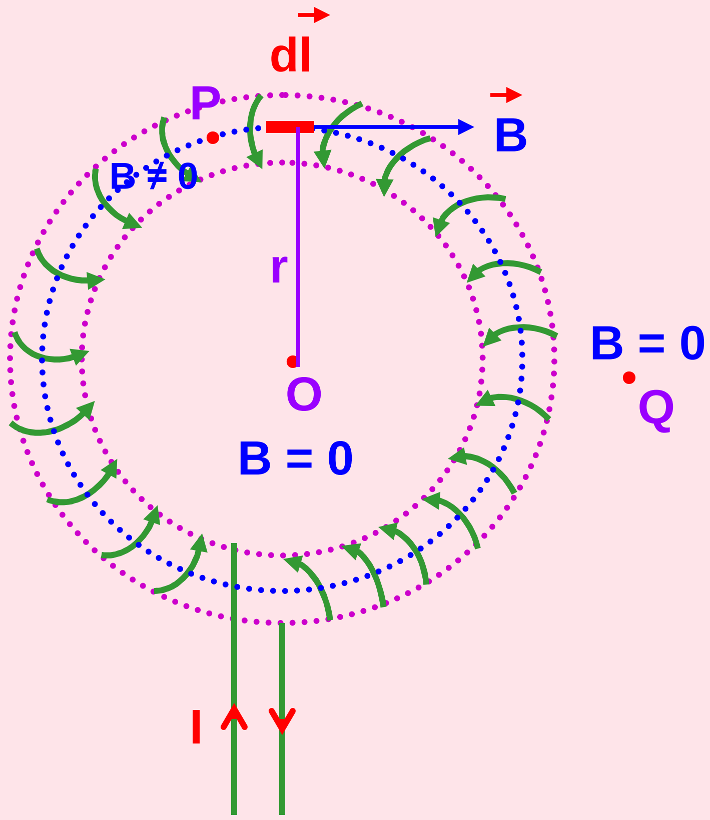
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_0$$

$$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl \cos 0°$$

$$= B \oint dl = B (2\pi r)$$

And $\mu_0 I_0 = \mu_0 n (2\pi r) I$

$$\therefore \quad \boxed{B = \mu_0 n I}$$



NOTE:

The magnetic field exists only in the tubular area bound by the coil and it does not exist in the area inside and outside the toroid.

i.e. B is zero at O and Q and non-zero at P.

MAGNETISM

1. **Bar Magnet and its properties**
2. **Current Loop as a Magnetic Dipole and Dipole Moment**
3. **Current Solenoid equivalent to Bar Magnet**
4. **Bar Magnet and it Dipole Moment**
5. **Coulomb's Law in Magnetism**
6. **Important Terms in Magnetism**
7. **Magnetic Field due to a Magnetic Dipole**
8. **Torque and Work Done on a Magnetic Dipole**
9. **Terrestrial Magnetism**
10. **Elements of Earth's Magnetic Field**
11. **Tangent Law**
12. **Properties of Dia-, Para- and Ferro-magnetic substances**
13. **Curie's Law in Magnetism**
14. **Hysteresis in Magnetism**

Magnetism:

- Phenomenon of attracting magnetic substances like iron, nickel, cobalt, etc.

• A body possessing the property of magnetism is called a magnet.

• A magnetic pole is a point near the end of the magnet where magnetism is concentrated.

• Earth is a natural magnet.

•The region around a magnet in which it exerts forces on other magnets and on objects made of iron is a magnetic field.

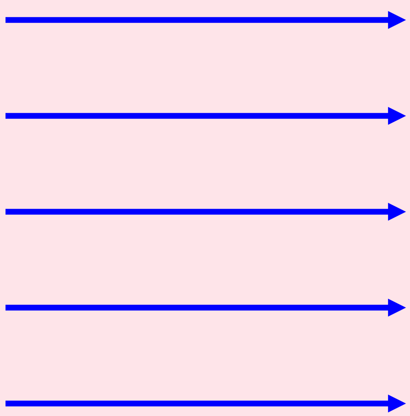
Properties of a bar magnet:

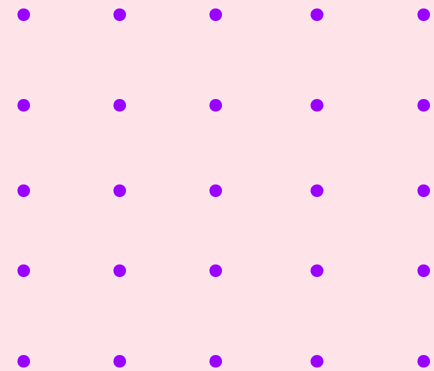
1. A freely suspended magnet aligns itself along North – South direction.
2. Unlike poles attract and like poles repel each other.
3. Magnetic poles always exist in pairs. i.e. Poles can not be separated.
4. A magnet can induce magnetism in other magnetic substances.
5. It attracts magnetic substances.

Repulsion is the surest test of magnetisation: A magnet attracts iron rod as well as opposite pole of other magnet. Therefore it is not a sure test of magnetisation.

But, if a rod is repelled with strong force by a magnet, then the rod is surely magnetised.

Representation of Uniform Magnetic Field:



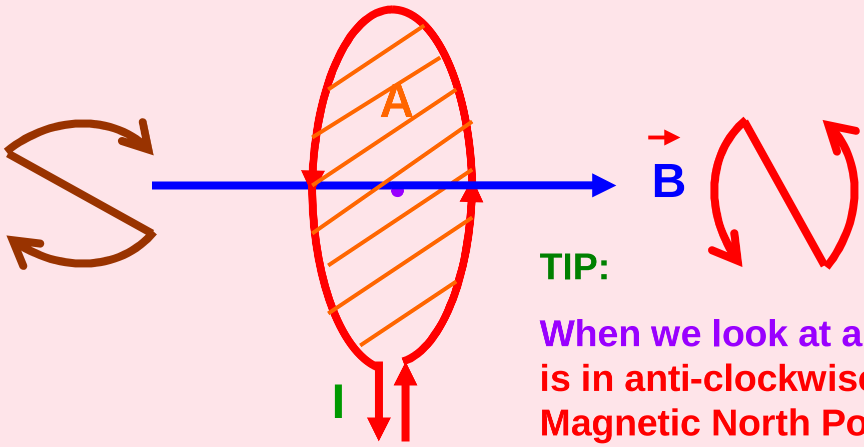


Uniform field on the plane of the diagram

Uniform field perpendicular & into the plane of the diagram

Uniform field perpendicular & emerging out of the plane of the diagram

Current Loop as a Magnetic Dipole & Dipole Moment:



Magnetic Dipole Moment is

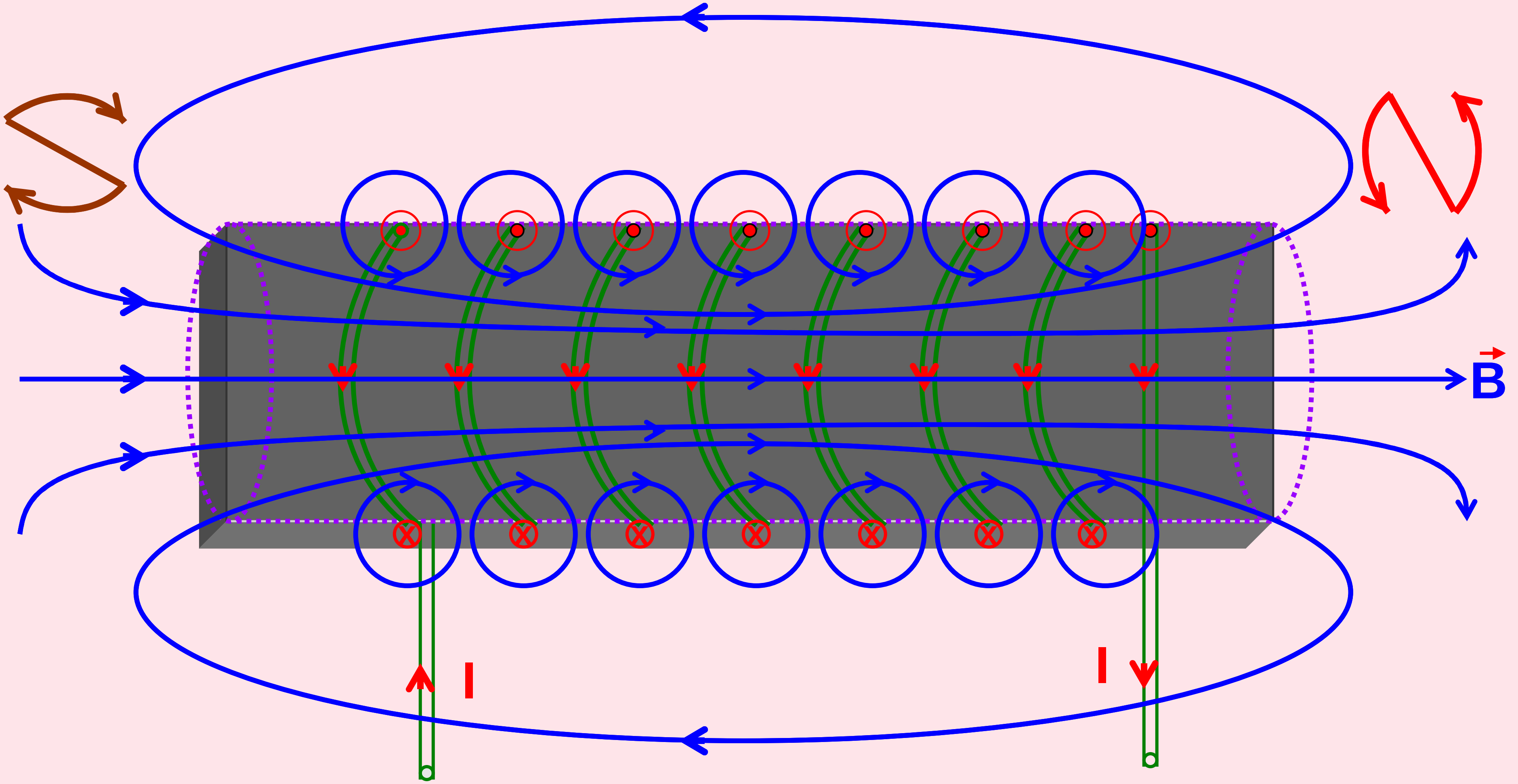
$$\vec{M} = I A \hat{n}$$

SI unit is A m^2.

TIP:

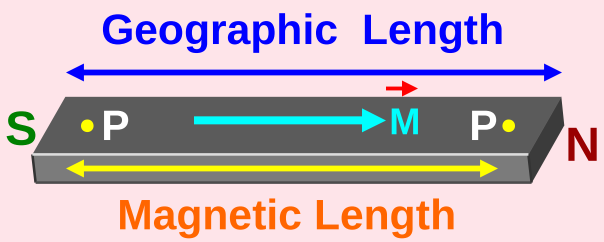
When we look at any one side of the loop carrying current, if the current is in anti-clockwise direction then that side of the loop behaves like Magnetic North Pole and if the current is in clockwise direction then that side of the loop behaves like Magnetic South Pole.

Current Solenoid as a Magnetic Dipole or Bar Magnet:



TIP: Play previous and next to understand the similarity of field lines.

Bar Magnet:



1. The line joining the poles of the magnet is called magnetic axis.
2. The distance between the poles of the magnet is called magnetic length of the magnet.
3. The distance between the ends of the magnet is called the geometrical length of the magnet.
4. The ratio of magnetic length and geometrical length is nearly 0.84.

Magnetic Dipole & Dipole Moment:

A pair of magnetic poles of equal and opposite strengths separated by a finite distance is called a magnetic dipole.

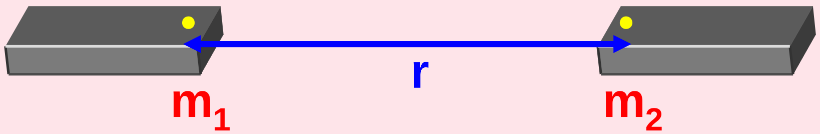
The magnitude of dipole moment is the product of the pole strength m and the separation 2l between the poles.

<u>Magnetic Dipole Moment</u> is $\vec{M} = m.2l.\hat{l}$ SI unit of pole strength is A.m

The direction of the dipole moment is from South pole to North Pole along the axis of the magnet.

Coulomb's Law in Magnetism:

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.



$F \ \alpha \ m_1 m_2$

$\alpha \ r^2$

$$F = \frac{k \ m_1 m_2}{r^2}$$

or

$$F = \frac{\mu_0 \ m_1 m_2}{4\pi \ r^2}$$

(where $k = \mu_0 / 4\pi$ is a constant and $\mu_0 = 4\pi \times 10^{-7}$ T m A-1)

In <u>vector form</u>

$$\vec{F} = \frac{\mu_0 \ m_1 m_2 \ \hat{r}}{4\pi \ r^2}$$

$$\vec{F} = \frac{\mu_0 \ m_1 m_2 \ \vec{r}}{4\pi \ r^3}$$

Magnetic Intensity or Magnetising force (H):

i) Magnetic Intensity at a point is the force experienced by a north pole of unit pole strength placed at that point due to pole strength of the given magnet. $H = B / \mu$

ii) It is also defined as the magnetomotive force per unit length.

iii) It can also be defined as the degree or extent to which a magnetic field can magnetise a substance.

iv) It can also be defined as the force experienced by a unit positive charge flowing with unit velocity in a direction normal to the magnetic field.

v) Its SI unit is ampere-turns per linear metre.

vi) Its cgs unit is oersted.

Magnetic Field Strength or Magnetic Field or Magnetic Induction or Magnetic Flux Density (B):

i) Magnetic Flux Density is the number of magnetic lines of force passing normally through a unit area of a substance. $B = \mu H$

ii) Its SI unit is weber-m^{-2} or Tesla (T).

iii) Its cgs unit is gauss. 1 gauss = 10^{-4} Tesla

Magnetic Flux (Φ):

i) It is defined as the number of magnetic lines of force passing normally through a surface.

ii) Its SI unit is weber.

Relation between B and H:

$B = \mu H$ (where μ is the permeability of the medium)

Magnetic Permeability (μ):

It is the degree or extent to which magnetic lines of force can pass enter a substance.

Its SI unit is $T\ m\ A^{-1}$ or $wb\ A^{-1}\ m^{-1}$ or $H\ m^{-1}$

Relative Magnetic Permeability (μ_r):

It is the ratio of magnetic flux density in a material to that in vacuum.

It can also be defined as the ratio of absolute permeability of the material to that in vacuum.

$\mu_r = B / B_0$ or $\mu_r = \mu / \mu_0$

Intensity of Magnetisation: (I):

i) It is the degree to which a substance is magnetised when placed in a magnetic field.

ii) It can also be defined as the magnetic dipole moment (M) acquired per unit volume of the substance (V).

iii) It can also be defined as the pole strength (m) per unit cross-sectional area (A) of the substance.

iv) $I = M / V$

v) $I = m(2l) / A(2l) = m / A$

vi) SI unit of Intensity of Magnetisation is $A\ m^{-1}$.

Magnetic Susceptibility (c_m):

i) It is the property of the substance which shows how easily a substance can be magnetised.

ii) It can also be defined as the ratio of intensity of magnetisation (I) in a substance to the magnetic intensity (H) applied to the substance.

iii) $c_m = I / H$

Susceptibility has no unit.

Relation between Magnetic Permeability (μ_r) & Susceptibility (c_m):

$$\mu_r = 1 + c_m$$

Magnetic Field due to a Magnetic Dipole (Bar Magnet):

i) At a point on the axial line of the magnet:

$$B_P = \frac{\mu_0 \, 2 M x}{4\pi (x^2 - l^2)^2}$$

If l << x, then

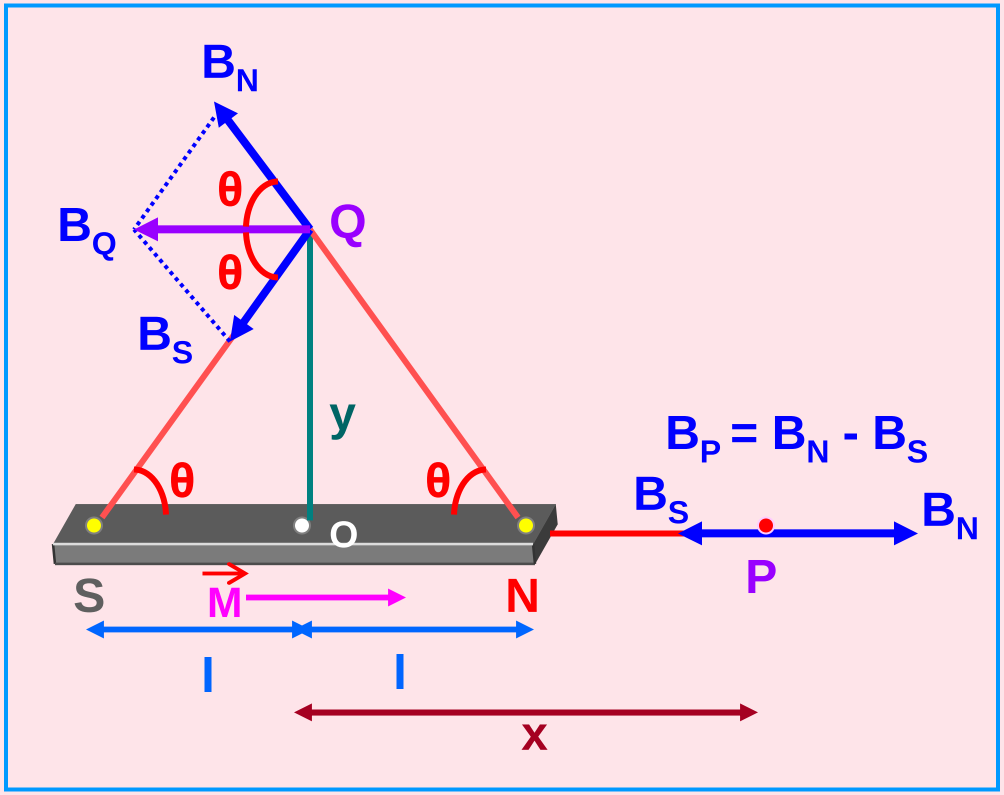
$$B_P \approx \frac{\mu_0 \, 2 M}{4\pi \, x^3}$$

ii) At a point on the equatorial line of the magnet:

$$B_Q = \frac{\mu_0 \, M}{4\pi (y^2 + l^2)^{3/2}}$$

If l << y, then

$$B_P \approx \frac{\mu_0 \, M}{4\pi \, y^3}$$

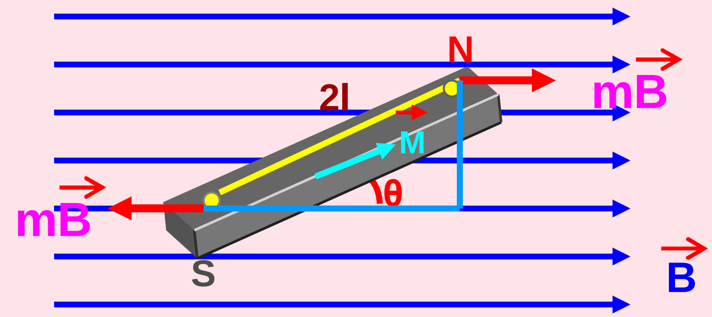


Magnetic Field at a point on the axial line acts along the dipole moment vector.

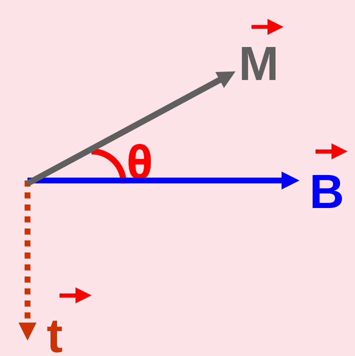
Magnetic Field at a point on the equatorial line acts opposite to the dipole moment vector.

Torque on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

The forces of magnitude mB act opposite to each other and hence net force acting on the bar magnet due to external uniform magnetic field is zero. So, there is no translational motion of the magnet.



However the forces are along different lines of action and constitute a couple. Hence the magnet will rotate and experience torque.



Torque = Magnetic Force x ⊥ distance

$t = mB\,(2l \sin\theta)$

$= M\,B \sin\theta$

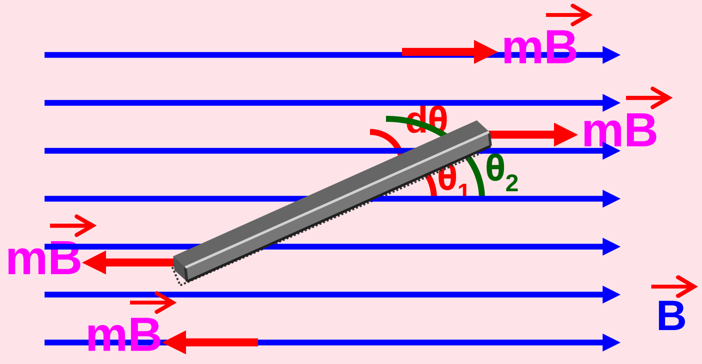
$\vec{t} = \vec{M} \times \vec{B}$

Direction of Torque is perpendicular and into the plane containing $\vec{M}$ and $\vec{B}$.

Work done on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

$dW = \tau\, d\theta$

$= M\, B \sin\theta\, d\theta$



$$W = \int_{\theta_1}^{\theta_2} M\, B \sin\theta\, d\theta$$

$$W = M\, B\, (\cos\theta_1 - \cos\theta_2)$$

If Potential Energy is arbitrarily taken zero when the dipole is at 90°, then P.E in rotating the dipole and inclining it at an angle θ is

Potential Energy = $- M\, B \cos\theta$

Note:

Potential Energy can be taken zero arbitrarily at any position of the dipole.

Terrestrial Magnetism:

i) **Geographic Axis** is a straight line passing through the geographical poles of the earth. It is the axis of rotation of the earth. It is also known as polar axis.

ii) **Geographic Meridian** at any place is a vertical plane passing through the geographic north and south poles of the earth.

iii) **Geographic Equator** is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distances from the geographic poles.

iv) **Magnetic Axis** is a straight line passing through the magnetic poles of the earth. It is inclined to Geographic Axis nearly at an angle of 17°.

v) **Magnetic Meridian** at any place is a vertical plane passing through the magnetic north and south poles of the earth.

vi) **Magnetic Equator** is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distances from the magnetic poles.

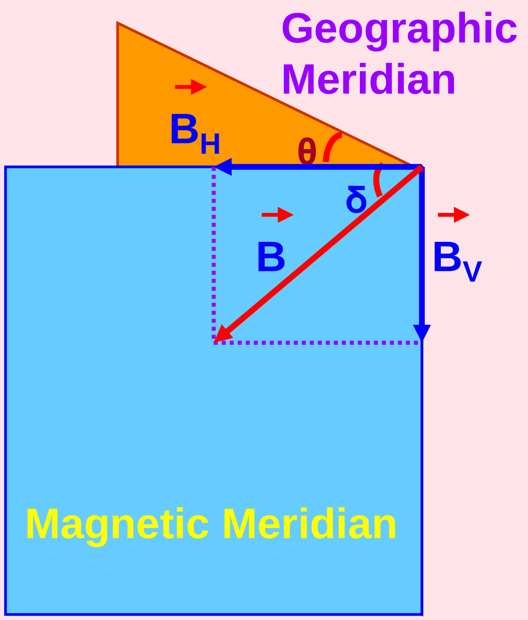
Declination (θ):

The angle between the magnetic meridian and the geographic meridian at a place is Declination at that place.

It varies from place to place.

Lines shown on the map through the places that have the same declination are called isogonic line.

Line drawn through places that have zero declination is called an agonic line.



Dip or Inclination (δ):

The angle between the horizontal component of earth's magnetic field and the earth's resultant magnetic field at a place is Dip or Inclination at that place.

It is zero at the equator and 90° at the poles.

Lines drawn up on a map through places that have the same dip are called isoclinic lines.

The line drawn through places that have zero dip is known as an aclinic line. It is the magnetic equator.

Horizontal Component of Earth's Magnetic Field (B_H):

The total intensity of the earth's magnetic field does not lie in any horizontal plane. Instead, it lies along the direction at an angle of dip (δ) to the horizontal. The component of the earth's magnetic field along the horizontal at an angle δ is called Horizontal Component of Earth's Magnetic Field.

$$B_H = B \cos \delta$$

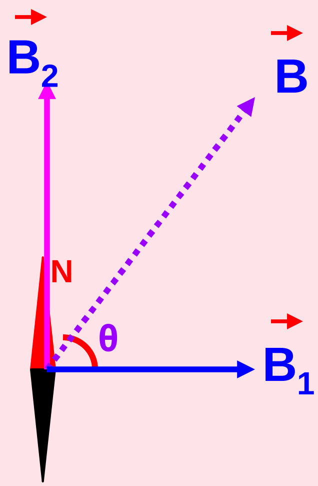
Similarly Vertical Component is $B_V = B \sin \delta$

such that $B = \sqrt{B_H^2 + B_V^2}$

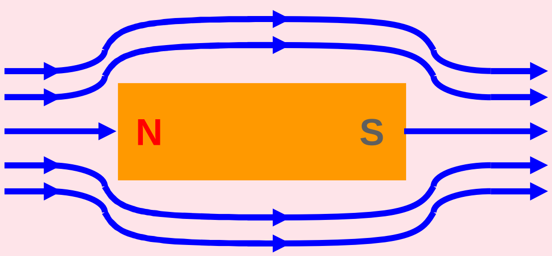
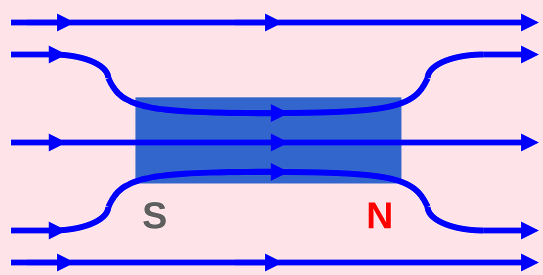
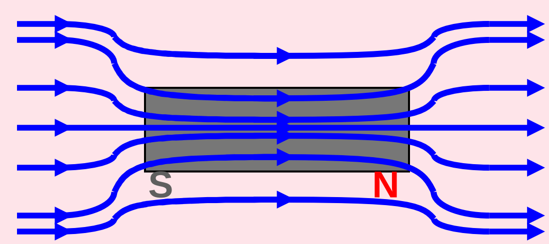
Tangent Law:

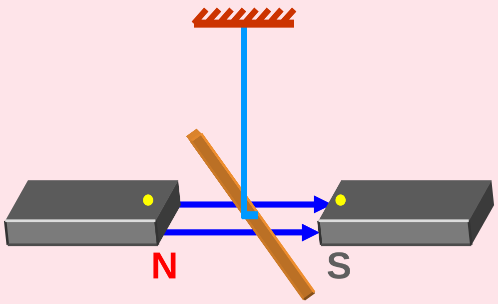
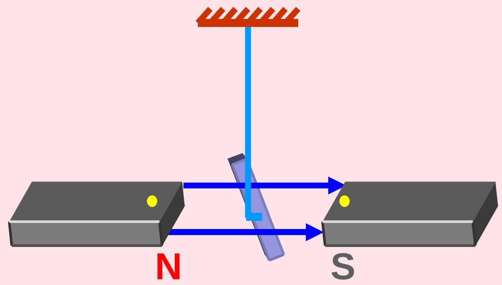
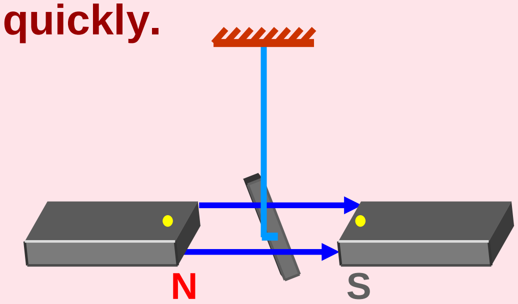
If a magnetic needle is suspended in a region where two uniform magnetic fields are perpendicular to each other, the needle will align itself along the direction of the resultant field of the two fields at an angle θ such that the tangent of the angle is the ratio of the two fields.

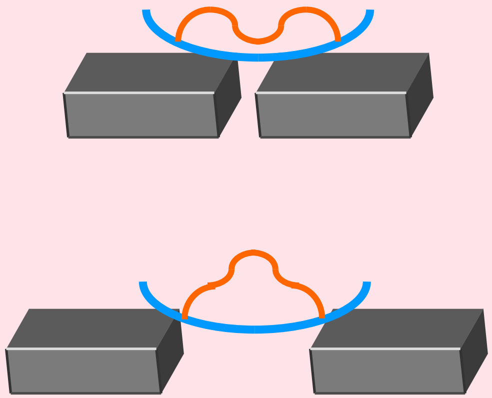
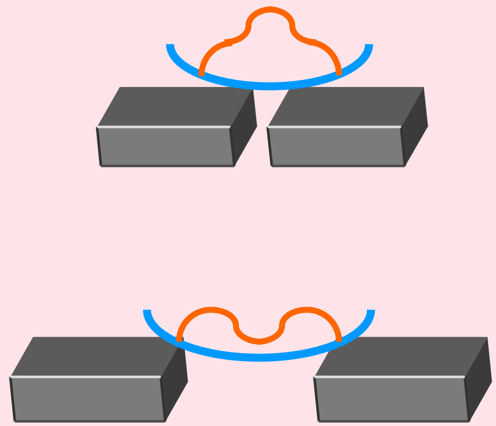
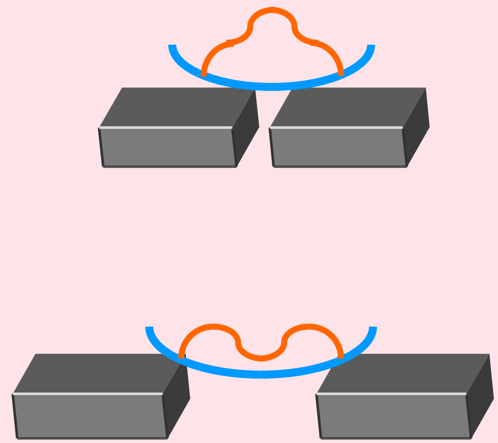
$$\tan \theta = B_2 / B_1$$



Comparison of Dia, Para and Ferro Magnetic materials:

DIA	PARA	FERRO
1. Diamagnetic substances are those substances which are feebly repelled by a magnet. Eg. Antimony, Bismuth, Copper, Gold, Silver, Quartz, Mercury, Alcohol, water, Hydrogen, Air, Argon, etc.	Paramagnetic substances are those substances which are feebly attracted by a magnet. Eg. Aluminium, Chromium, Alkali and Alkaline earth metals, Platinum, Oxygen, etc.	Ferromagnetic substances are those substances which are strongly attracted by a magnet. Eg. Iron, Cobalt, Nickel, Gadolinium, Dysprosium, etc.
2. When placed in magnetic field, the lines of force tend to avoid the substance. 	The lines of force prefer to pass through the substance rather than air. 	The lines of force tend to crowd into the specimen. 

2. When placed in non-uniform magnetic field, it moves from stronger to weaker field (feeble repulsion).	When placed in non-uniform magnetic field, it moves from weaker to stronger field (feeble attraction).	When placed in non-uniform magnetic field, it moves from weaker to stronger field (strong attraction).
3. When a diamagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction perpendicular to the field. 	When a paramagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction parallel to the field. 	When a paramagnetic rod is freely suspended in a uniform magnetic field, it aligns itself in a direction parallel to the field very quickly. 

4. If diamagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects away from the centre when the magnetic poles are closer and collects at the centre when the magnetic poles are farther.	If paramagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects at the centre when the magnetic poles are closer and collects away from the centre when the magnetic poles are farther.	If ferromagnetic liquid taken in a watch glass is placed in uniform magnetic field, it collects at the centre when the magnetic poles are closer and collects away from the centre when the magnetic poles are farther.
		

5. When a diamagnetic substance is placed in a magnetic field, it is weakly magnetised in the direction opposite to the inducing field.	When a paramagnetic substance is placed in a magnetic field, it is weakly magnetised in the direction of the inducing field.	When a ferromagnetic substance is placed in a magnetic field, it is strongly magnetised in the direction of the inducing field.
6. Induced Dipole Moment (M) is a small – ve value.	Induced Dipole Moment (M) is a small + ve value.	Induced Dipole Moment (M) is a large + ve value.
7. Intensity of Magnetisation (I) has a small – ve value.	Intensity of Magnetisation (I) has a small + ve value.	Intensity of Magnetisation (I) has a large + ve value.
8. Magnetic permeability μ is always less than unity.	Magnetic permeability μ is more than unity.	Magnetic permeability μ is large i.e. much more than unity.

9. Magnetic susceptibility c_m has a small – ve value.	Magnetic susceptibility c_m has a small + ve value.	Magnetic susceptibility c_m has a large + ve value.
10. They do not obey Curie's Law. i.e. their properties do not change with temperature.	They obey Curie's Law. They lose their magnetic properties with rise in temperature.	They obey Curie's Law. At a certain temperature called Curie Point, they lose ferromagnetic properties and behave like paramagnetic substances.

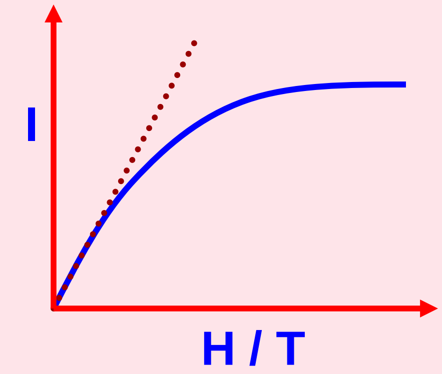
Curie's Law:

Magnetic susceptibility of a material varies inversely with the absolute temperature.

$I \alpha H / T$ or $I / H \alpha 1 / T$

$c_m \alpha 1 / T$

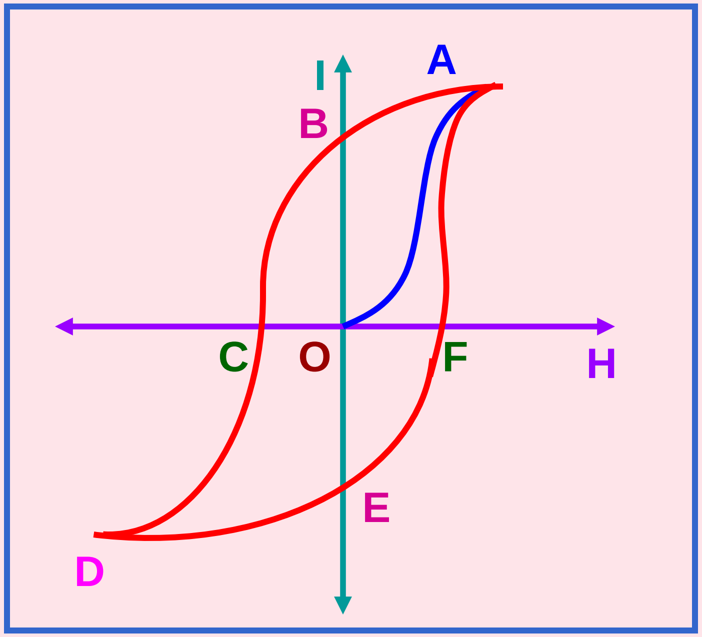
$c_m = C / T$ (where C is Curie constant)



Curie temperature for iron is 1000 K, for cobalt 1400 K and for nickel 600 K.

Hysteresis Loop or Magnetisation Curve:

Intensity of Magnetisation (I) increases with increase in Magnetising Force (H) initially through OA and reaches saturation at A.

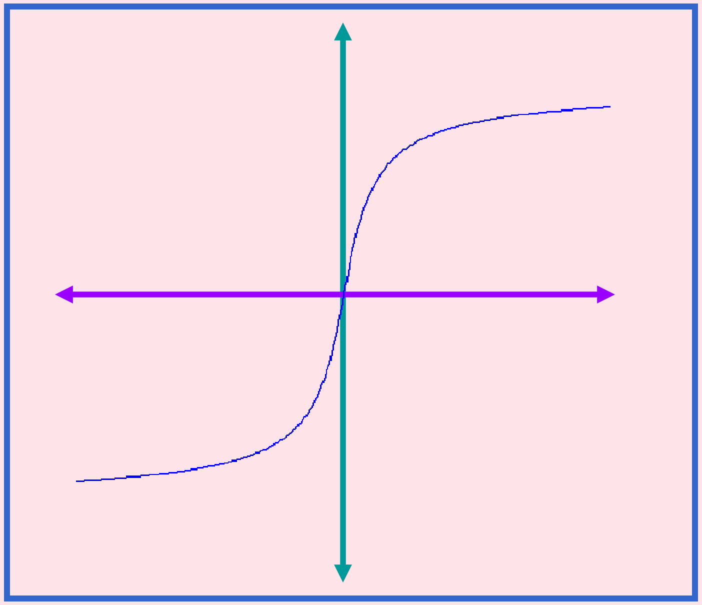


When H is decreased, I decreases but it does not come to zero at H = 0.

The residual magnetism (I) set up in the material represented by OB is called Retentivity.

To bring I to zero (to demagnetise completely), opposite (negative) magnetising force is applied. This magetising force represented by OC is called coercivity.

After reaching the saturation level D, when the magnetising force is reversed, the curve closes to the point A completing a cycle.



The loop ABCDEFA is called Hysteresis Loop.

The area of the loop gives the loss of energy due to the cycle of magnetisation and demagnetisation and is dissipated in the form of heat.

The material (like iron) having thin loop is used for making temporary magnets and that with thick loop (like steel) is used for permanent magnets.

ELECTROMAGNETIC INDUCTION

1. Magnetic Flux
2. Faraday’s Experiments
3. Faraday’s Laws of Electromagnetic Induction
4. Lenz’s Law and Law of Conservation of Energy
5. Expression for Induced emf based on both laws
6. Methods of producing induced emf
   a) By changing Magnetic Field
   b) By changing the Area of the Coil (Motional emf)
   c) By changing the Relative Orientation of the coil with the Magnetic Field
7. Eddy Currents
8. Self Induction and Self Inductance
9. Mutual Induction and Mutual Inductance
10. Additional Information

Magnetic Flux (Φ):

Magnetic Flux through any surface is the number of magnetic lines of force passing normally through that surface.

It can also be defined as the product of the area of the surface and the component of the magnetic field normal to that surface.

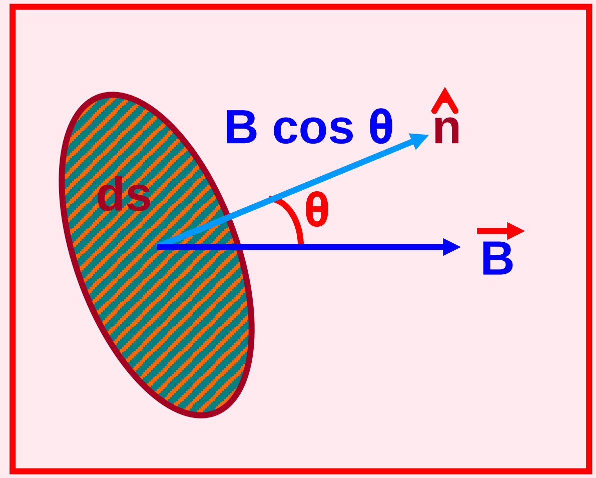
$d\Phi = \vec{B} \cdot \vec{ds} = \vec{B} \cdot ds\, \hat{n}$

$d\Phi = B \cdot ds \cos \theta$

$\Phi = \vec{B} \cdot \vec{A} = \vec{B} \cdot A\, \hat{n}$

$\Phi = B \cdot A \cos \theta$

Direction of $\vec{ds}$ is along the normal to the surface and $\hat{n}$ is unit normal vector.



Positive Flux:
Magnetic Flux is positive for $0° \leq \theta < 90°$ & $270° < \theta \leq 360°$

Zero Flux:
Magnetic Flux is zero for $\theta = 90°$ & $\theta = 270°$

Negative Flux:
Magnetic Flux is negative for $90° < \theta < 270°$

Flux is maximum when $\theta = 0°$ and is $\Phi = B \cdot A$

$$\Phi = B \cdot A \cos \theta$$

Magnetic Flux across a coil can be changed by changing :

1) the strength of the magnetic field B

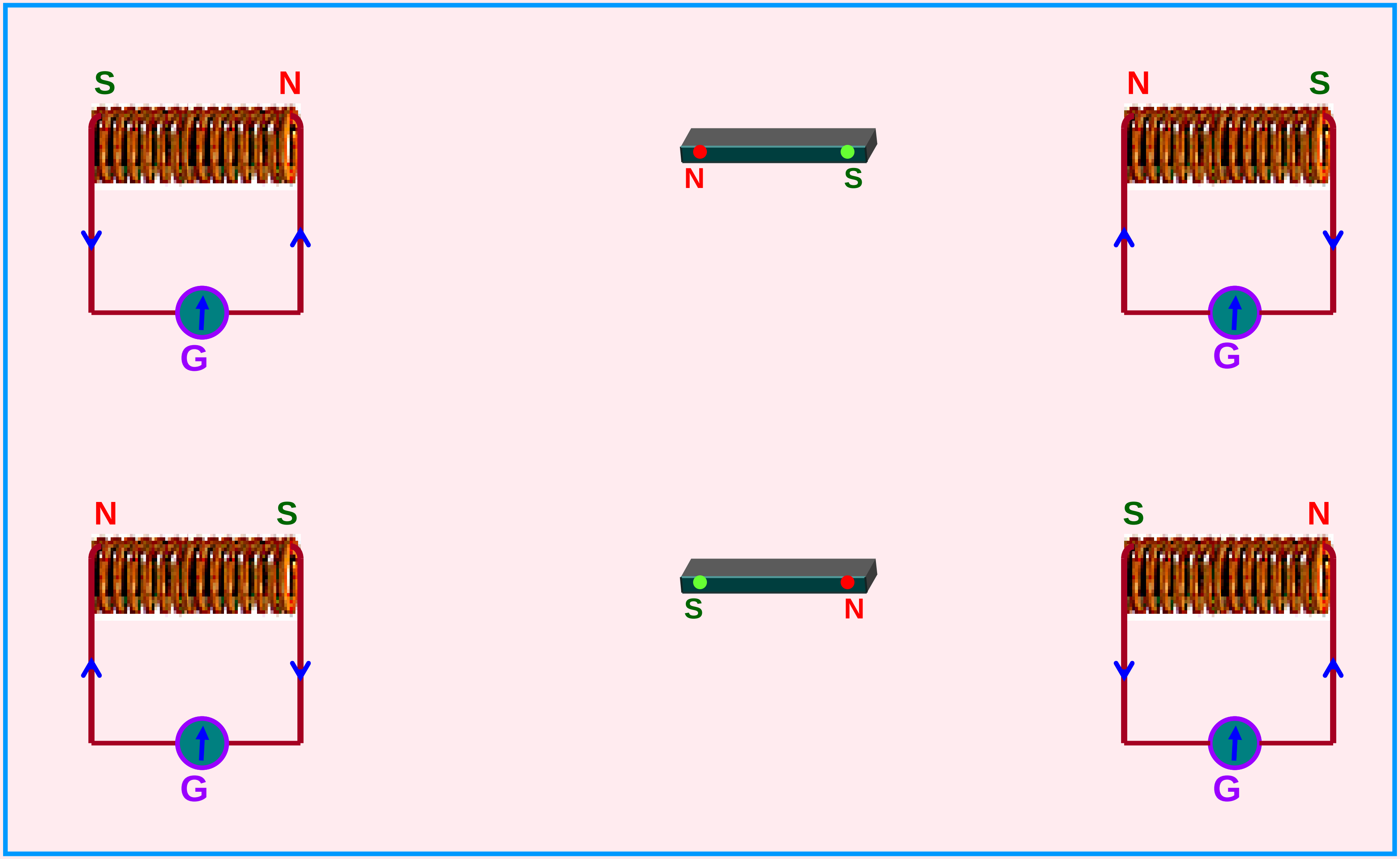
2) the area of cross section of the coil A

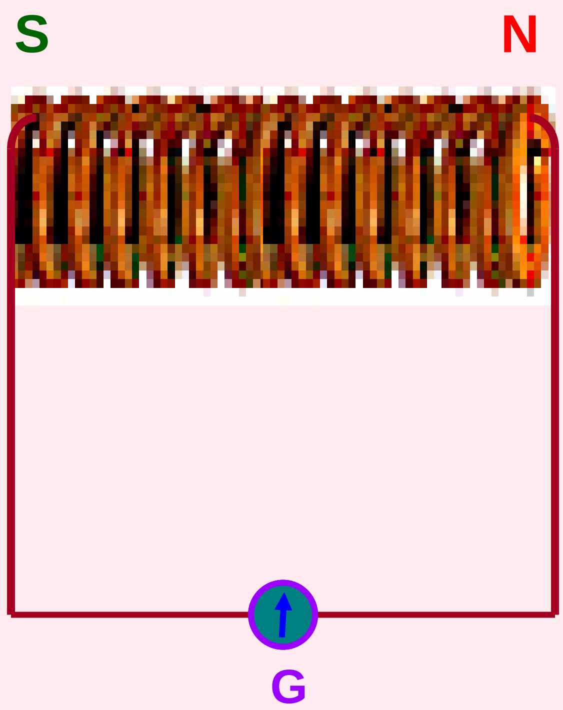
3) the orientation of the coil with magnetic field θ or

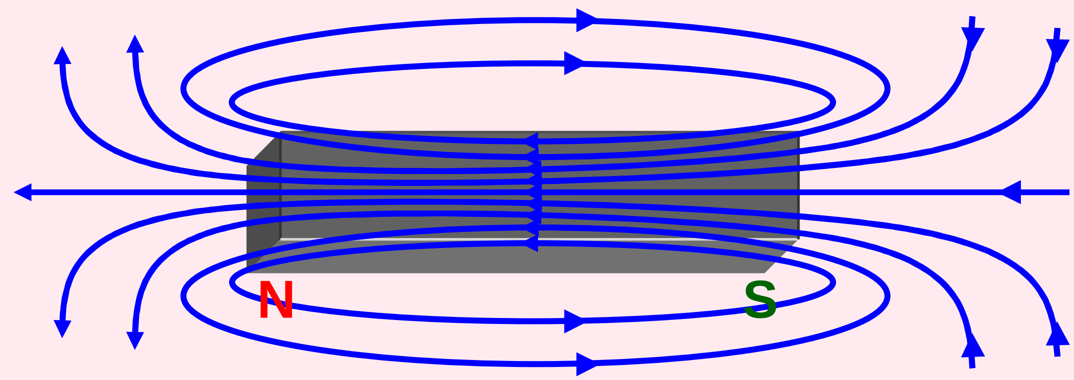
4) any of the combination of the above

* Magnetic flux is a scalar quantity.
* SI unit of magnetic flux is weber or tesla-metre^2 or (wb or Tm^2).
* cgs unit of magnetic flux is maxwell.
* 1 maxwell = 10^{-8} weber
* Magnetic flux (associated normally) per unit area is called Magnetic Flux Density or Strength of Magnetic Field or Magnetic Induction (B).

Faraday's Experiment - 1:







Magnetic flux linked with the coil changes relative to the positions of the coil and the magnet due to the magnetic lines of force cutting at different angles at the same cross sectional area of the coil.

Observe:

i) the relative motion between the coil and the magnet

ii) the induced polarities of magnetism in the coil

iii) the direction of current through the galvanometer and hence the deflection in the galvanometer

iv) that the induced current (e.m.f) is available only as long as there is relative motion between the coil and the magnet

Note:

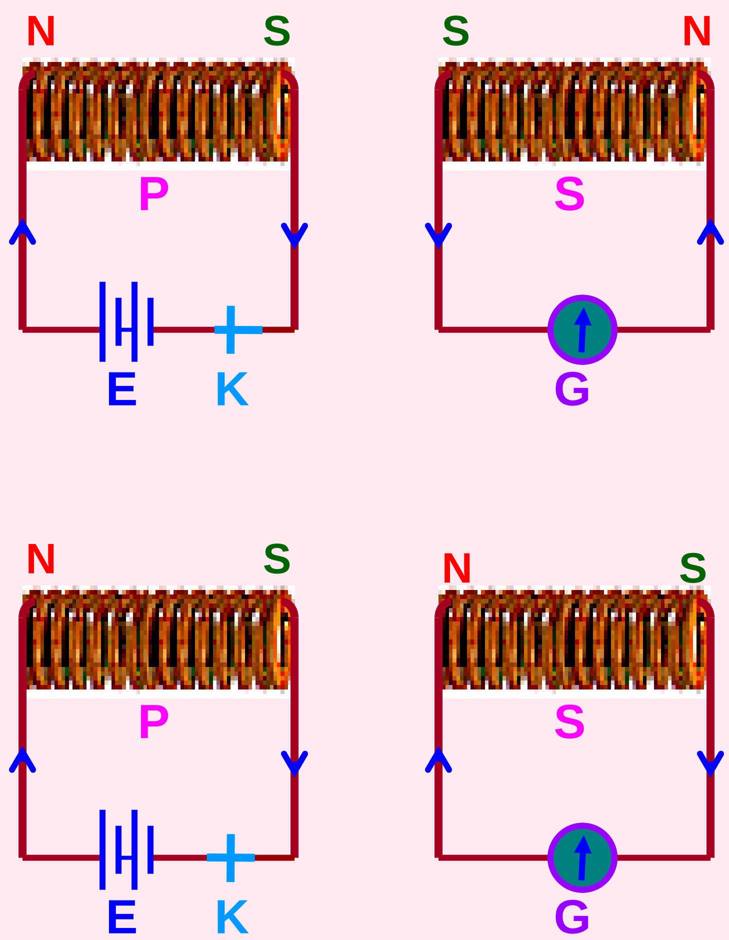
i) coil can be moved by fixing the magnet

ii) both the coil and magnet can be moved (towards each other or away from each other) i.e. there must be a relative velocity between them

iii) magnetic flux linked with the coil changes relative to the positions of the coil and the magnet

iv) current and hence the deflection is large if the relative velocity between the coil and the magnet and hence the rate of change of flux across the coil is more

Faraday's Experiment - 2:



When the primary circuit is closed current grows from zero to maximum value.

During this period changing, current induces changing magnetic flux across the primary coil.

This changing magnetic flux is linked across the secondary coil and induces e.m.f (current) in the secondary coil.

Induced e.m.f (current) and hence deflection in galvanometer lasts only as long as the current in the primary coil and hence the magnetic flux in the secondary coil change.

When the primary circuit is open current decreases from maximum value to zero.

During this period changing current induces changing magnetic flux across the primary coil.

This changing magnetic flux is linked across the secondary coil and induces current (e.m.f) in the secondary coil.

However, note that the direction of current in the secondary coil is reversed and hence the deflection in the galvanometer is opposite to the previous case.

Faraday's Laws of Electromagnetic Induction:

I Law:

Whenever there is a change in the magnetic flux linked with a circuit, an emf and hence a current is induced in the circuit. However, it lasts only so long as the magnetic flux is changing.

II Law:

The magnitude of the induced emf is directly proportional to the rate of change of magnetic flux linked with a circuit.

$E \propto d\Phi / dt \Longrightarrow E = k\, d\Phi / dt \Longrightarrow E = d\Phi / dt \Longrightarrow E = (\Phi_2 - \Phi_1) / t$

(where k is a constant and units are chosen such that k = 1)

Lenz's Law:

The direction of the induced emf or induced current is such that it opposes the change that is producing it.

i.e. If the current is induced due to motion of the magnet, then the induced current in the coil sets itself to stop the motion of the magnet.

If the current is induced due to change in current in the primary coil, then induced current is such that it tends to stop the change.

Lenz's Law and Law of Conservation of Energy:

According to Lenz's law, the induced emf opposes the change that produces it. It is this opposition against which we perform mechanical work in causing the change in magnetic flux. Therefore, mechanical energy is converted into electrical energy. Thus, Lenz's law is in accordance with the law of conservation of energy.

If, however, the reverse would happen (i.e. the induced emf does not oppose or aids the change), then a little change in magnetic flux would produce an induced current which would help the change of flux further thereby producing more current. The increased emf would then cause further change of flux and it would further increase the current and so on. This would create energy out of nothing which would violate the law of conservation of energy.

Expression for Induced emf based on both the laws:

$E = - d\Phi / dt$

$E = - (\Phi 2 – \Phi 1) / t$

And for ‘N’ no. of turns of the coil,

$E = - N\, d\Phi / dt$

$E = - N (\Phi 2 – \Phi 1) / t$

Expression for Induced current:

$I = - d\Phi / (R\, dt)$

Expression for Charge:

$dq / dt = - d\Phi / (R\, dt)$

$dq = - d\Phi / R$

Note:

Induced emf does not depend on resistance of the circuit where as the induced current and induced charge depend on resistance.

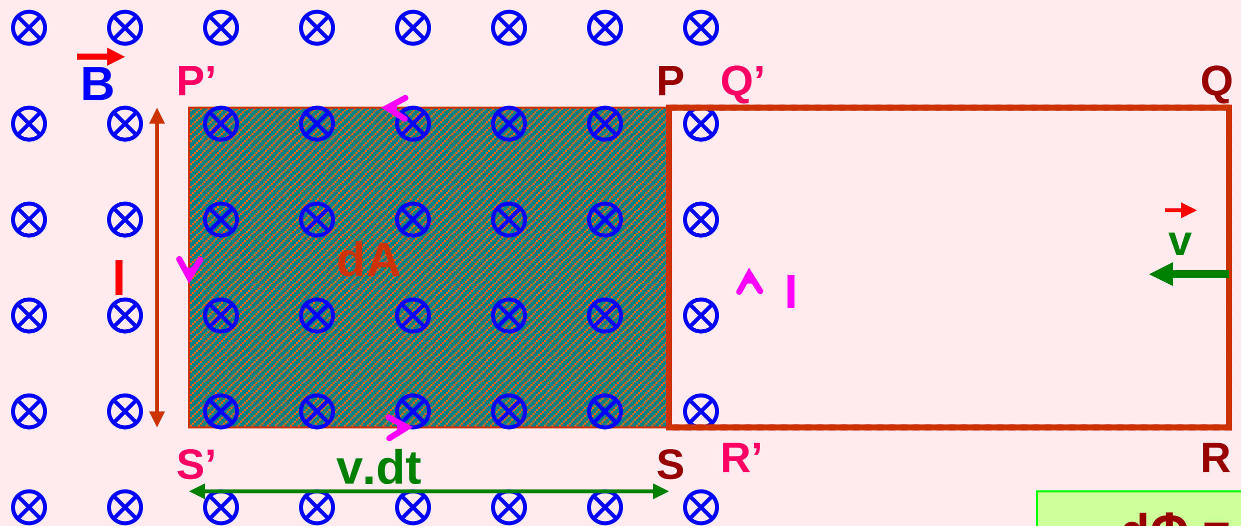
Methods of producing Induced emf:

1. **By changing Magnetic Field B:**

Magnetic flux Φ can be changed by changing the magnetic field B and hence emf can be induced in the circuit (as done in Faraday’s Experiments).

2. By changing the area of the coil A available in Magnetic Field:

Magnetic flux Φ can be changed by changing the area of the loop A which is acted upon by the magnetic field B and hence emf can be induced in the circuit.



The loop PQRS is slided into uniform and perpendicular magnetic field. The change (increase) in area of the coil under the influence of the field is dA in time dt. This causes an increase in magnetic flux dΦ.

$d\Phi = B.dA$

$= B.l.v.dt$

$E = - d\Phi / dt$

$\therefore E = - Blv$

The induced emf is due to motion of the loop and so it is called 'motional emf'.

If the loop is pulled out of the magnetic field, then $E = Blv$

The direction of induced current is anticlockwise in the loop. i.e. P'S'R'Q'P' by Fleming's Right Hand Rule or Lenz's Rule.

According Lenz's Rule, the direction of induced current is such that it opposes the cause of changing magnetic flux.

Here, the cause of changing magnetic flux is due to motion of the loop and increase in area of the coil in the uniform magnetic field.

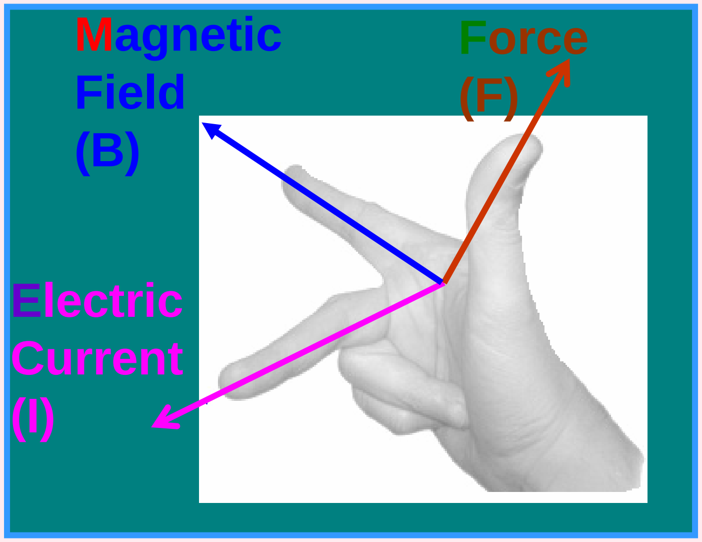
Therefore, this motion of the loop is to be opposed. So, the current is setting itself such that by Fleming's Left Hand Rule, the conductor arm PS experiences force to the right whereas the loop is trying to move to the left.

Against this force, mechanical work is done which is converted into electrical energy (induced current).

NOTE: If the loop is completely inside the boundary of magnetic field, then there will not be any change in magnetic flux and so there will not be induced current in the loop.

Fleming's Right Hand Rule:

If the central finger, fore finger and thumb of right hand are stretched mutually perpendicular to each other and the fore finger points to magnetic field, thumb points in the direction of motion (force), then central finger points to the direction of induced current in the conductor.



3. By changing the orientation of the coil (θ) in Magnetic Field:

Magnetic flux Φ can be changed by changing the relative orientation of the loop (θ) with the magnetic field B and hence emf can be induced in the circuit.

$\Phi = N\,B\,A\cos\theta$

At time t, with angular velocity ω,

$\theta = \omega t$ (at t = 0, loop is assumed to be perpendicular to the magnetic field and θ = 0°)

$\therefore\ \Phi = N\,B\,A\cos\omega t$

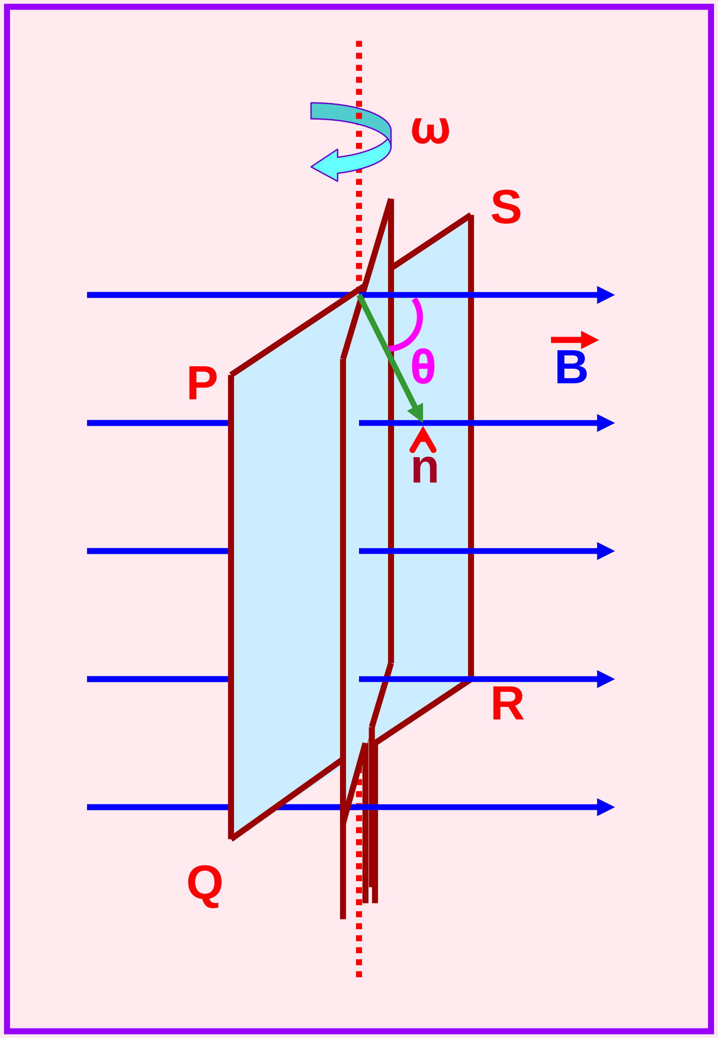
Differentiating w.r.t. t,

$d\Phi / dt = -NBA\omega \sin\omega t$

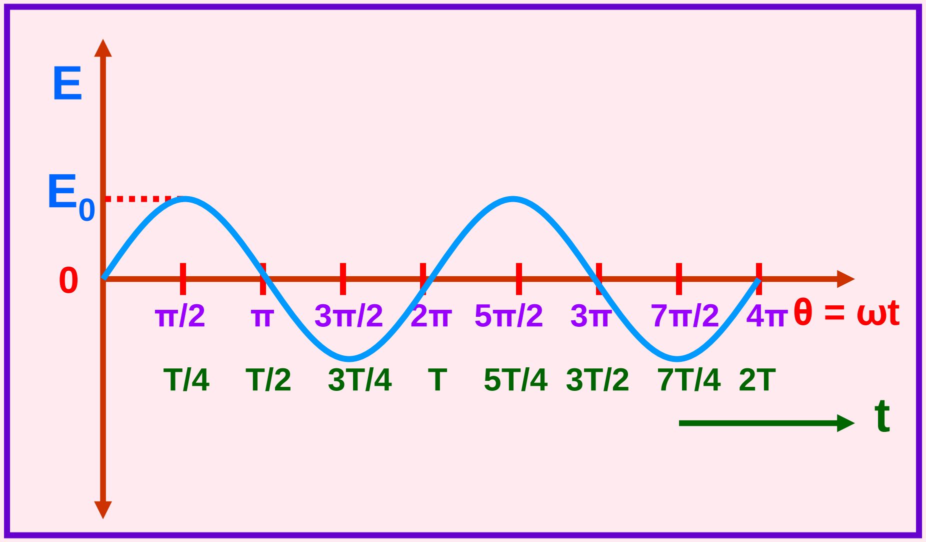
$E = -d\Phi / dt$

$E = NBA\omega \sin\omega t$

$E = E_0 \sin\omega t$ (where $E_0 = NBA\omega$ is the maximum emf)



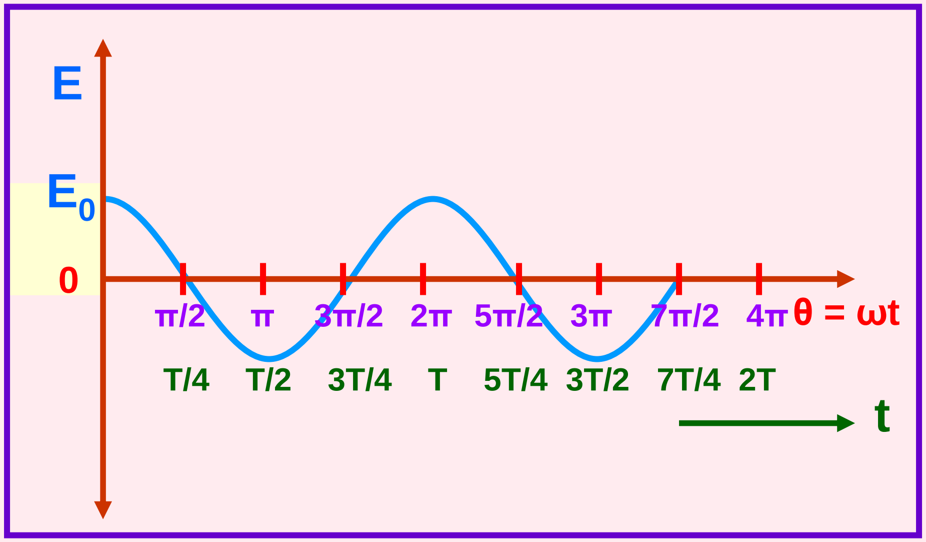
The emf changes continuously in magnitude and periodically in direction w.r.t. time giving rise to alternating emf.



If initial position of the coil is taken as 0°, i.e. normal to the coil is at 90° with the magnetic field, then

θ becomes θ + π/2 or ωt + π/2

$\therefore E = E_0 \cos \omega t$

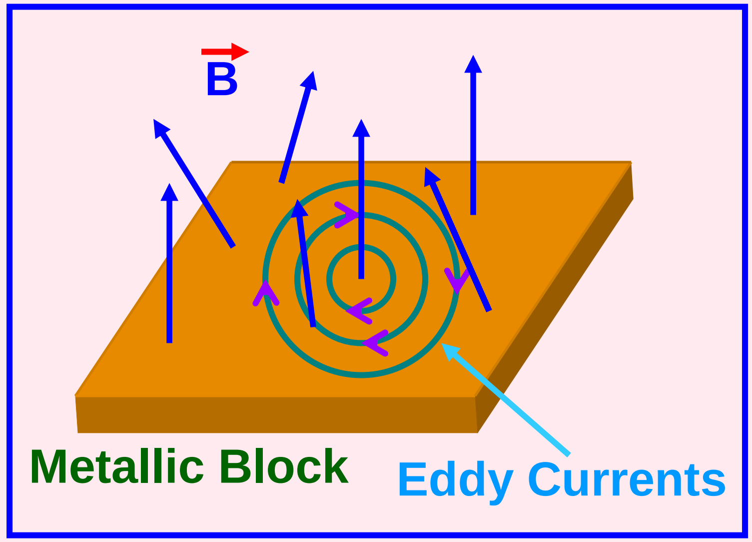


So, alternating emf and consequently alternating current can be expressed in sin or cos function.

This method of inducing emf is the basic principle of generators.

Eddy Currents or Foucault Currents:

The induced circulating (looping) currents produced in a solid metal due to change in magnetic field (magnetic flux) in the metal are called eddy currents.



Applications of Eddy Currents:

1. In induction furnace eddy currents are used for melting iron ore, etc.
2. In speedometer eddy currents are used to measure the instantaneous speed of the vehicle.
3. In dead beat galvanometer eddy currents are used to stop the damping of the coil in a shorter interval.
4. In electric brakes of the train eddy currents are produced to stop the rotation of the axle of the wheel.
5. In energy meters (watt – meter) eddy currents are used to measure the consumption of electric energy.
6. In diathermy eddy currents are used for localised heating of tissues in human bodies.

Self Induction:

Self Induction is the phenomenon of inducing emf in the self coil due to change in current and hence the change in magnetic flux in the coil.

The induced emf opposes the growth or decay of current in the coil and hence delays the current to acquire the maximum value.

Self induction is also called inertia of electricity as it opposes the growth or decay of current.

Self Inductance:

$\Phi \propto I$ or $\Phi = LI$

(where L is the constant of proportionality and is known as Self Inductance or co-efficient of self induction)

If $I = 1$, then $L = \Phi$

Thus, self inductance is defined as the magnetic flux linked with a coil when unit current flows through it.

Also, $E = - d\Phi / dt$ or $E = - L (dI / dt)$

If $dI / dt = 1$, then $L = E$

Thus, self inductance is defined as the induced emf set up in the coil through which the rate of change of current is unity.

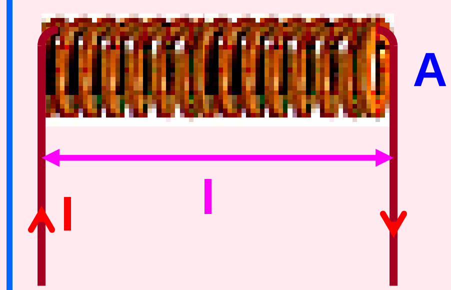
SI unit of self inductance is henry (H).

Self inductance is said to be 1 henry when 1 A current in a coil links magnetic flux of 1 weber.

or

Self inductance is said to be 1 henry when unit rate of change of current (1 A / s) induces emf of 1 volt in the coil.

Self inductance of a solenoid:



Magnetic Field due to the solenoid is

$B = \mu_0 nI$

Magnetic Flux linked across one turn of the coil is

Φ per turn $= B\,A = \mu_0 nIA = \mu_0 NIA / l$

Magnetic Flux linked across N turns of the coil is

$\Phi = \mu_0 N^2 IA / l$

But, $\Phi = LI$

So, $L = \mu_0 N^2 A / l = \mu_0 n^2 Al$

Energy in Inductor:

Small work done dW in establishing a current I in the coil in time dt is $dW = -EI\,dt$

$dW = LI\,dI$ (since $E = -L(dI / dt)$)

$$W = \int_0^{I_0} L\,I\,dI = \frac{1}{2} L I_0^2$$

Mutual Induction:

Mutual Induction is the phenomenon of inducing emf in the secondary coil due to change in current in the primary coil and hence the change in magnetic flux in the secondary coil.

Mutual Inductance:

$\Phi_{21} \propto I_1$ or $\Phi_{21} = MI_1$

(where M is the constant of proportionality and is known as Mutual Inductance or co-efficient of mutual induction)

If $I_1 = 1$, then $M = \Phi$

Thus, mutual inductance is defined as the magnetic flux linked with the secondary coil when unit current flows through the primary coil.

Also, $E_2 = - d\Phi_{21} / dt$ or $E_2 = - M (dI_1 / dt)$

If $dI_1 / dt = 1$, then $M = E$

Thus, mututal inductance is defined as the induced emf set up in the secondary coil when the rate of change of current in primary coil is unity.

SI unit of mututal inductance is henry (H).

Mutual inductance is said to be 1 henry when 1 A current in the primary coil links magnetic flux of 1 weber across the secondary coil. or

Mutual inductance is said to be 1 henry when unit rate of change of current (1 A / s) in primary coil induces emf of 1 volt in the secondary coil.

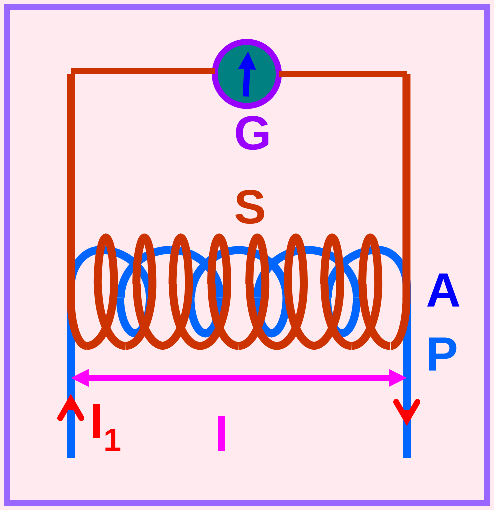
Mutual inductance of two long co-axial solenoids:

Magnetic Field due to primary solenoid is

$B_1 = \mu_0 n_1 I_1$

Magnetic Flux linked across one turn of the secondary solenoid is

Φ_{21} per turn $= B_1 A = \mu_0 n_1 I_1 A = \mu_0 N_1 I_1 A / l$



Magnetic Flux linked across N turns of the secondary solenoid is

$$\Phi_{21} = \mu_0 N_1 N_2 I_1 A / l$$

But, $\Phi_{21} = M_{21} I_1$

$$M_{21} = \mu_0 N_1 N_2 A / l = \mu_0 n_1 n_2 A l$$

IIIly $\quad M_{12} = \mu_0 N_1 N_2 A / l = \mu_0 n_1 n_2 A l$

∴ For two long co-axial solenoids of same length and cross-sectional area, the mutual inductance is same and leads to principle of reciprocity.

$$M = M_{12} = M_{21}$$

Additional Information:

1) If the two solenoids are wound on a magnetic core of relative permeability μ_r, then

   $M = \mu_0 \mu_r N_1 N_2 A / l$

2) If the solenoids S_1 and S_2 have no. of turns N_1 and N_2 of different radii r_1 and r_2 ($r_1 < r_2$), then

   $M = \mu_0 \mu_r N_1 N_2 (\pi r_1^2) / l$

3) Mutual inductance depends also on the relative placement of the solenoids.

4) Co-efficient of Coupling (K) between two coils having self-inductance L_1 and L_2 and mutual inductance M is

   $K = M / (\sqrt{L_1 L_2})$ Generally, $K < 1$

5) If L_1 and L_2 are in series, then $L = L_1 + L_2$

6) If L_1 and L_2 are in parallel, then $(1/L) = (1/L_1) + (1/L_2)$

ALTERNATING CURRENTS

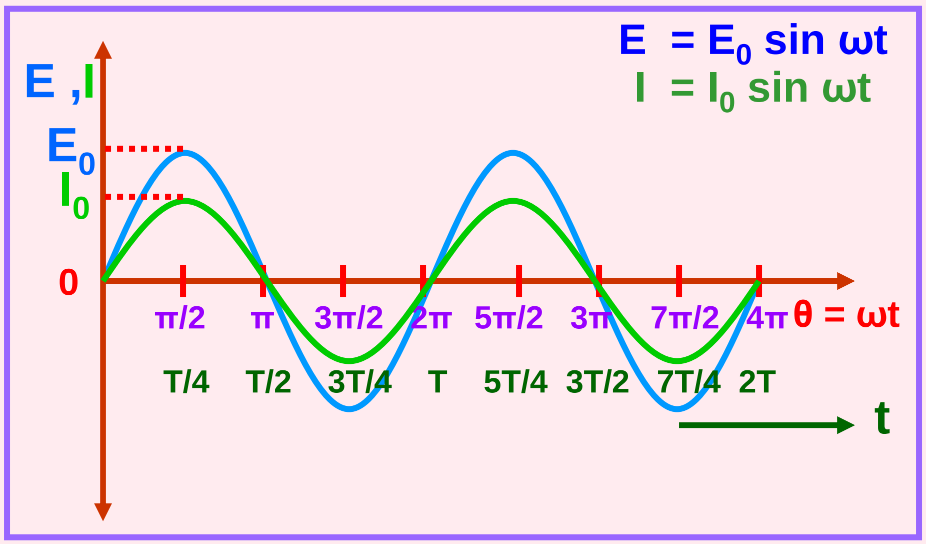
1. Alternating EMF and Current
2. Average or Mean Value of Alternating EMF and Current
3. Root Mean Square Value of Alternating EMF and Current
4. A C Circuit with Resistor
5. A C Circuit with Inductor
6. A C Circuit with Capacitor
7. A C Circuit with Series LCR – Resonance and Q-Factor
8. Graphical Relation between Frequency vs X_L, X_C
9. Power in LCR A C Circuit
10. Watt-less Current
11. L C Oscillations
12. Transformer
13. A.C. Generator

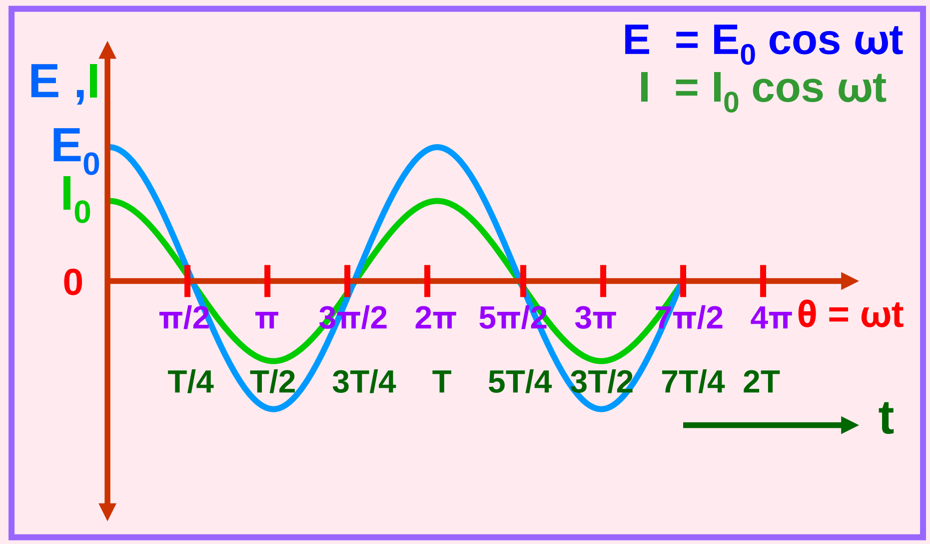
Alternating emf:

Alternating emf is that emf which continuously changes in magnitude and periodically reverses its direction.

Alternating Current:

Alternating current is that current which continuously changes in magnitude and periodically reverses its direction.





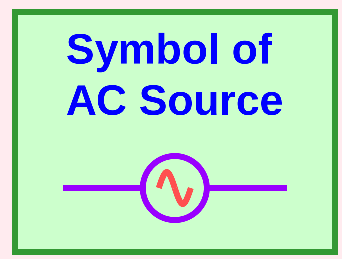
E, I – Instantaneous value of emf and current

E_0, I_0 – Peak or maximum value or amplitude of emf and current

ω – Angular frequency

t – Instantaneous time

ωt – Phase



Average or Mean Value of Alternating Current:

Average or Mean value of alternating current over half cycle is that steady current which will send the same amount of charge in a circuit in the time of half cycle as is sent by the given alternating current in the same circuit in the same time.

$dq = I\,dt = I_0 \sin \omega t\,dt$

$$q = \int_0^{T/2} I_0 \sin \omega t\,dt$$

$q = 2 I_0 / \omega \quad = \quad 2 I_0 T / 2\pi \quad = \quad I_0 T / \pi$

Mean Value of AC, $I_m = I_{av} \;=\; q / (T/2)$

$$I_m = I_{av} \;=\; 2 I_0 / \pi \;=\; 0.637\, I_0 \;=\; 63.7\ \%\ I_0$$

Average or Mean Value of Alternating emf:

$$E_m = E_{av} \;=\; 2 E_0 / \pi \;=\; 0.637\, E_0 \;=\; 63.7\ \%\ E_0$$

Note: Average or Mean value of alternating current or emf is zero over a cycle as the + ve and – ve values get cancelled.

Root Mean Square or Virtual or Effective Value of Alternating Current:

Root Mean Square (rms) value of alternating current is that steady current which would produce the same heat in a given resistance in a given time as is produced by the given alternating current in the same resistance in the same time.

$$dH = I^2R\,dt = I_0^2\,R\,\sin^2 \omega t\,dt$$

$$H = \int_0^T I_0^2\,R\,\sin^2 \omega t\,dt$$

$$H = I_0^2\,RT / 2$$ (After integration, ω is replaced with 2 π / T)

If I_v be the virtual value of AC, then

$$H = I_v^{\,2}\,RT$$

$\therefore$ $I_v = I_{rms} = I_{eff} = I_0 / \sqrt{2} = 0.707\,I_0 = 70.7\,\%\,I_0$

Root Mean Square or Virtual or Effective Value of Alternating emf:

$E_v = E_{rms} = E_{eff} = E_0 / \sqrt{2} = 0.707\,E_0 = 70.7\,\%\,E_0$

Note:

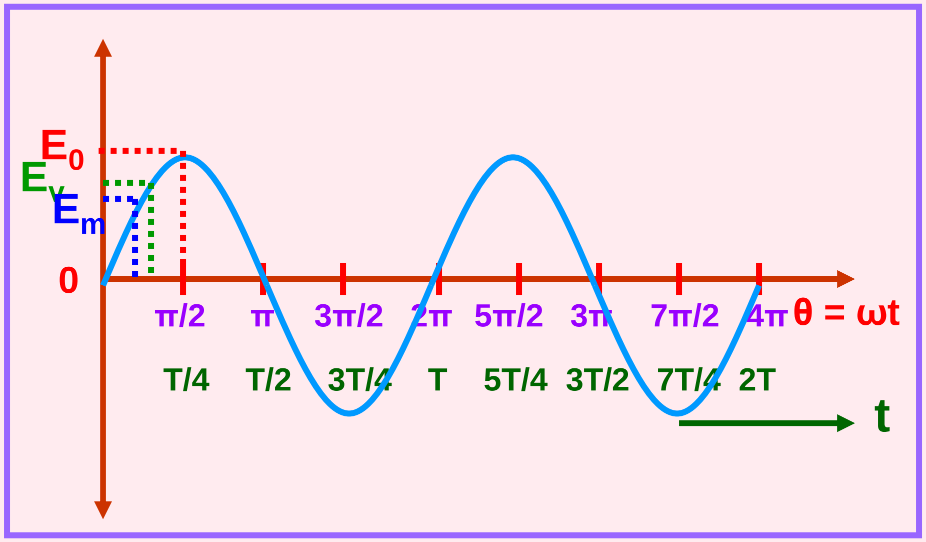
1. Root Mean Square value of alternating current or emf can be calculated over any period of the cycle since it is based on the heat energy produced.

2. Do not use the above formulae if the time interval under the consideration is less than one period.

Relative Values Peak, Virtual and Mean Values of Alternating emf:

$E_m = E_{av} = 0.637\ E_0$

$E_v = E_{rms} = E_{eff} = 0.707\ E_0$



Tips:

1. The given values of alternating emf and current are virtual values unless otherwise specified.

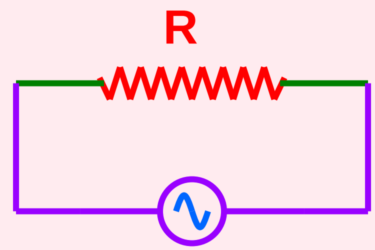
   i.e. 230 V AC means $E_v = E_{rms} = E_{eff} = 230$ V

2. AC Ammeter and AC Voltmeter read the rms values of alternating current and voltage respectively.

   They are called as 'hot wire meters'.

3. The scale of DC meters is linearly graduated where as the scale of AC meters is not evenly graduated because $H \propto I^2$

AC Circuit with a Pure Resistor:



$E = E_0 \sin \omega t$

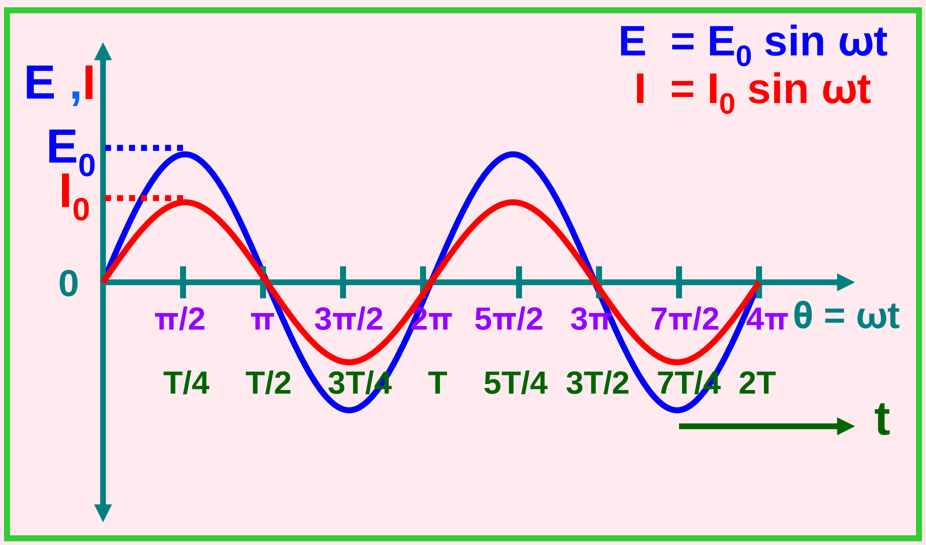
$E = E_0 \sin \omega t$

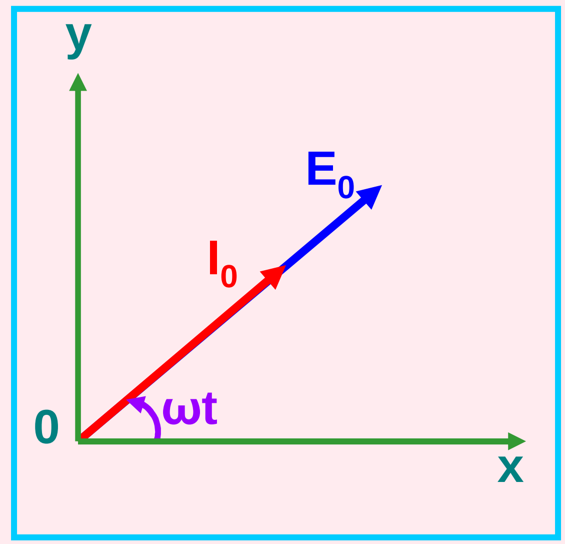
$I = E / R$

$= (E_0 / R) \sin \omega t$

$I = I_0 \sin \omega t$ (where $I_0 = E_0 / R$ and $R = E_0 / I_0$)

Emf and current are in same phase.

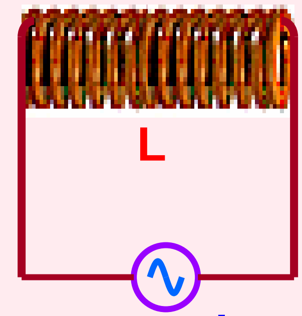




AC Circuit with a Pure Inductor:

$E = E_0 \sin \omega t$

Induced emf in the inductor is $- L\,(dI / dt)$



$E = E_0 \sin \omega t$

In order to maintain the flow of current, the applied emf must be equal and opposite to the induced emf.

$$\therefore \quad E = L\,(dI / dt)$$

$$E_0 \sin \omega t = L\,(dI / dt)$$

$$dI = (E_0 / L) \sin \omega t \, dt$$

⇒

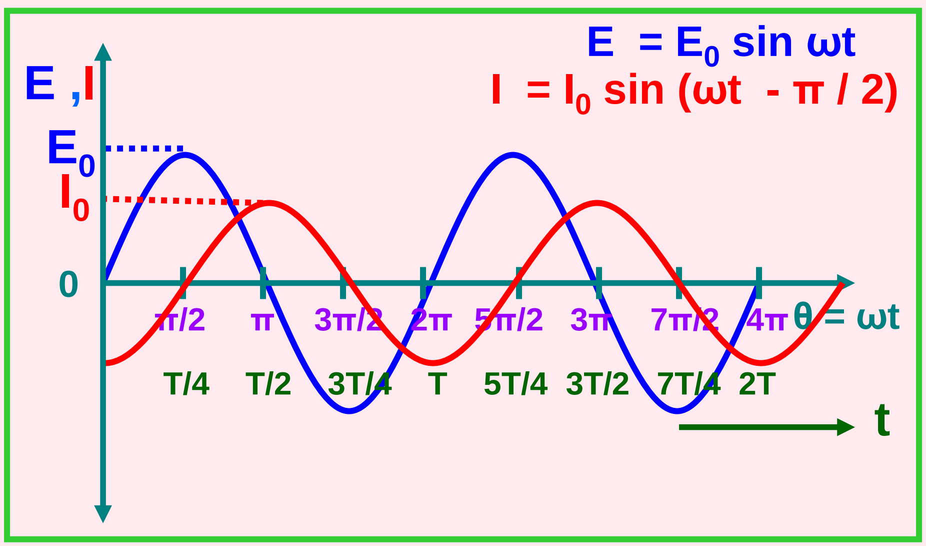
$$I = \int (E_0 / L) \sin \omega t \, dt$$

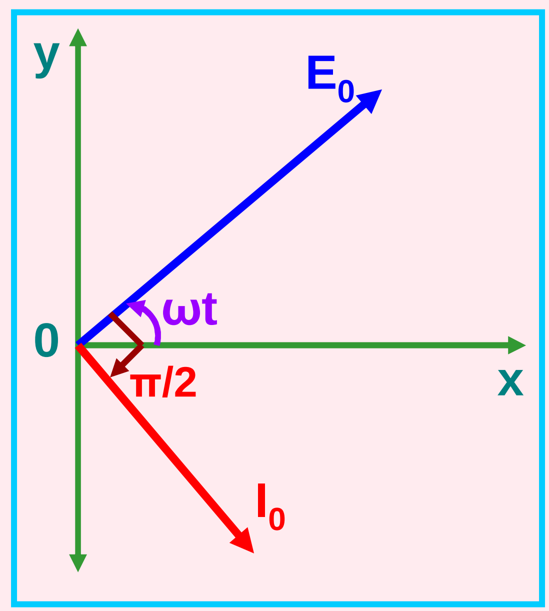
$$I = (E_0 / \omega L)\,(- \cos \omega t)$$

$$I = I_0 \sin (\omega t - \pi / 2)$$

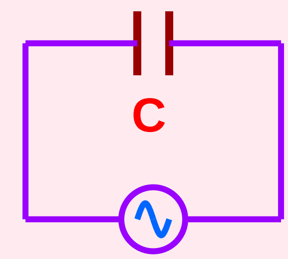
(where $I_0 = E_0 / \omega L$ and $X_L = \omega L = E_0 / I_0$) Current lags behind emf by $\pi/2$ rad.

X_L is Inductive Reactance. Its SI unit is ohm.





AC Circuit with a Capacitor:



$E = E_0 \sin \omega t$

$q = CE = CE_0 \sin \omega t$

$I = dq / dt$

$= (d / dt) [CE_0 \sin \omega t]$

$I = [E_0 / (1 / \omega C)] (\cos \omega t)$

$I = I_0 \sin (\omega t + \pi / 2)$

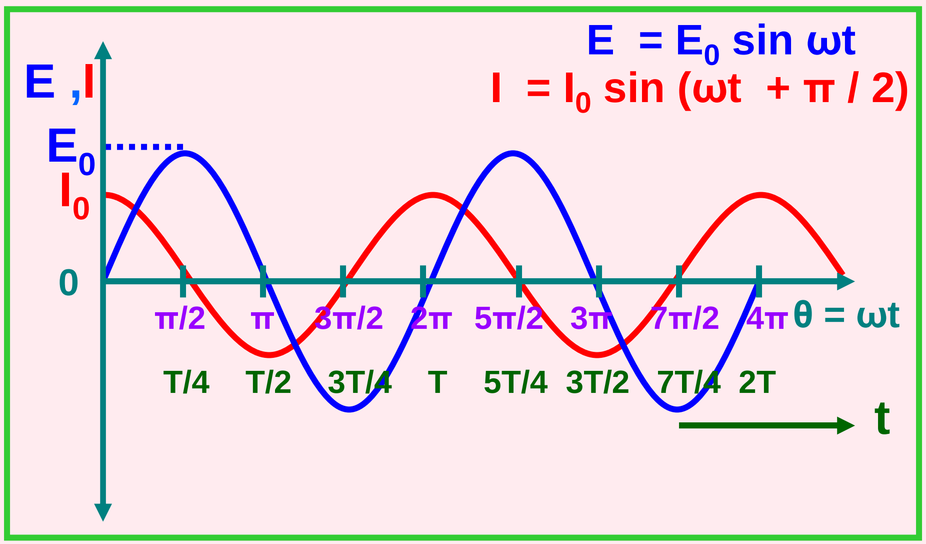
(where $I_0 = E_0 / (1 / \omega C)$ and

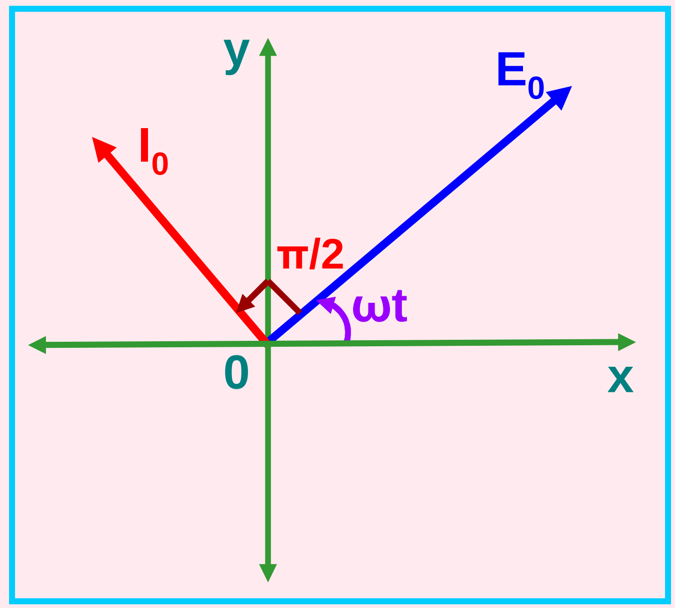
$X_C = 1 / \omega C = E_0 / I_0$)

X_C is Capacitive Reactance.

Its SI unit is ohm.

Current leads the emf by π/2 radians.



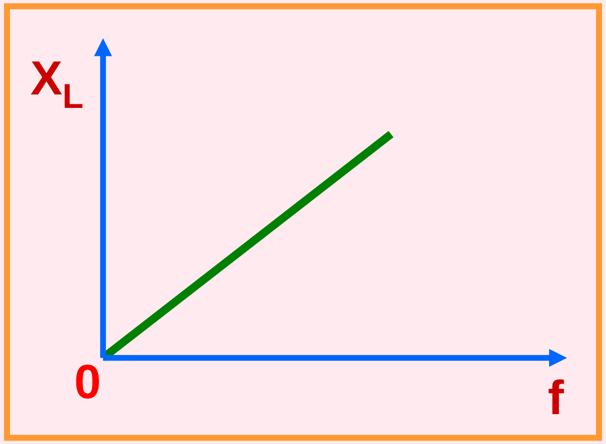


Variation of X_L with Frequency:

$I_0 = E_0 / \omega L$ and $X_L = \omega L$

X_L is Inductive Reactance and $\omega = 2\pi f$

$X_L = 2\pi f L$ i.e. $X_L \propto f$

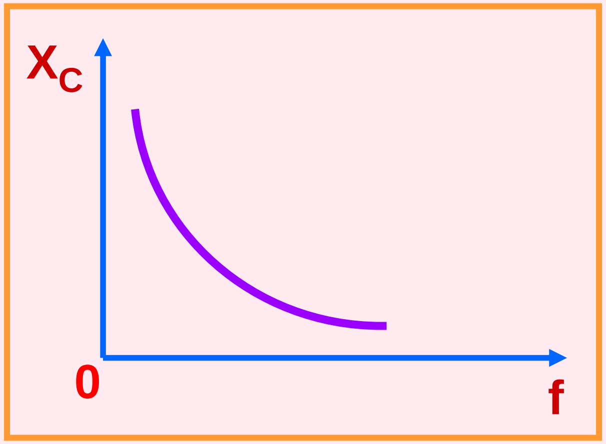


Variation of X_C with Frequency:

$I_0 = E_0 / (1/\omega C)$ and $X_C = 1 / \omega C$

X_C is Inductive Reactance and $\omega = 2\pi f$

$X_C = 1 / 2\pi f C$ i.e. $X_C \propto 1 / f$



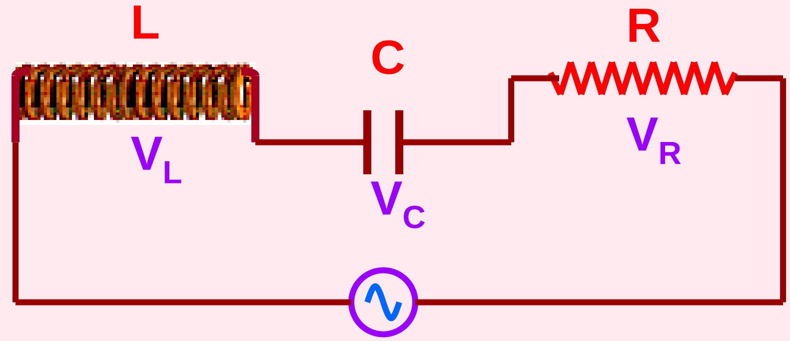
TIPS:

1) Inductance (L) can not decrease Direct Current. It can only decrease Alternating Current.
2) Capacitance (C) allows AC to flow through it but blocks DC.

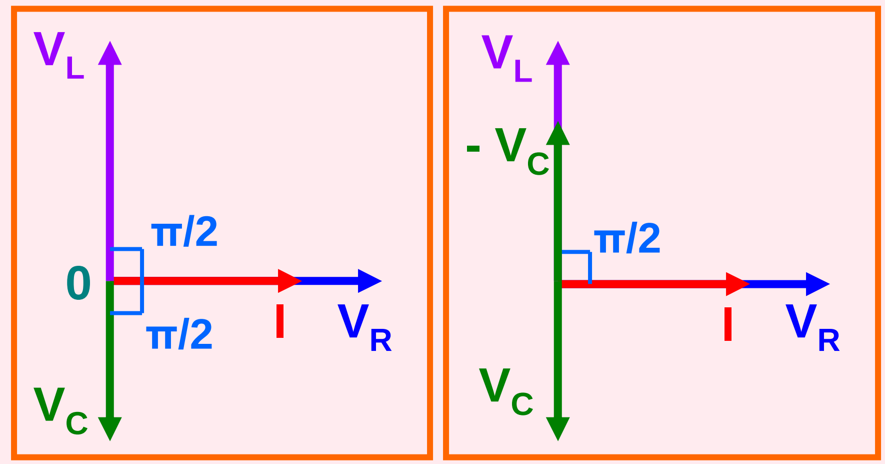
AC Circuit with L, C, R in Series Combination:

The applied emf appears as Voltage drops V_R, V_L and V_C across R, L and C respectively.

1) In R, current and voltage are in phase.
2) In L, current lags behind voltage by π/2
3) In C, current leads the voltage by π/2



$E = E_0 \sin \omega t$

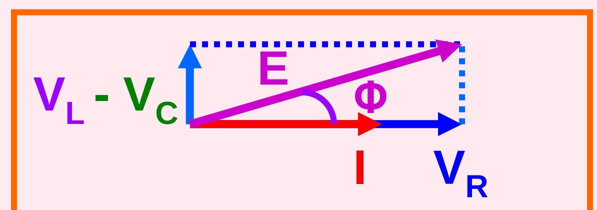


$$E = \sqrt{[V_R^2 + (V_L - V_C)^2]}$$

$$I = \frac{E}{\sqrt{[R^2 + (X_L - X_C)^2]}}$$

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$

$$Z = \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}$$



$$E = \sqrt{[V_R^2 + (V_L - V_C)^2]}$$

$$\tan \Phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \tan \Phi = \frac{\omega L - 1/\omega C}{R}$$

$$\tan \Phi = \frac{X_L - X_C}{R}$$ or $$\tan \Phi = \frac{\omega L - 1/\omega C}{R}$$

Special Cases:

Case I: When $X_L > X_C$ i.e. $\omega L > 1/\omega C$,

$\tan \Phi$ = +ve or Φ is +ve

The current lags behind the emf by phase angle Φ and the LCR circuit is inductance - dominated circuit.

Case II: When $X_L < X_C$ i.e. $\omega L < 1/\omega C$,

$\tan \Phi$ = -ve or Φ is -ve

The current leads the emf by phase angle Φ and the LCR circuit is capacitance - dominated circuit.

Case III: When $X_L = X_C$ i.e. $\omega L = 1/\omega C$,

$\tan \Phi = 0$ or Φ is $0°$

The current and the emf are in same phase. The impedance does not depend on the frequency of the applied emf. LCR circuit behaves like a purely resistive circuit.

Resonance in AC Circuit with L, C, R:

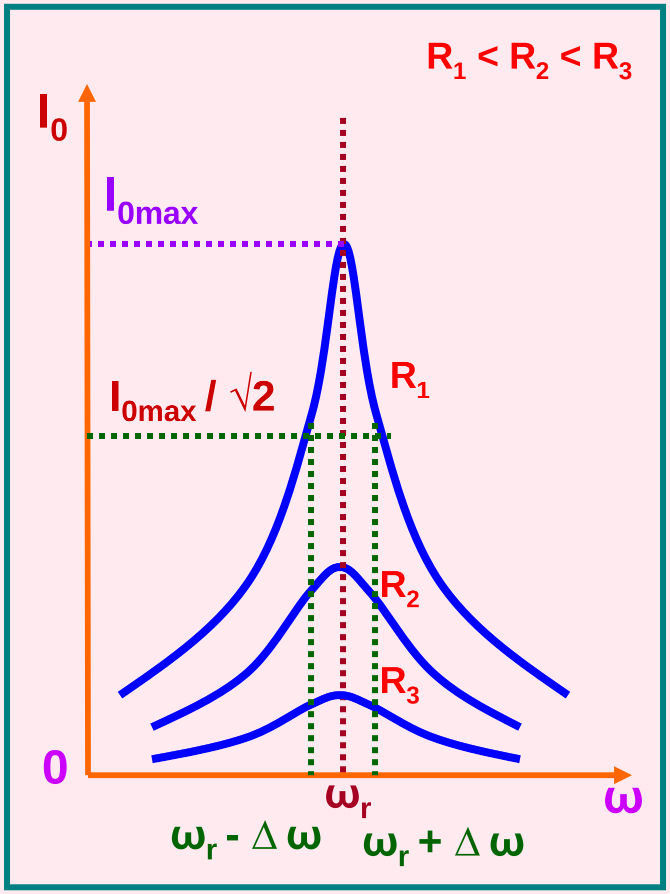
When $X_L = X_C$ i.e. $\omega L = 1/\omega C$, $\tan \Phi = 0$ or Φ is 0° and

$Z = \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}$ becomes $Z_{min} = R$ and $I_{0max} = E / R$

i.e. The impedance offered by the circuit is minimum and the current is maximum. This condition is called resonant condition of LCR circuit and the frequency is called resonant frequency.

At resonant angular frequency ω_r,

$\omega_r L = 1/\omega_r C$ or $\omega_r = 1 / \sqrt{LC}$ or $f_r = 1 / (2\pi \sqrt{LC})$



Resonant Curve & Q - Factor:

Band width = $2 \Delta \omega$

Quality factor (Q – factor) is defined as the ratio of resonant frequency to band width.

$$Q = \omega_r / 2 \Delta \omega$$

It can also be defined as the ratio of potential drop across either the inductance or the capacitance to the potential drop across the resistance.

$Q = V_L / V_R$ or $Q = V_C / V_R$

or $Q = \omega_r L / R$ or $Q = 1 / \omega_r CR$

Power in AC Circuit with L, C, R:

$E = E_0 \sin \omega t$

$I = I_0 \sin (\omega t + \Phi)$ (where Φ is the phase angle between emf and current)

Instantaneous Power = E I

$= E_0 I_0 \sin \omega t \sin (\omega t + \Phi)$

$= E_0 I_0 [\sin^2 \omega t \cos\Phi + \sin \omega t \cos\omega t \cos\Phi]$

If the instantaneous power is assumed to be constant for an infinitesimally small time dt, then the work done is

$dW = E_0 I_0 [\sin^2 \omega t \cos\Phi + \sin \omega t \cos\omega t \cos\Phi]$

Work done over a complete cycle is

$$W = \int_0^T E_0 I_0 [\sin^2 \omega t \cos\Phi + \sin \omega t \cos\omega t \cos\Phi]\, dt$$

$W = E_0 I_0 \cos \Phi \times T / 2$

Average Power over a cycle is $P_{av} = W / T$

$P_{av} = (E_0 I_0 / 2) \cos \Phi$

$P_{av} = (E_0/\sqrt{2})(I_0/\sqrt{2}) \cos \Phi$

$P_{av} = E_v I_v \cos \Phi$

(where $\cos \Phi = R / Z$

$= R / \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}$

is called Power Factor)

$P_{av} = E_v\, I_v \cos \Phi$

Power in AC Circuit with R:

In R, current and emf are in phase.

$\Phi = 0°$

$P_{av} = E_v\, I_v \cos \Phi = E_v\, I_v \cos 0° = E_v\, I_v$

Power in AC Circuit with L:

In L, current lags behind emf by π/2.

$\Phi = -\ \pi/2$

$P_{av} = E_v\, I_v \cos(-\pi/2) = E_v\, I_v\, (0) = 0$

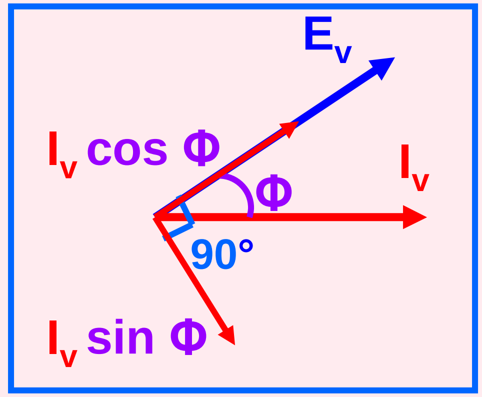
Power in AC Circuit with C:

In C, current leads emf by π/2.

$\Phi = +\ \pi/2$

$P_{av} = E_v\, I_v \cos(\pi/2) = E_v\, I_v\, (0) = 0$

Wattless Current or Idle Current:



The component $I_v \cos \Phi$ generates power with E_v.

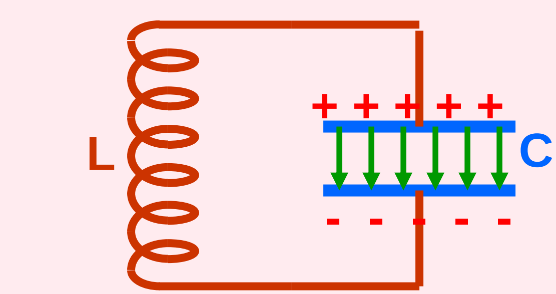
However, the component $I_v \sin \Phi$ does not contribute to power along E_v and hence power generated is zero. This component of current is called wattless or idle current.

$P = E_v\, I_v \sin \Phi \cos 90° = 0$

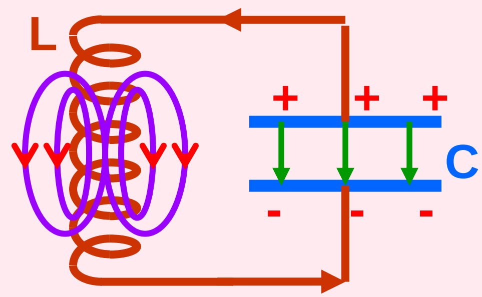
Note:

Power (Energy) is not dissipated in Inductor and Capacitor and hence they find a lot of practical applications and in devices using alternating current.

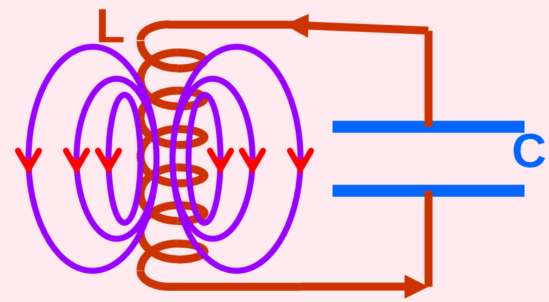
L C Oscillations:



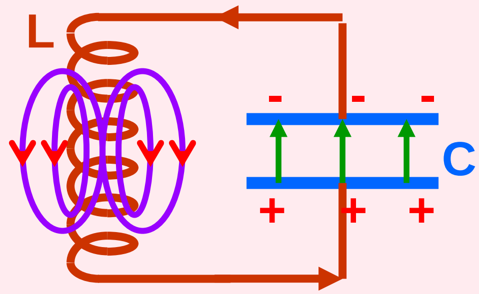
At t = 0, U_E=Max. & U_B=0



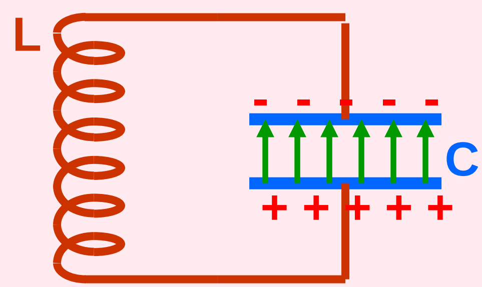
At t = T/8, U_E = U_B



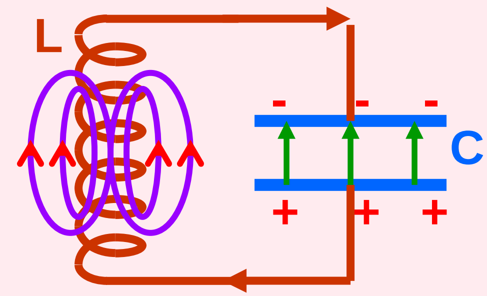
At t = 2T/8, U_E=0 & U_B=Max.



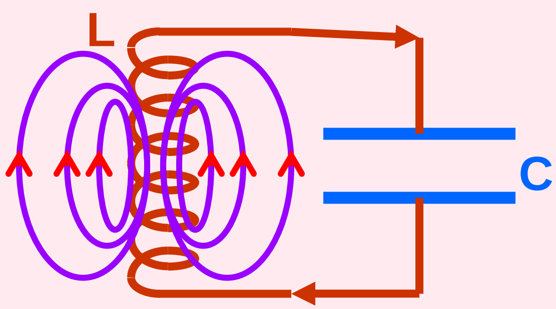
At t =3T/8, U_E = U_B



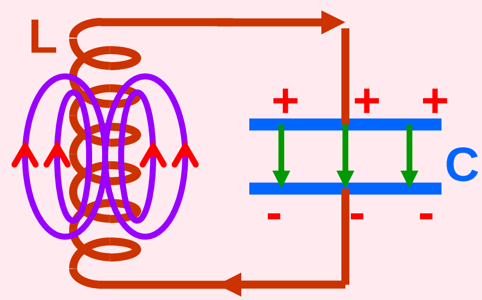
At t = 4T/8, U_E=Max. & U_B=0



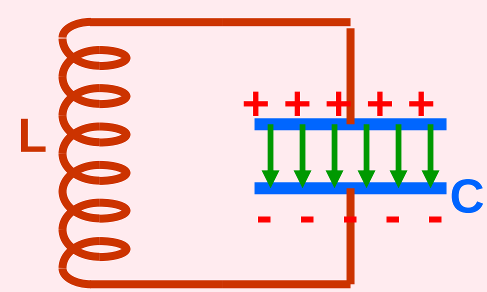
At t =5T/8, U_E = U_B



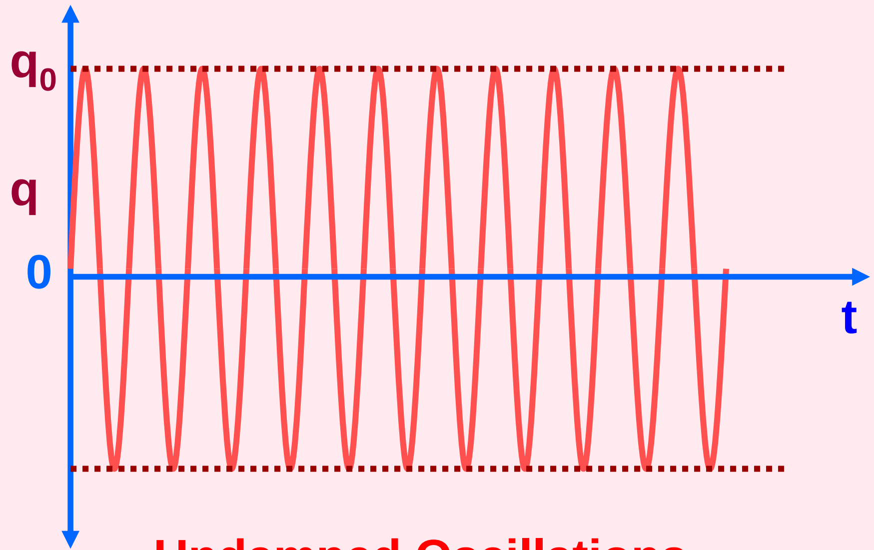
At t = 6T/8, U_E=0 & U_B=Max.



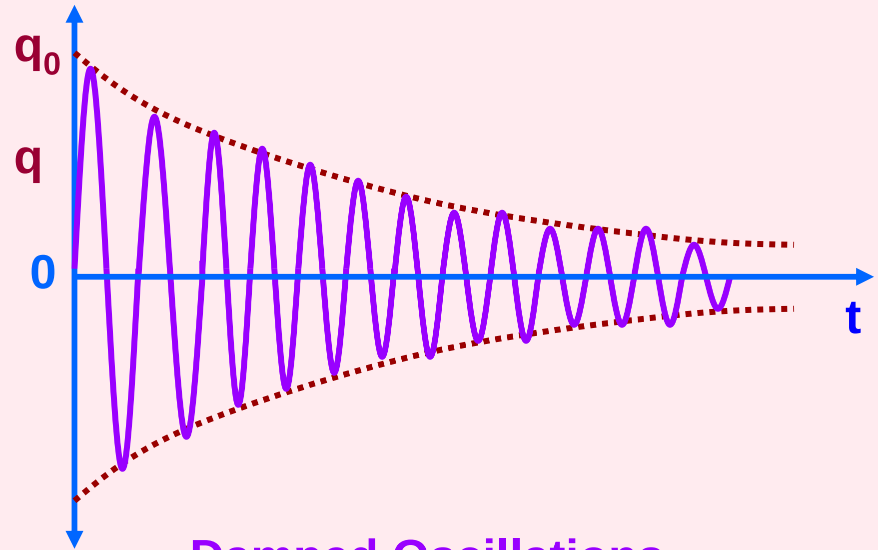
At t =7T/8, U_E = U_B



At t =T, U_E=Max. & U_B=0



Undamped Oscillations



Damped Oscillations

If q be the charge on the capacitor at any time t and dI / dt the rate of change of current, then

$$L\, dI / dt + q / C = 0$$

or $$L\,(d^2q / dt^2) + q / C = 0$$

or $$d^2q / dt^2 + q / (LC) = 0$$

Putting $1 / LC = \omega^2$

$$d^2q / dt^2 + \omega^2 q = 0$$

The final equation represents Simple Harmonic Electrical Oscillation with ω as angular frequency.

So, $$\omega = 1 / \sqrt{LC}$$

or $$f = \frac{1}{2\pi \sqrt{LC}}$$

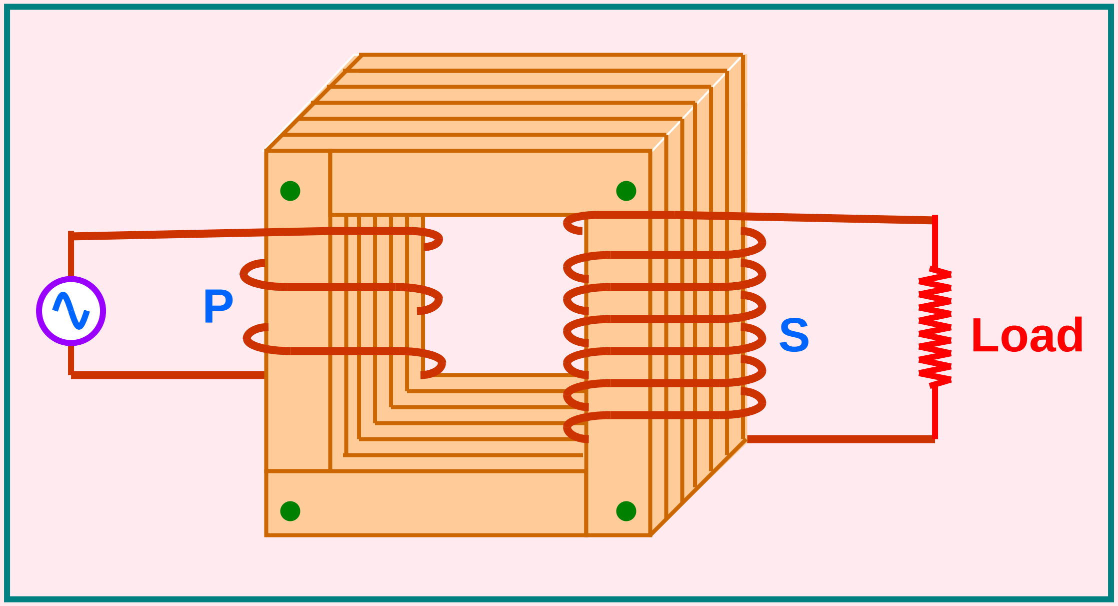
Transformer:

Transformer is a device which converts lower alternating voltage at higher current into higher alternating voltage at lower current.

Principle:

Transformer is based on Mutual Induction.

It is the phenomenon of inducing emf in the secondary coil due to change in current in the primary coil and hence the change in magnetic flux in the secondary coil.



Theory:

$E_P = - N_P \, d\Phi / dt$

$E_S = - N_S \, d\Phi / dt$

$E_S / E_P = N_S / N_P = K$

(where K is called Transformation Ratio or Turns Ratio)

For an ideal transformer,

Output Power = Input Power

$E_S I_S = E_P I_P$

$E_S / E_P = I_P / I_S$

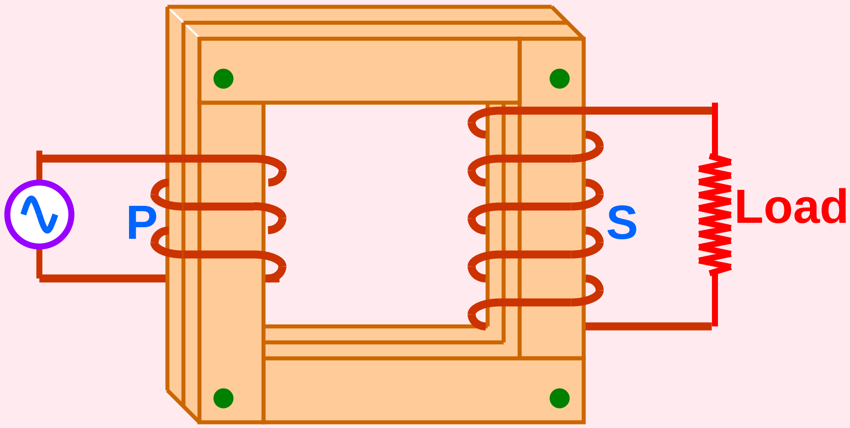
$E_S / E_P = I_P / I_S = N_S / N_P$

Efficiency (η):

$\eta = E_S I_S / E_P I_P$

For an ideal transformer η is 100%

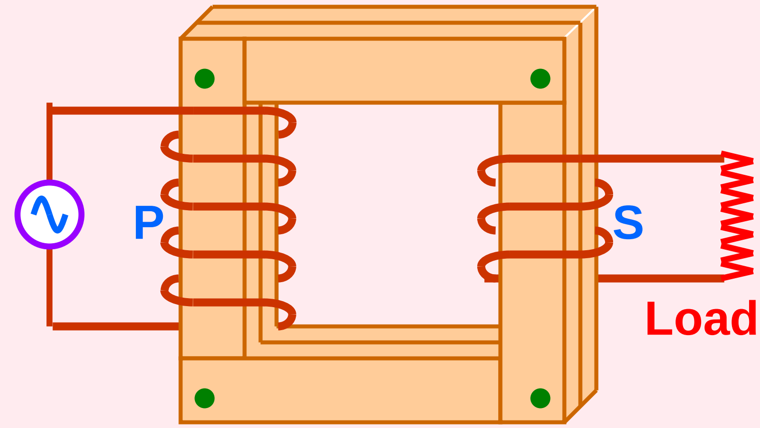
Step - up Transformer:



$N_S > N_P$ i.e. $K > 1$

$E_S > E_P$ & $I_S < I_P$

Step - down Transformer:



$N_S < N_P$ i.e. $K < 1$

$E_S < E_P$ & $I_S > I_P$

Energy Losses in a Transformer:

1. **Copper Loss:** Heat is produced due to the resistance of the copper windings of Primary and Secondary coils when current flows through them.

   This can be avoided by using thick wires for winding.

2. **Flux Loss:** In actual transformer coupling between Primary and Secondary coil is not perfect. So, a certain amount of magnetic flux is wasted.

   Linking can be maximised by winding the coils over one another.

3. **Iron Losses:**

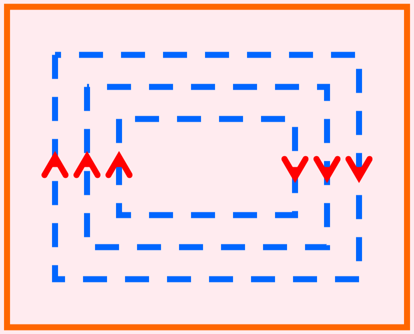
a) **Eddy Currents Losses:**

When a changing magnetic flux is linked with the iron core, eddy currents are set up which in turn produce heat and energy is wasted.

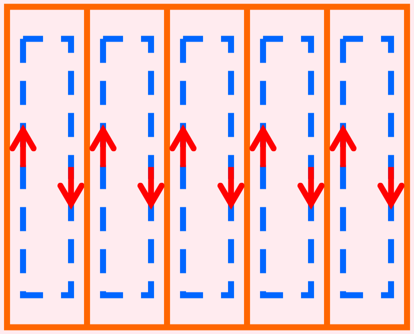
Eddy currents are reduced by using laminated core instead of a solid iron block because in laminated core the eddy currents are confined with in the lamination and they do not get added up to produce larger current. In other words their paths are broken instead of continuous ones.

b) **Hysteresis Loss:**

When alternating current is passed, the iron core is magnetised and demagnetised repeatedly over the cycles and some energy is being lost in the process.



Solid Core

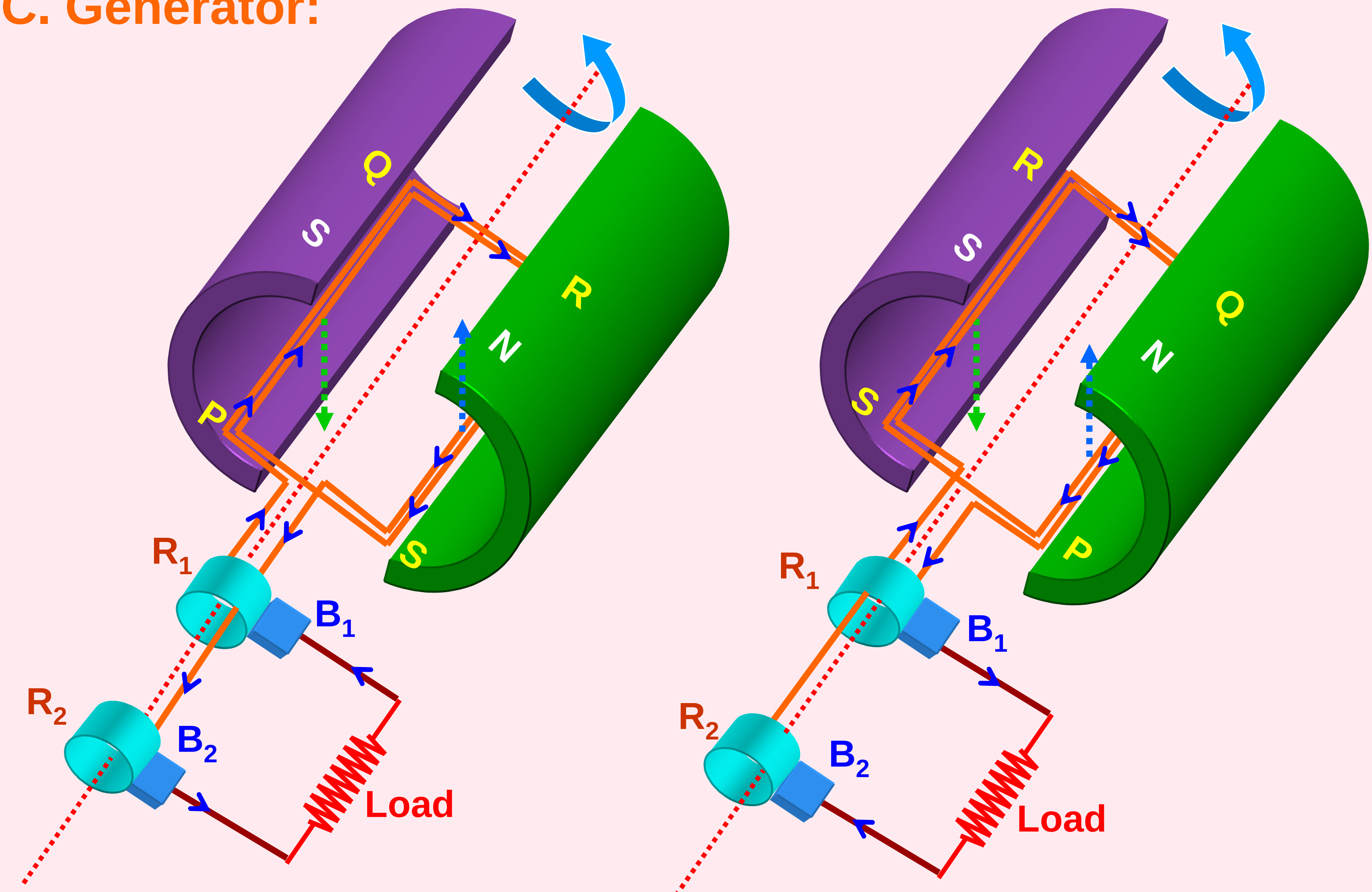


Laminated Core

This can be minimised by using suitable material with thin hysteresis loop.

4. **Losses due to vibration of core:** Some electrical energy is lost in the form of mechanical energy due to vibration of the core and humming noise due to magnetostriction effect.

A.C. Generator:



A.C. Generator or A.C. Dynamo or Alternator is a device which converts mechanical energy into alternating current (electrical energy).

Principle:

A.C. Generator is based on the principle of Electromagnetic Induction.

Construction:

(i) Field Magnet with poles N and S
(ii) Armature (Coil) PQRS
(iii) Slip Rings (R_1 and R_2)
(iv) Brushes (B_1 and B_2)
(v) Load

Working:

Let the armature be rotated in such a way that the arm PQ goes down and RS comes up from the plane of the diagram. Induced emf and hence current is set up in the coil. By Fleming's Right Hand Rule, the direction of the current is $PQRSR_2B_2B_1R_1P$.

After half the rotation of the coil, the arm PQ comes up and RS goes down into the plane of the diagram. By Fleming's Right Hand Rule, the direction of the current is $PR_1B_1B_2R_2SRQP$.

If one way of current is taken +ve, then the reverse current is taken –ve.

Therefore the current is said to be alternating and the corresponding wave is sinusoidal.

Theory:

$\Phi = N\,B\,A \cos\theta$

At time t, with angular velocity ω,

$\theta = \omega t$ (at t = 0, loop is assumed to be perpendicular to the magnetic field and θ = 0°)

∴ $\Phi = N\,B\,A \cos\omega t$

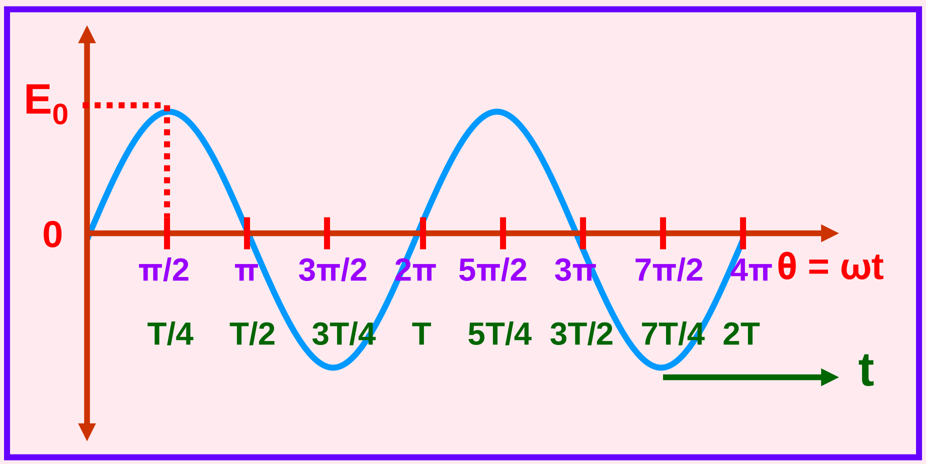
Differentiating w.r.t. t,

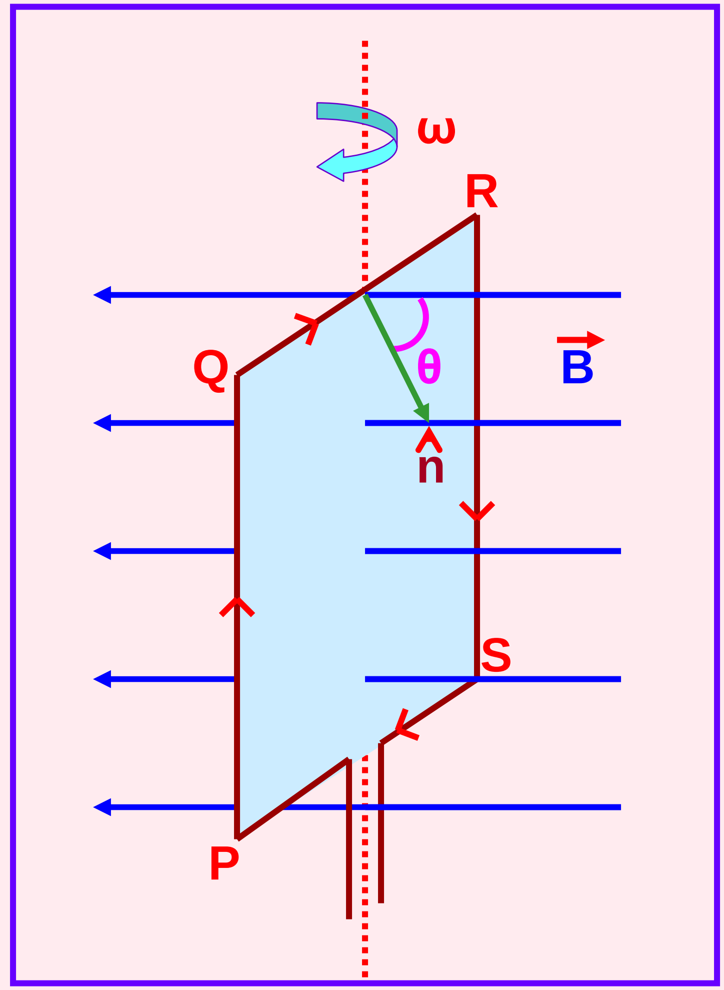
$d\Phi / dt = - NBA\omega \sin\omega t$

$E = - d\Phi / dt$

$E = NBA\omega \sin\omega t$

$E = E_0 \sin\omega t$ (where $E_0 = NBA\omega$)





ELECTROMAGNETIC WAVES

1. Electromagnetic Waves
2. Properties of Electromagnetic Waves
3. Hertz Experiment
4. Electromagnetic Spectrum
   - Wavelength and Frequency Range
   - Sources and Uses

Electromagnetic Waves:

For a region where there are no charges and conduction current, Faraday's and Ampere's laws take the symmetrical form:

$$\oint_l \vec{E} \, . \, \vec{dl} = - \frac{d\Phi_B}{dt} \quad \text{and} \quad \oint_l \vec{B} \, . \, \vec{dl} = - \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

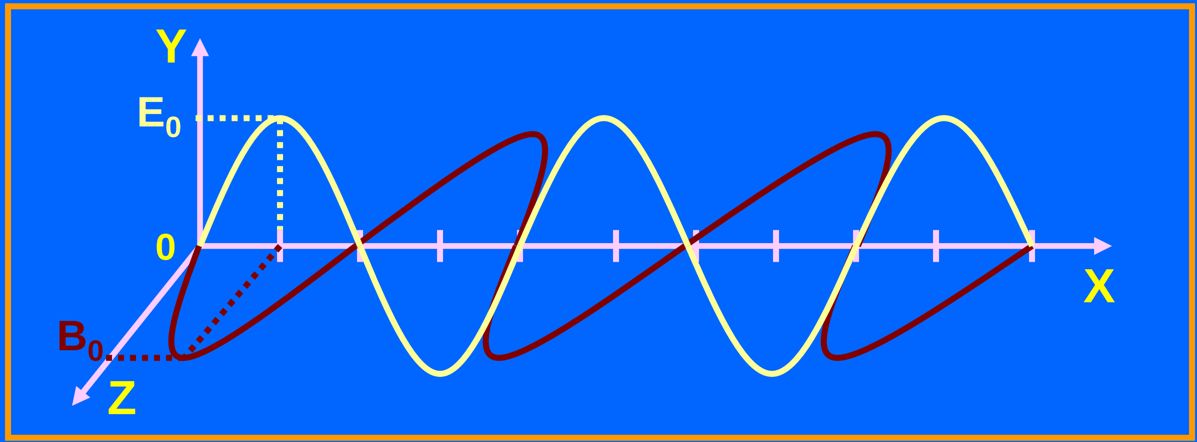
It can also be shown that time – varying electric field produces space – varying magnetic field and time – varying magnetic field produces space – varying electric field with the equations:

$$\frac{jE_y}{jx} = - \frac{jB_z}{jt} \quad \text{and} \quad \frac{jB_z}{jx} = - \mu_0 \varepsilon_0 \frac{jE_y}{jt}$$

Electric and magnetic fields are sources to each other.

Electromagnetic wave is a wave in which electric and magnetic fields are perpendicular to each other and also perpendicular to the direction of propagation of wave.

Properties of Electromagnetic Waves:



1. Variations in both electric and magnetic fields occur simultaneously. Therefore, they attain their maxima and minima at the same place and at the same time.

2. The direction of electric and magnetic fields are mutually perpendicular to each other and as well as to the direction of propagation of wave.

3. The electric field vector E and magnetic field vector B are related by $c = E_0 / B_0$ where E_0 and B_0 are the amplitudes of the respective fields and c is speed of light.

4. The velocity of electromagnetic waves in free space, $c = 1 / \sqrt{\mu_0 \varepsilon_0}$

5. The velocity of electromagnetic waves in a material medium $= 1 / \sqrt{\mu\varepsilon}$ where μ and ε are absolute permeability and absolute permitivity of the material medium.

6. Electromagnetic waves obey the principle of superposition.

7. Electromagnetic waves carry energy as they propagate through space. This energy is divided equally between electric and magnetic fields.

8. Electromagnetic waves can transfer energy as well as momentum to objects placed on their paths.

9. For discussion of optical effects of EM wave, more significance is given to Electric Field, E. Therefore, electric field is called 'light vector'.

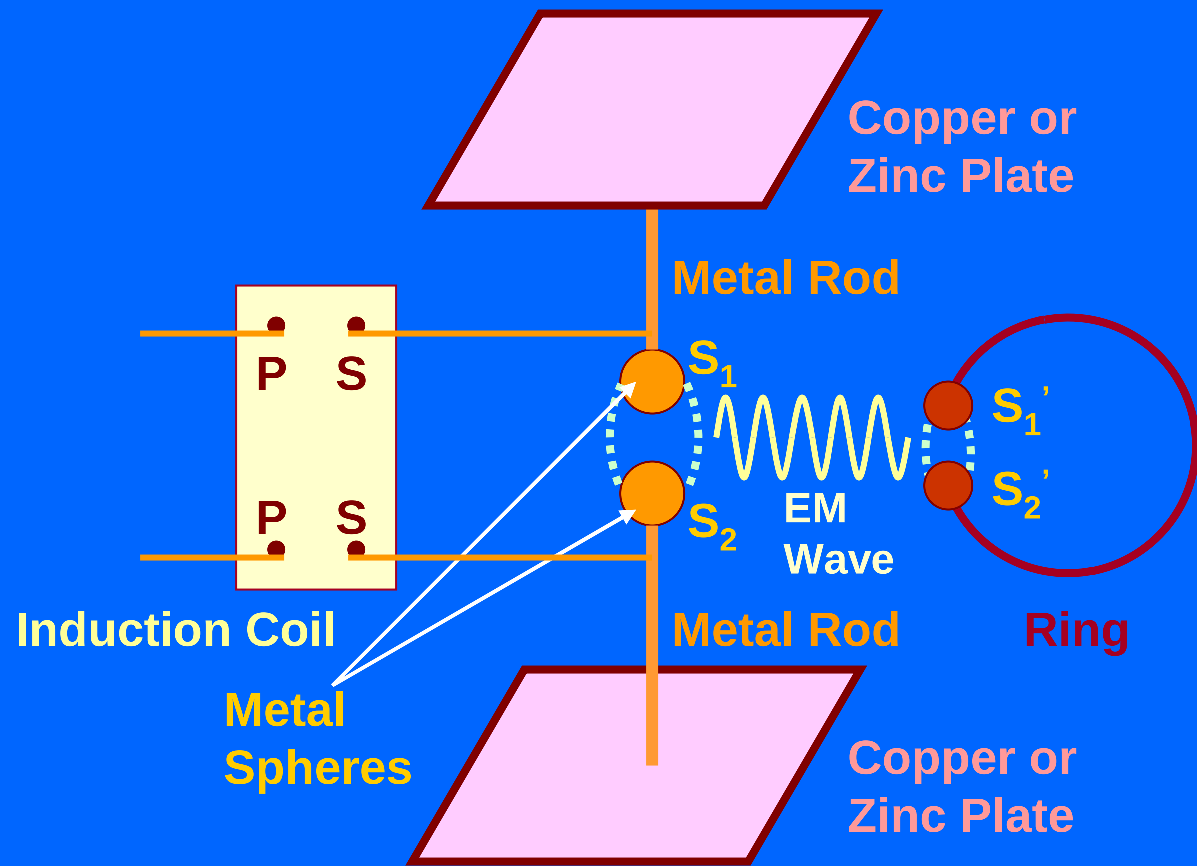
10. Electromagnetic waves do not require material medium to travel.

11. An oscillating charge which has non-zero acceleration can produce electromagnetic waves.

Hertz Experiment:

The copper or zinc plates are kept parallel separated by 60 cm. The metal spheres are slided over the metal rods to have a gap of 2 to 3 cm. Induction coil supplies high voltage of several thousand volts.

The plates and the rods (with spheres) constitute an LC combination.



An open metallic ring of diameter 0.70 m having small metallic spheres acts as a detector.

This constitutes another LC combination whose frequency can be varied by varying its diameter.

Due to high voltage, the air in the small gap between the spheres gets ionised. This provides the path for the discharge of the plates. A spark begins to pass between the spheres.

A very high frequency oscillations of charges occur on the plates. This results in high frequency oscillating electric field in the vertical gap S_1S_2. Consequently, an oscillating magnetic field of the same frequency is set up in the horizontal plane and perpendicular to the gap between the spheres.

These oscillating electric and magnetic fields constitute electromagnetic waves. The electromagnetic waves produced are radiated from the spark gap.

The detector is held in a position such that the magnetic field produced by the oscillating current is perpendicular to the plane of the coil. The resultant electric field induced by the oscillating magnetic field causes the ionisation of air in the gap between the spheres. So, a conducting path becomes available for the induced current to flow across the gap. This causes sparks to appear at the narrow gap.

It was observed that this spark was most intense when the spheres S_1S_2 and $S_1'S_2'$ were parallel to each other. This was a clear evidence of the polarisation of the electromagnetic waves.

Hertz was able to produce electromagnetic waves of wavelength nearly 6 m. After seven years, J.C. Bose succeeded in producing the em waves of wavelength ranging from 25 mm to 5 mm.

Electromagnetic Spectrum:

S. No.	EM Wave	Range of λ	Range of ν	Source	Use
1	Radio Wave	A few km to 0.3 m	A few Hz to 10^9 Hz	Oscillating electronic circuits	Radio and TV broadcasting
2	Microwave	0.3 m to 10^{-3} m	10^9 Hz to 3×10^{11} Hz	Oscillating electronic circuits	Radar, analysis of fine details of atomic and molecular structures & Microwave oven
3	Infra Red wave	10^{-3} m to 7.8×10^{-7} m	3×10^{11} Hz to 4×10^{14} Hz	Molecules and hot bodies	Industry, medicine, astronomy, night vision device, green house, revealing secret writings on ancient walls, etc.
4	Light or Visible Spectrum	7.8×10^{-7} m to 3.8×10^{-7} m	4×10^{14} Hz to 8×10^{14} Hz	Atoms and molecules when electrons are excited	Optics and Optical Instruments, Vision, photography, etc.

S. No.	EM Wave	Range of λ	Range of v	Source	Use
5	Ultra Violet Rays	3.8×10^{-7} m to 6×10^{-10} m	8×10^{14} Hz to 3×10^{17} Hz	Atoms and molecules in electrical discharges and Sun	Medical application, sterilization, killing bacteria and germs in food stuff, detection of invisible writing, forged documents, finger print, etc.
6	X - Rays	10^{-9} m to 6×10^{-12} m	3×10^{17} Hz to 5×10^{19} Hz	Inner or more tightly bound electrons in atoms	X-ray photography, treatment of cancer, skin disease & tumor, locating cracks and flaws in finished metallic objects, detection of smuggled goods in bags of a person, study of crystal structure, etc.
7	γ-Rays	They overlap the upper limit of the X-Ray. 10^{-10} m to 10^{-14} m	3×10^{18} Hz to 3×10^{22} Hz	Radioactive substances	Information about structure of nuclei, astronomical research, etc.

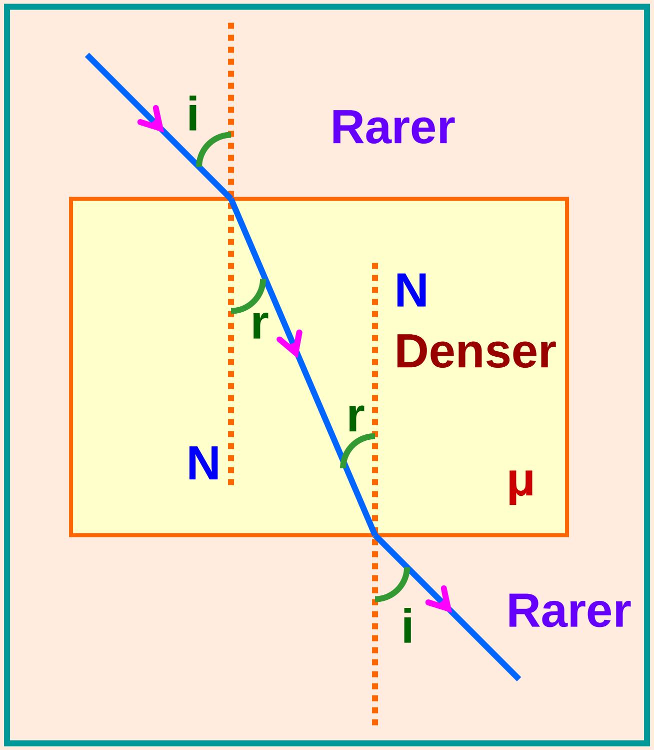
RAY OPTICS - I

1. Refraction of Light
2. Laws of Refraction
3. Principle of Reversibility of Light
4. Refraction through a Parallel Slab
5. Refraction through a Compound Slab
6. Apparent Depth of a Liquid
7. Total Internal Reflection
8. Refraction at Spherical Surfaces - Introduction
9. Assumptions and Sign Conventions
10. Refraction at Convex and Concave Surfaces
11. Lens Maker’s Formula
12. First and Second Principal Focus
13. Thin Lens Equation (Gaussian Form)
14. Linear Magnification

Refraction of Light:

Refraction is the phenomenon of change in the path of light as it travels from one medium to another (when the ray of light is incident obliquely).

It can also be defined as the phenomenon of change in speed of light from one medium to another.



Laws of Refraction:

I Law: The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray all lie in the same plane.

II Law: For a given pair of media and for light of a given wavelength, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. (Snell's Law)

$$\mu = \frac{\sin i}{\sin r}$$

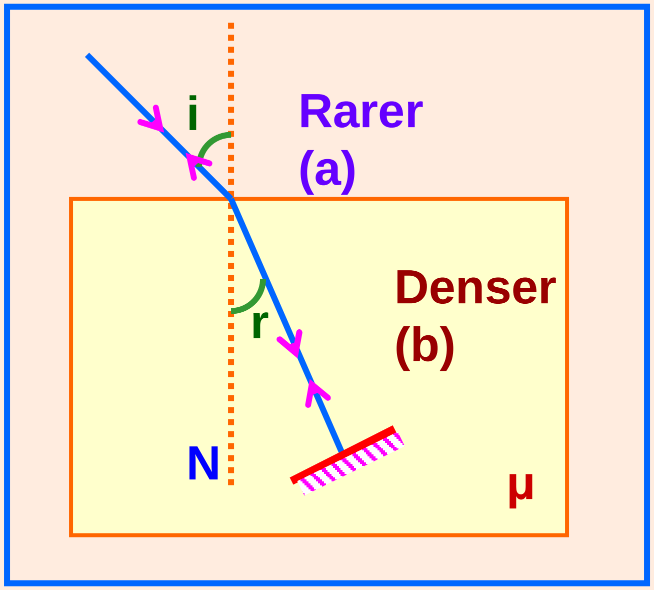
(The constant μ is called refractive index of the medium, i is the angle of incidence and r is the angle of refraction.)

TIPS:

1. μ of optically rarer medium is lower and that of a denser medium is higher.
2. μ of denser medium w.r.t. rarer medium is more than 1 and that of rarer medium w.r.t. denser medium is less than 1. ($\mu_{air} = \mu_{vacuum} = 1$)
3. In refraction, the velocity and wavelength of light change.
4. In refraction, the frequency and phase of light do not change.
5. ${}_a\mu_m = c_a / c_m$ and ${}_a\mu_m = \lambda_a / \lambda_m$

Principle of Reversibility of Light:

$${}_a\mu_b = \frac{\sin i}{\sin r} \qquad {}_b\mu_a = \frac{\sin r}{\sin i}$$



$${}_a\mu_b \times {}_b\mu_a = 1 \quad \text{or} \quad {}_a\mu_b = 1 / {}_b\mu_a$$

If a ray of light, after suffering any number of reflections and/or refractions has its path reversed at any stage, it travels back to the source along the same path in the opposite direction.

A natural consequence of the principle of reversibility is that the image and object positions can be interchanged. These positions are called conjugate positions.

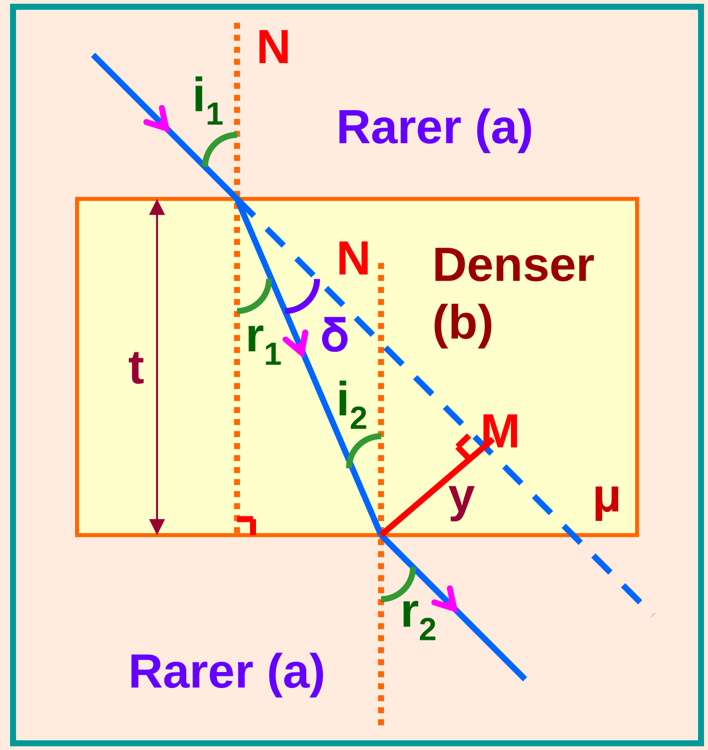
Refraction through a Parallel Slab:

$${}_a\mu_b = \frac{\sin i_1}{\sin r_1} \qquad {}_b\mu_a = \frac{\sin i_2}{\sin r_2}$$

But ${}_a\mu_b \times {}_b\mu_a = 1$

$$\therefore \frac{\sin i_1}{\sin r_1} \times \frac{\sin i_2}{\sin r_2} = 1$$

It implies that $i_1 = r_2$ and $i_2 = r_1$
since $i_1 \neq r_1$ and $i_2 \neq r_2$.



Lateral Shift:

$$y = \frac{t \sin \delta}{\cos r_1} \quad \text{or} \quad y = \frac{t \sin(i_1 - r_1)}{\cos r_1}$$

Special Case:

If i_1 is very small, then r_1 is also very small.
i.e. $\sin(i_1 - r_1) = i_1 - r_1$ and $\cos r_1 = 1$

$$\therefore \quad y = t\,(i_1 - r_1) \quad \text{or} \quad y = t\, i_1(1 - 1/{}_a\mu_b)$$

Refraction through a Compound Slab:

$${}_a\mu_b = \frac{\sin i_1}{\sin r_1}$$

$${}_b\mu_c = \frac{\sin r_1}{\sin r_2}$$

$${}_c\mu_a = \frac{\sin r_2}{\sin i_1}$$

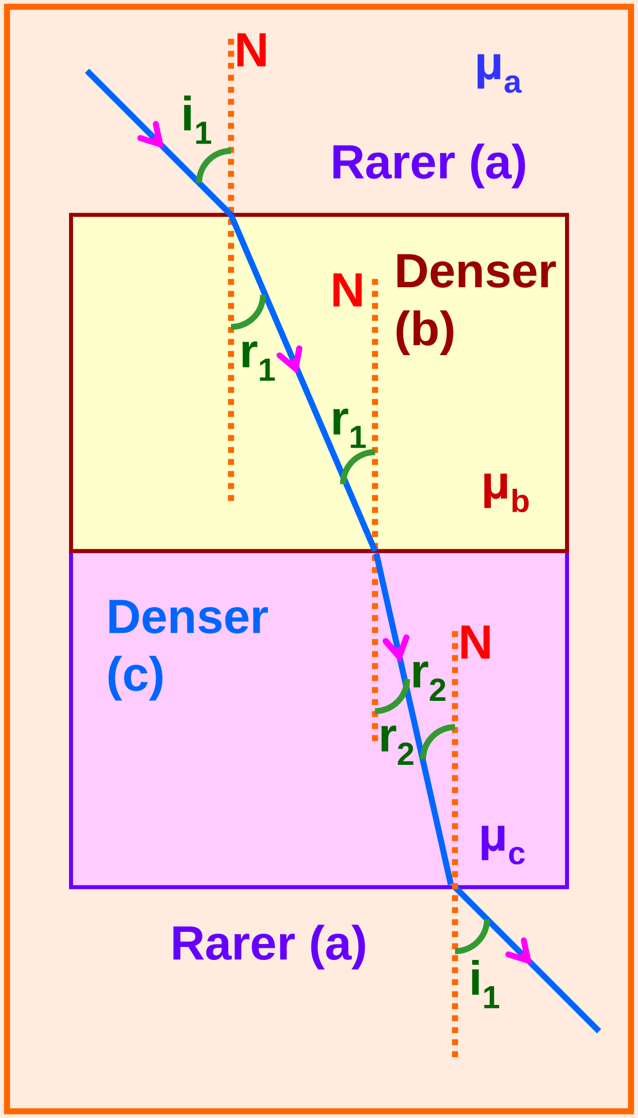
$${}_a\mu_b \times {}_b\mu_c \times {}_c\mu_a = 1$$

or

$${}_a\mu_b \times {}_b\mu_c = {}_a\mu_c$$

or

$${}_b\mu_c = {}_a\mu_c \,/\, {}_a\mu_b$$

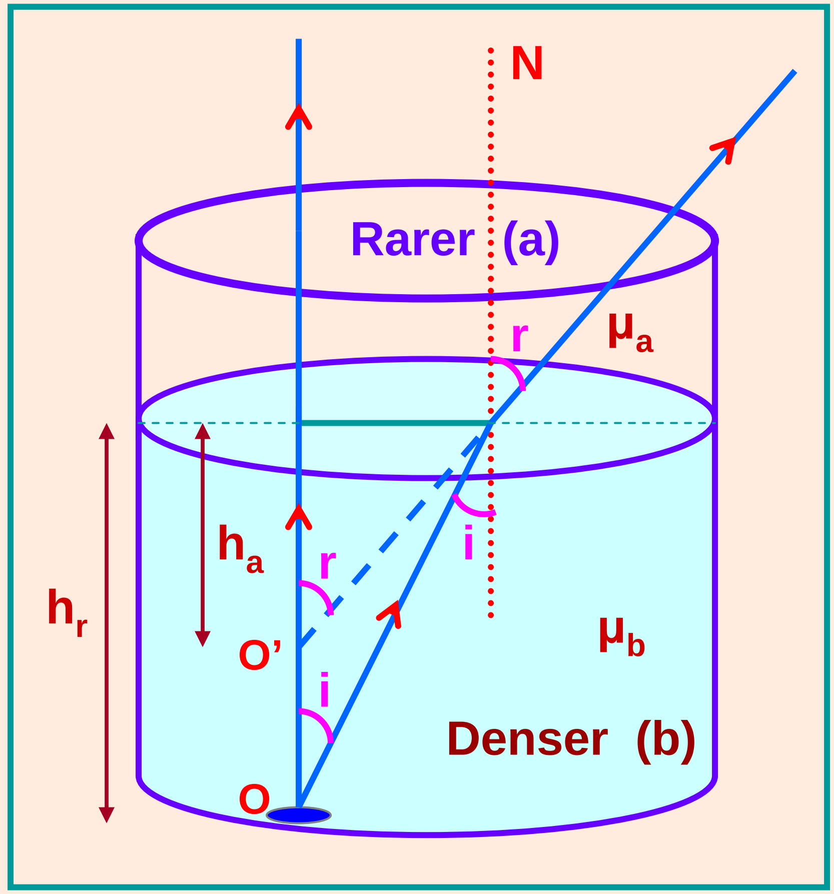


$\mu_c > \mu_b$

Apparent Depth of a Liquid:

$$ {}_b\mu_a = \frac{\sin i}{\sin r} \quad \text{or} \quad {}_a\mu_b = \frac{\sin r}{\sin i} $$

$$ {}_a\mu_b = \frac{h_r}{h_a} = \frac{\text{Real depth}}{\text{Apparent depth}} $$



Apparent Depth of a Number of Immiscible Liquids:

$$ h_a = \sum_{i=1}^{n} h_i / \mu_i $$

Apparent Shift:

Apparent shift = $h_r - h_a = h_r - (h_r / \mu)$

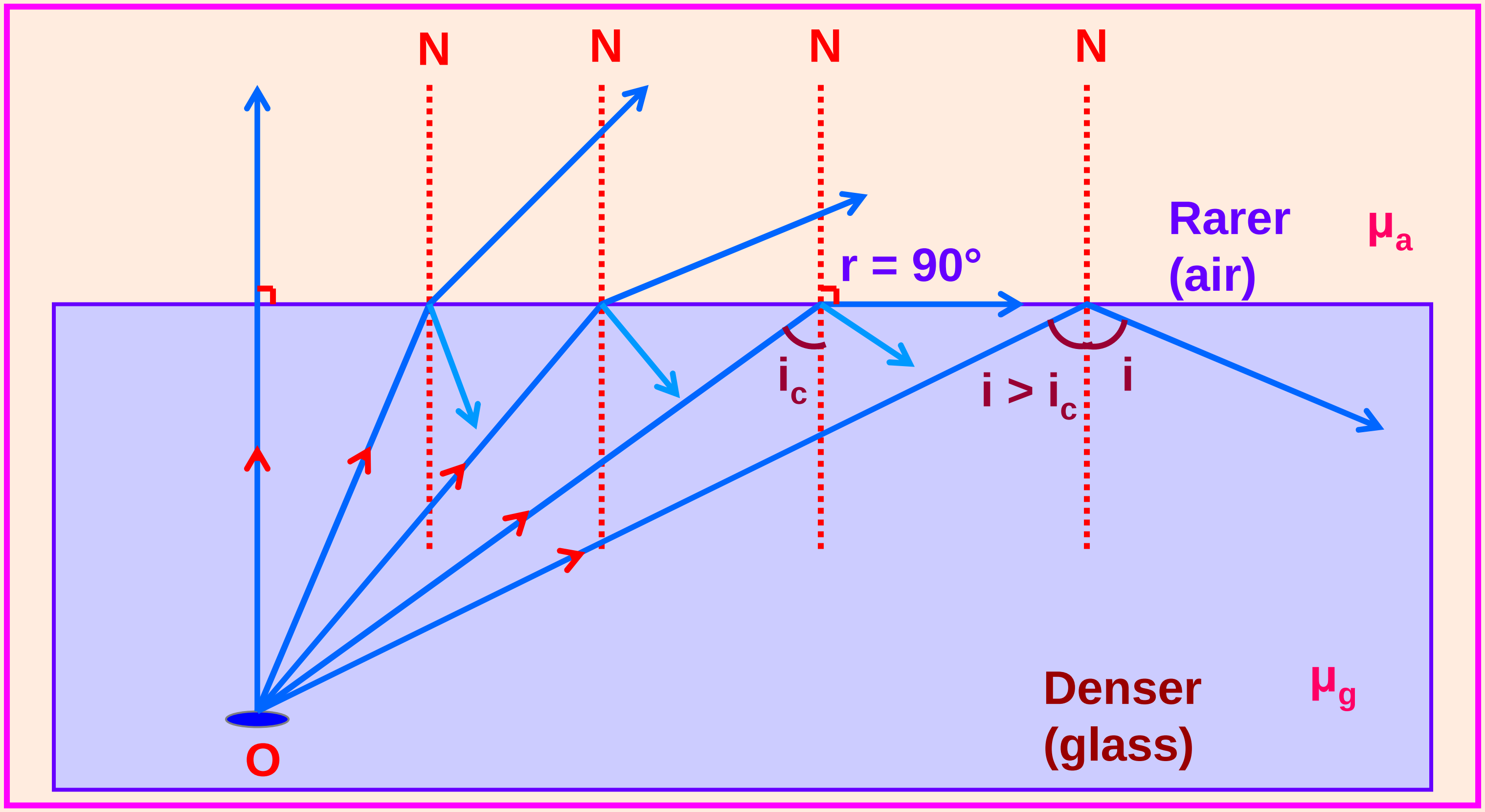
$= h_r [1 - 1/\mu]$

TIPS:

1. If the observer is in rarer medium and the object is in denser medium then $h_a < h_r$. (To a bird, the fish appears to be nearer than actual depth.)
2. If the observer is in denser medium and the object is in rarer medium then $h_a > h_r$. (To a fish, the bird appears to be farther than actual height.)

Total Internal Reflection:

Total Internal Reflection (TIR) is the phenomenon of complete reflection of light back into the same medium for angles of incidence greater than the critical angle of that medium.



Conditions for TIR:

1. **The incident ray must be in optically denser medium.**
2. **The angle of incidence in the denser medium must be greater than the critical angle for the pair of media in contact.**

Relation between Critical Angle and Refractive Index:

Critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

$$ {}_g\mu_a = \frac{\sin i}{\sin r} = \frac{\sin i_c}{\sin 90°} = \sin i_c $$

or $$ {}_a\mu_g = \frac{1}{{}_g\mu_a} $$ $\therefore$ $$ {}_a\mu_g = \frac{1}{\sin i_c} $$ or $$ \sin i_c = \frac{1}{{}_a\mu_g} $$ Also $$ \sin i_c = \frac{\lambda_g}{\lambda_a} $$

Red colour has maximum value of critical angle and Violet colour has minimum value of critical angle since,

$$ \sin i_c = \frac{1}{{}_a\mu_g} = \frac{1}{a + (b/\lambda^2)} $$

Applications of T I R:

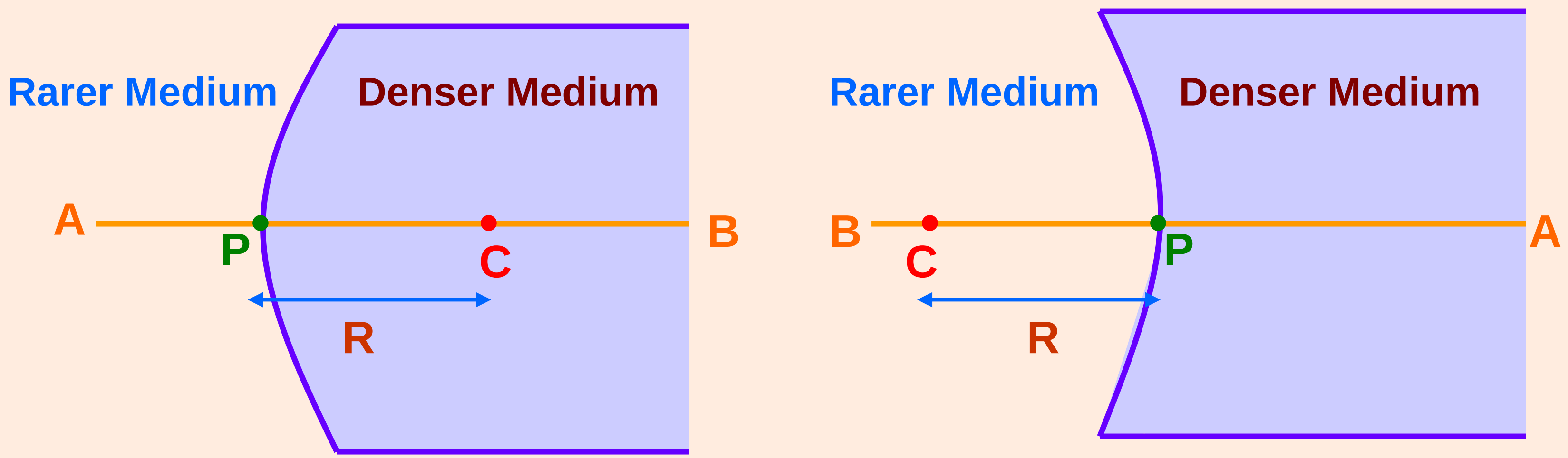
1. Mirage formation
2. Looming
3. Totally reflecting Prisms
4. Optical Fibres
5. Sparkling of Diamonds

Spherical Refracting Surfaces:

A spherical refracting surface is a part of a sphere of refracting material.

A refracting surface which is convex towards the rarer medium is called convex refracting surface.

A refracting surface which is concave towards the rarer medium is called concave refracting surface.



APCB – Principal Axis
C – Centre of Curvature
P – Pole
R – Radius of Curvature

Assumptions:

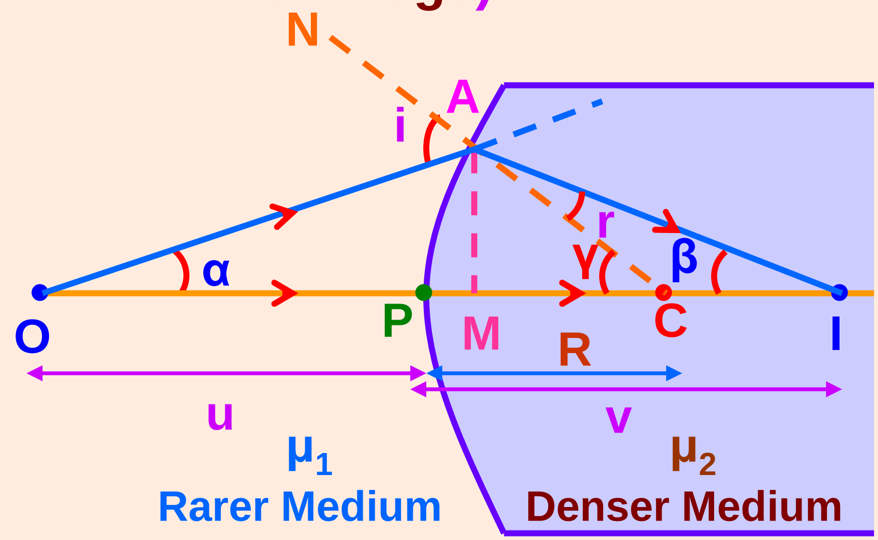
1. Object is the point object lying on the principal axis.
2. The incident and the refracted rays make small angles with the principal axis.
3. The aperture (diameter of the curved surface) is small.

New Cartesian Sign Conventions:

1. The incident ray is taken from left to right.
2. All the distances are measured from the pole of the refracting surface.
3. The distances measured along the direction of the incident ray are taken positive and against the incident ray are taken negative.
4. The vertical distances measured from principal axis in the upward direction are taken positive and in the downward direction are taken negative.

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Real Image)



$i = \alpha + \gamma$

$\gamma = r + \beta$ or $r = \gamma - \beta$

$\tan \alpha = \frac{MA}{MO}$ or $\alpha = \frac{MA}{MO}$

$\tan \beta = \frac{MA}{MI}$ or $\beta = \frac{MA}{MI}$

$\tan \gamma = \frac{MA}{MC}$ or $\gamma = \frac{MA}{MC}$

According to Snell's law,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ or $\frac{i}{r} = \frac{\mu_2}{\mu_1}$ or $\mu_1 i = \mu_2 r$

Substituting for i, r, α, β and γ, replacing M by P and rearranging,

$$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Applying sign conventions with values,

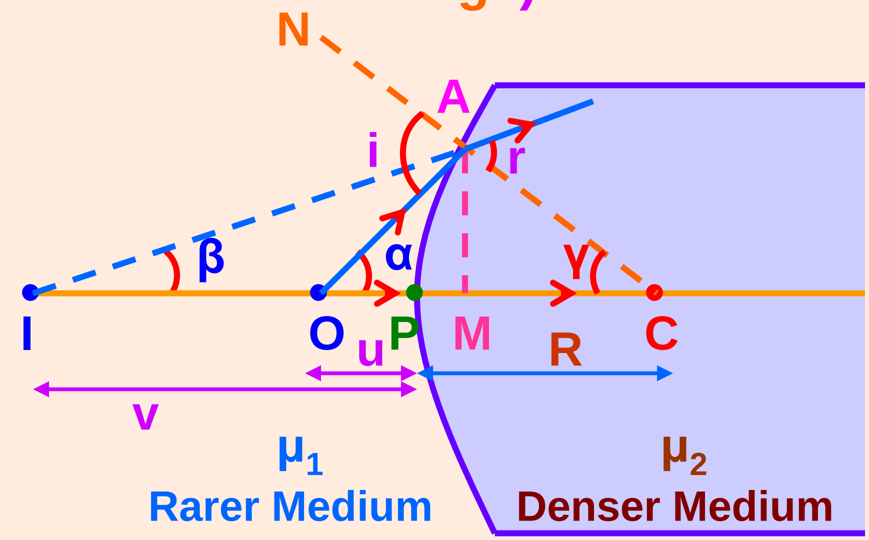
PO = - u, PI = + v and PC = + R

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Virtual Image)

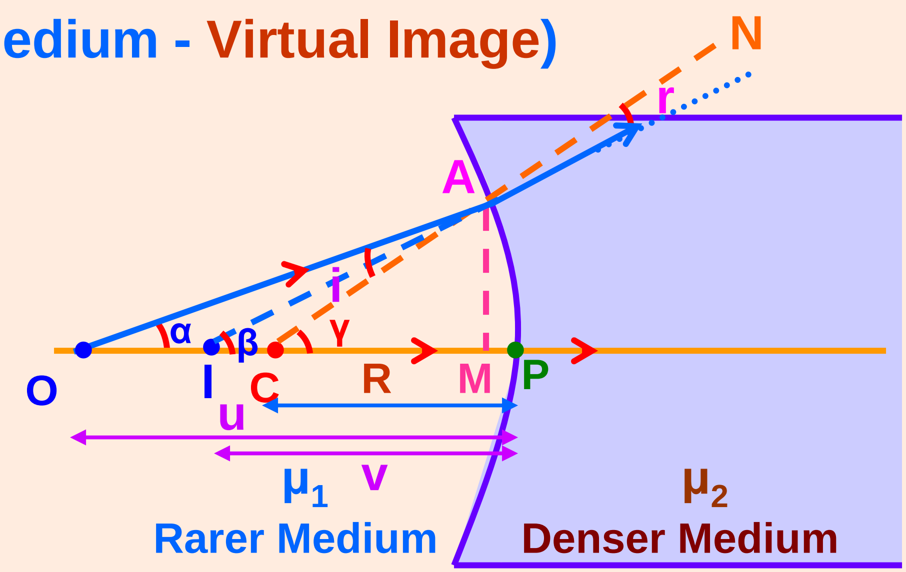
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Concave Surface:

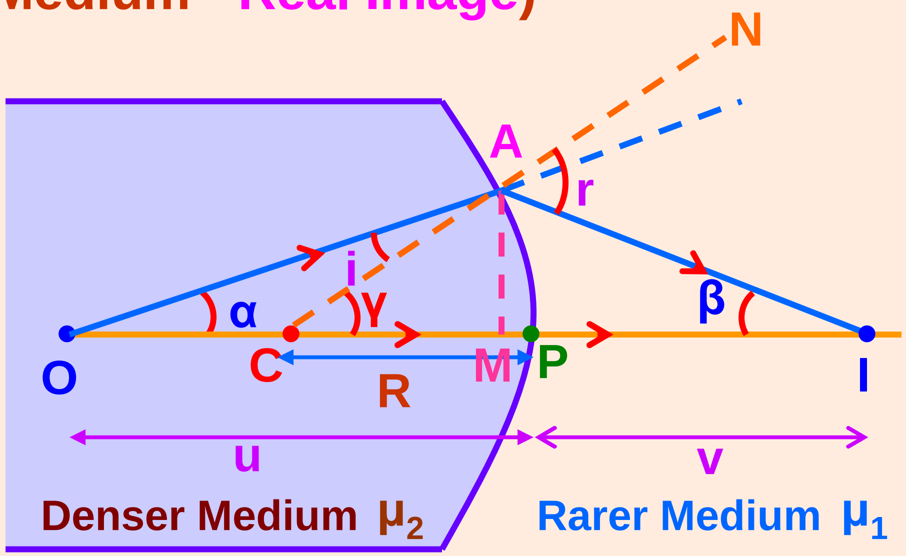
(From Rarer Medium to Denser Medium - Virtual Image)

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Real Image)



$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at Concave Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Note:

1. Expression for '**object in rarer medium**' is same for whether it is real or virtual image or convex or concave surface.

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

2. Expression for '**object in denser medium**' is same for whether it is real or virtual image or convex or concave surface.

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

3. However the values of u, v, R, etc. must be taken with proper sign conventions while solving the numerical problems.

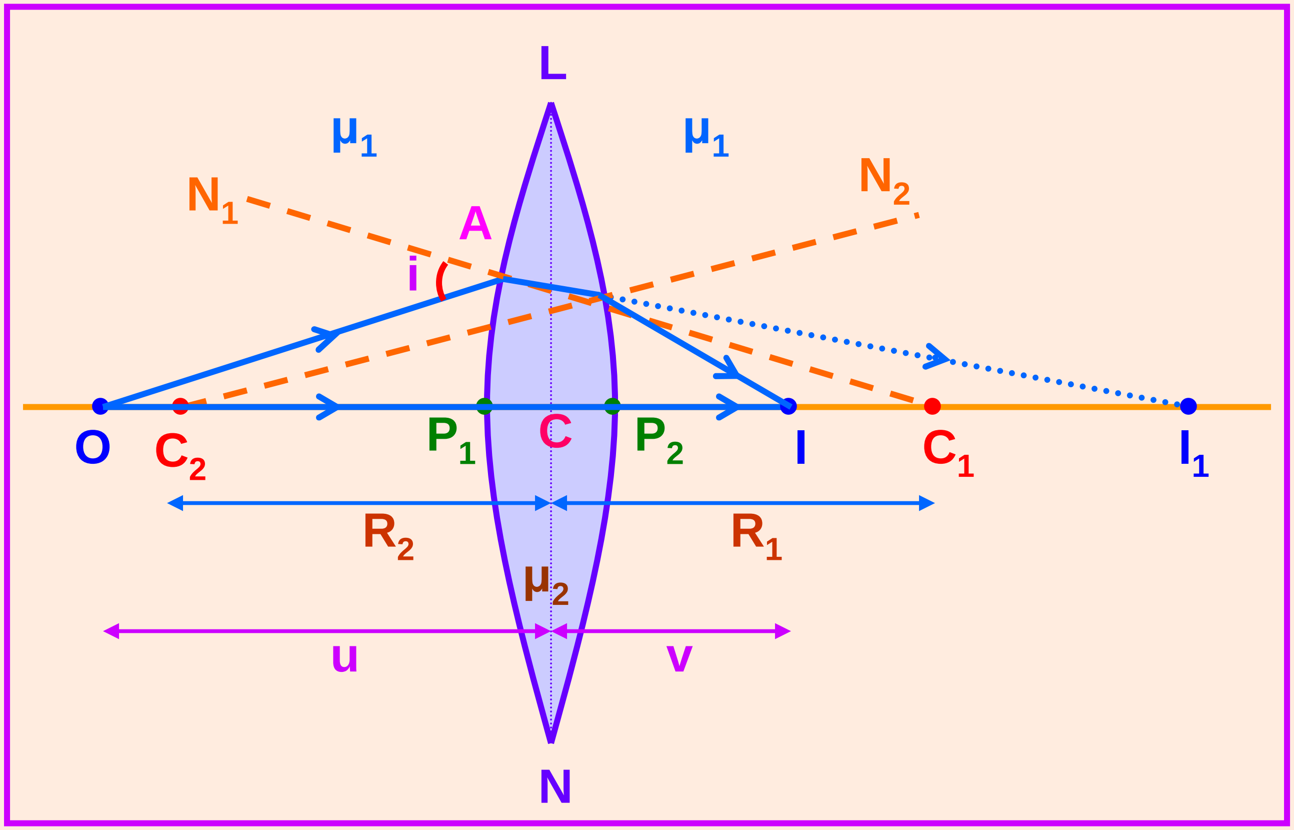
4. The refractive indices μ_1 and μ_2 get interchanged in the expressions.

Lens Maker's Formula:

For refraction at LP_1N,

$$\frac{\mu_1}{CO} + \frac{\mu_2}{CI_1} = \frac{\mu_2 - \mu_1}{CC_1}$$

(as if the image is formed in the denser medium)



For refraction at LP_2N,

$$\frac{\mu_2}{-CI_1} + \frac{\mu_1}{CI} = \frac{-(\mu_1 - \mu_2)}{CC_2}$$

(as if the object is in the denser medium and the image is formed in the rarer medium)

Combining the refractions at both the surfaces,

$$\frac{\mu_1}{CO} + \frac{\mu_1}{CI} = (\mu_2 - \mu_1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right)$$

Substituting the values with sign conventions,

$$\frac{1}{-u} + \frac{1}{v} = \frac{(\mu_2 - \mu_1)}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since $\mu_2 / \mu_1 = \mu$

$$\frac{1}{-u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{-u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When the object is kept at infinity, the image is formed at the principal focus.

i.e. $u = -\infty$, $v = +f$.

So,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

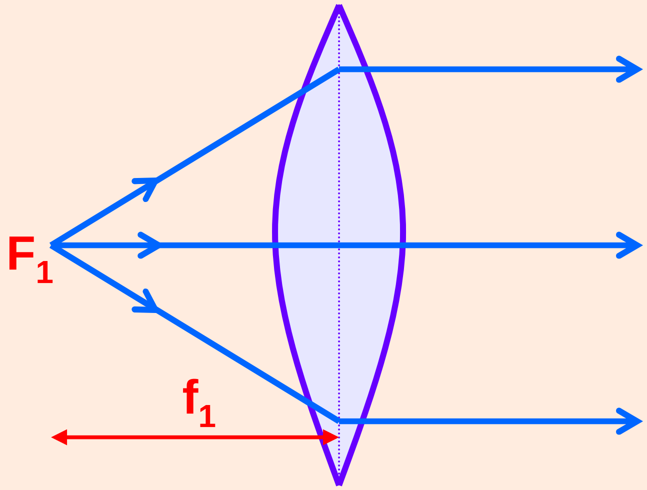
This equation is called 'Lens Maker's Formula'.

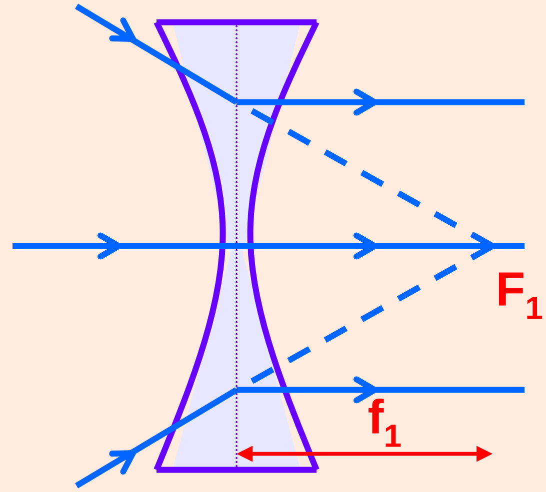
Also, from the above equations we get,

$$\frac{1}{-u} + \frac{1}{v} = \frac{1}{f}$$

First Principal Focus:

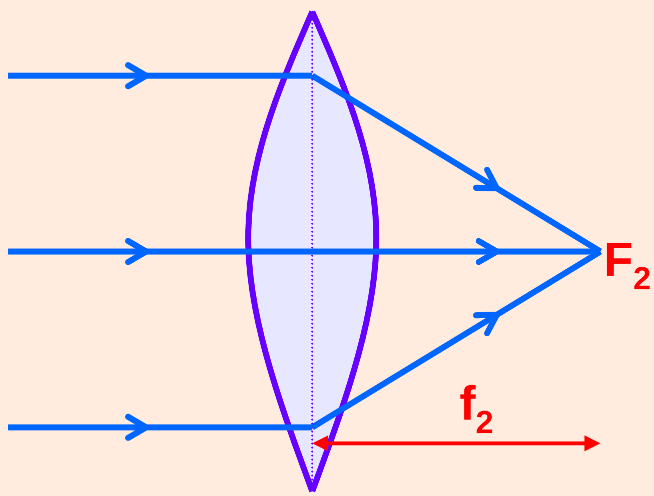
First Principal Focus is the point on the principal axis of the lens at which if an object is placed, the image would be formed at infinity.

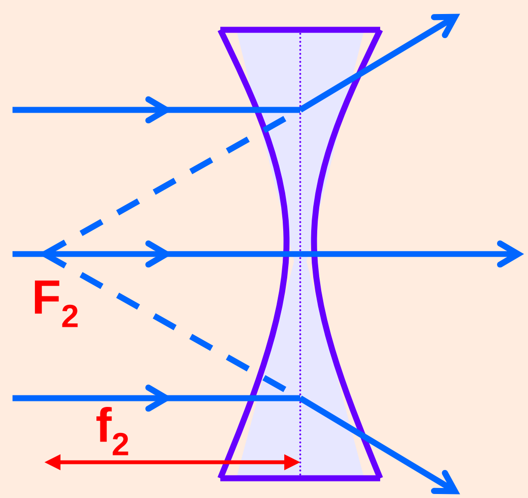




Second Principal Focus:

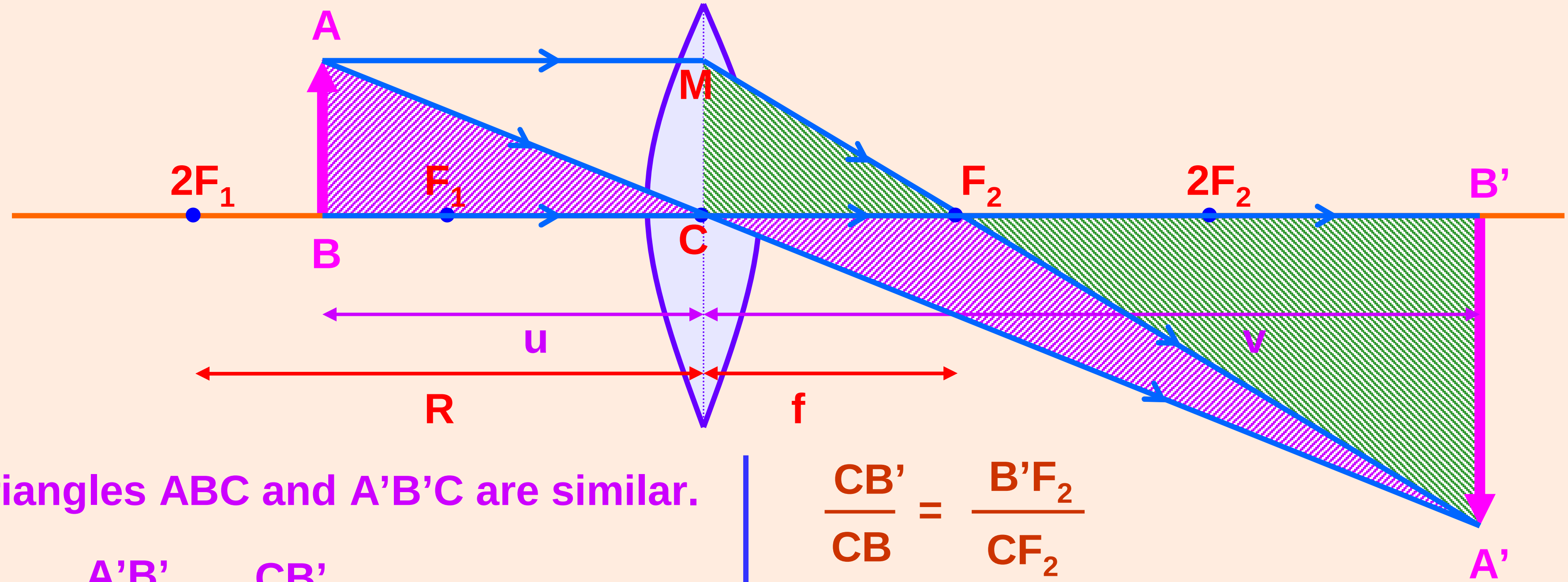
Second Principal Focus is the point on the principal axis of the lens at which the image is formed when the object is kept at infinity.





Thin Lens Formula (Gaussian Form of Lens Equation):

For Convex Lens:



Triangles ABC and A'B'C are similar.

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

Triangles MCF_2 and $A'B'F_2$ are similar.

$$\frac{A'B'}{MC} = \frac{B'F_2}{CF_2}$$

or $$\frac{A'B'}{AB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{CB' - CF_2}{CF_2}$$

According to new Cartesian sign conventions,

CB = - u, CB' = + v and CF_2 = + f.

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Linear Magnification:

Linear magnification produced by a lens is defined as the ratio of the size of the image to the size of the object.

$$m = \frac{I}{O}$$

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

According to new Cartesian sign conventions,

A'B' = + I, AB = - O, CB' = + v and CB = - u.

$$\frac{+I}{-O} = \frac{+v}{-u} \quad \text{or} \quad m = \frac{I}{O} = \frac{v}{u}$$

Magnification in terms of v and f:

$$m = \frac{f - v}{f}$$

Magnification in terms of v and f:

$$m = \frac{f}{f - u}$$

Power of a Lens:

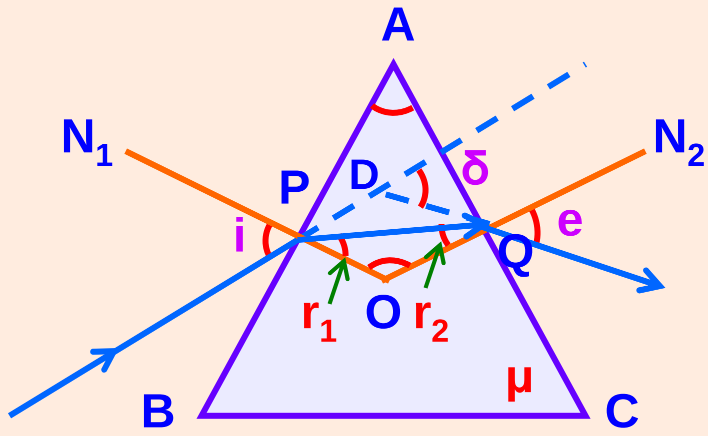
Power of a lens is its ability to bend a ray of light falling on it and is reciprocal of its focal length. When f is in metre, power is measured in Dioptre (D).

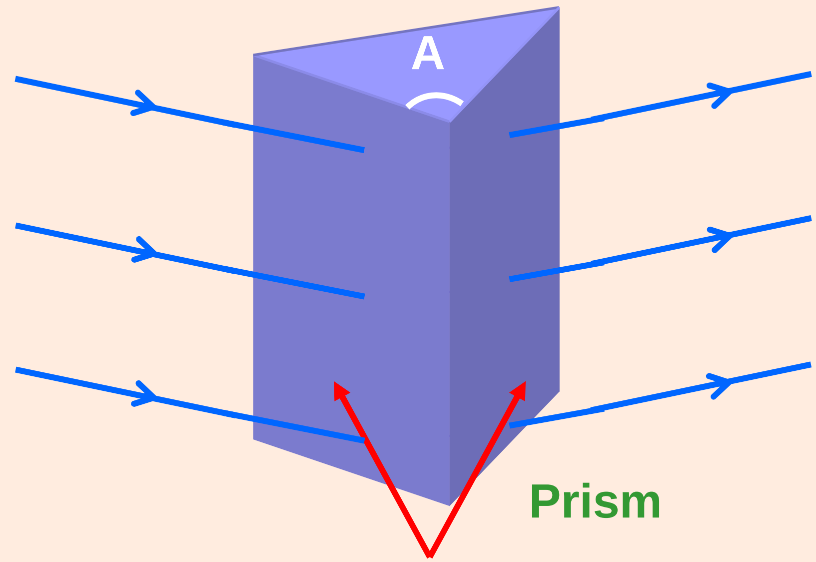
$$P = \frac{1}{f}$$

RAY OPTICS - II

1. Refraction through a Prism
2. Expression for Refractive Index of Prism
3. Dispersion
4. Angular Dispersion and Dispersive Power
5. Blue Colour of the Sky and Red Colour of the Sun
6. Compound Microscope
7. Astronomical Telescope (Normal Adjustment)
8. Astronomical Telescope (Image at LDDV)
9. Newtonian Telescope (Reflecting Type)
10. Resolving Power of Microscope and Telescope

Refraction of Light through Prism:





Refracting Surfaces

In quadrilateral APOQ,

$A + O = 180°$ …… .(1)

(since N_1 and N_2 are normal)

In triangle OPQ,

$r_1 + r_2 + O = 180°$ …… .(2)

In triangle DPQ,

$\delta = (i - r_1) + (e - r_2)$

$\delta = (i + e) - (r_1 + r_2)$ …….(3)

From (1) and (2),

$A = r_1 + r_2$

From (3),

$\delta = (i + e) - (A)$

or $i + e = A + \delta$

Sum of angle of incidence and angle of emergence is equal to the sum of angle of prism and angle of deviation.

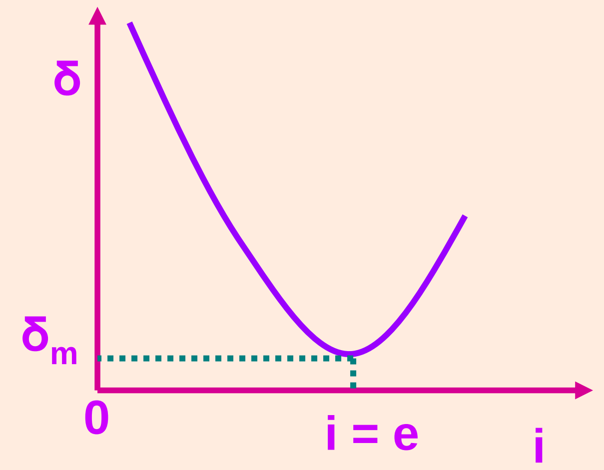
Variation of angle of deviation with angle of incidence:

When angle of incidence increases,
the angle of deviation decreases.

At a particular value of angle of incidence the angle of deviation becomes minimum and is called **'angle of minimum deviation'**.

At δ_m, $i = e$ and $r_1 = r_2 = r$ (say)

After minimum deviation, angle of deviation increases with angle of incidence.



Refractive Index of Material of Prism:

$A = r_1 + r_2$

$A = 2r$

$r = A / 2$

$i + e = A + \delta$

$2i = A + \delta_m$

$i = (A + \delta_m) / 2$

According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin i}{\sin r}$$

$$\therefore \quad \mu = \frac{\sin \frac{(A + \delta_m)}{2}}{\sin \frac{A}{2}}$$

Refraction by a Small-angled Prism for Small angle of Incidence:

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu = \frac{\sin e}{\sin r_2}$$

If i is assumed to be small, then r_1, r_2 and e will also be very small.
So, replacing sines of the angles by angles themselves, we get

$$\mu = \frac{i}{r_1} \quad \text{and} \quad \mu = \frac{e}{r_2}$$

$$i + e = \mu (r_1 + r_2) = \mu A$$

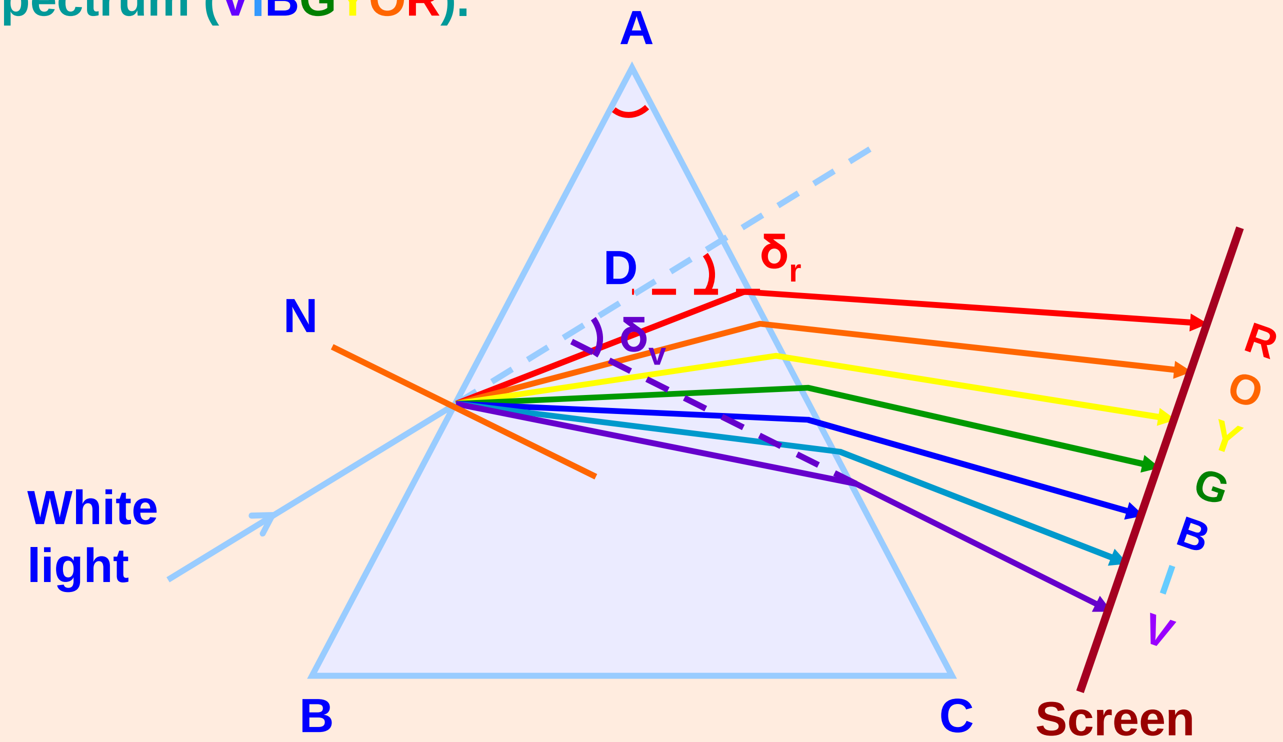
But $i + e = A + \delta$

So, $A + \delta = \mu A$

or $$\delta = A (\mu - 1)$$

Dispersion of White Light through Prism:

The phenomenon of splitting a ray of white light into its constituent colours (wavelengths) is called dispersion and the band of colours from violet to red is called spectrum (VIBGYOR).



Cause of Dispersion:

$$\mu_v = \frac{\sin i}{\sin r_v} \quad \text{and} \quad \mu_r = \frac{\sin i}{\sin r_r}$$

Since $\mu_v > \mu_r$, $r_r > r_v$

So, the colours are refracted at different angles and hence get separated.

Dispersion can also be explained on the basis of Cauchy's equation.

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$ (where a, b and c are constants for the material)

Since $\lambda_v < \lambda_r$, $\mu_v > \mu_r$

But $\delta = A(\mu - 1)$

Therefore, $\delta_v > \delta_r$

So, the colours get separated with different angles of deviation.
Violet is most deviated and Red is least deviated.

Angular Dispersion:

1. The difference in the deviations suffered by two colours in passing through a prism gives the angular dispersion for those colours.
2. The angle between the emergent rays of any two colours is called angular dispersion between those colours.
3. It is the rate of change of angle of deviation with wavelength. ($\Phi = d\delta / d\lambda$)

$\Phi = \delta_v - \delta_r$ or $\Phi = (\mu_v - \mu_r) A$

Dispersive Power:

The dispersive power of the material of a prism for any two colours is defined as the ratio of the angular dispersion for those two colours to the mean deviation produced by the prism.

It may also be defined as dispersion per unit deviation.

$$\omega = \frac{\Phi}{\delta}$$ where δ is the mean deviation and $$\delta = \frac{\delta_v + \delta_r}{2}$$

Also $$\omega = \frac{\delta_v - \delta_r}{\delta}$$ or $$\omega = \frac{(\mu_v - \mu_r) A}{(\mu_y - 1) A}$$ or $$\omega = \frac{(\mu_v - \mu_r)}{(\mu_y - 1)}$$

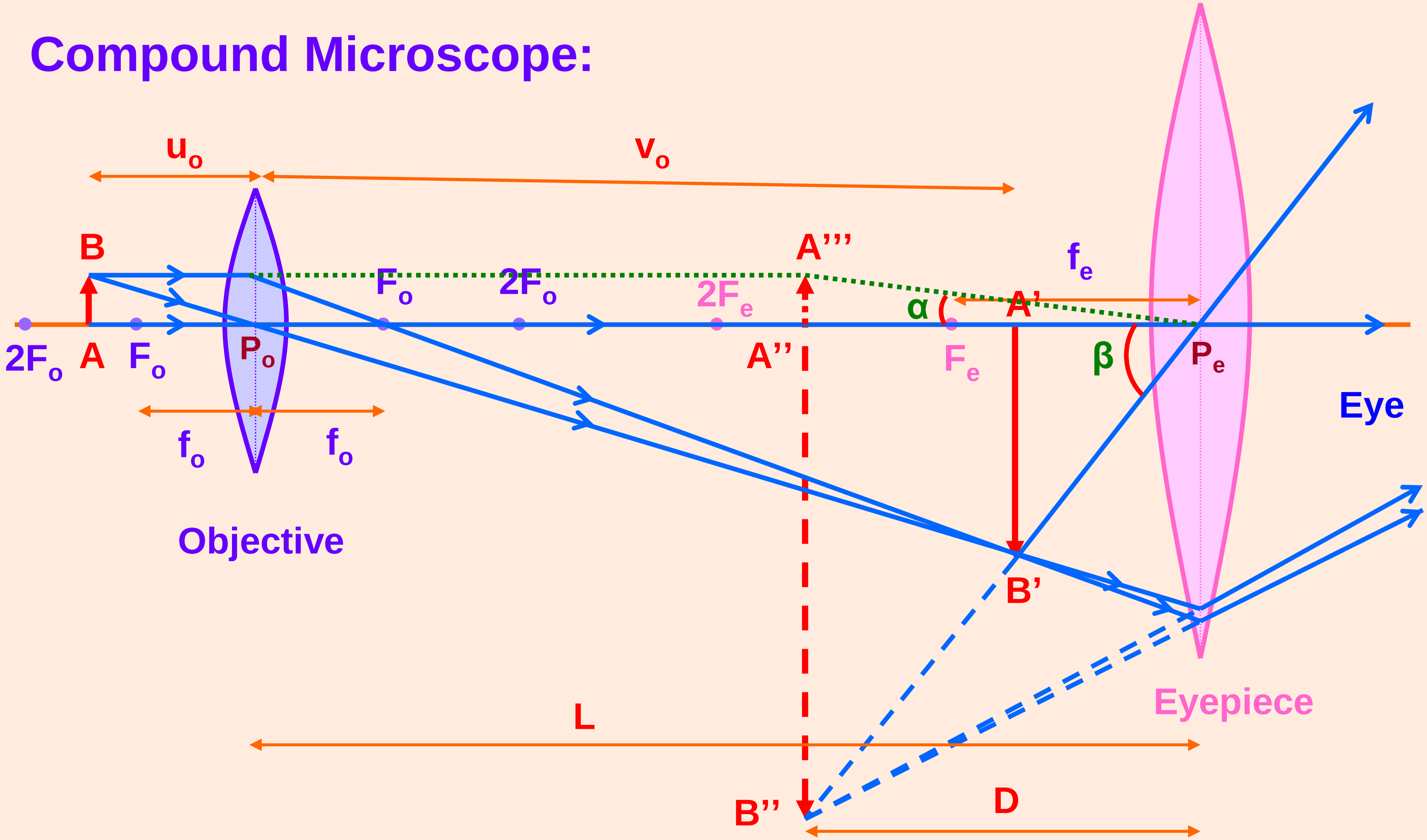
Scattering of Light – Blue colour of the sky and Reddish appearance of the Sun at Sun-rise and Sun-set:

The molecules of the atmosphere and other particles that are smaller than the longest wavelength of visible light are more effective in scattering light of shorter wavelengths than light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. (Rayleigh Effect)

Light from the Sun near the horizon passes through a greater distance in the Earth's atmosphere than does the light received when the Sun is overhead. The correspondingly greater scattering of short wavelengths accounts for the reddish appearance of the Sun at rising and at setting.

When looking at the sky in a direction away from the Sun, we receive scattered sunlight in which short wavelengths predominate giving the sky its characteristic bluish colour.

Compound Microscope:



Objective: The converging lens nearer to the object.

Eyepiece: The converging lens through which the final image is seen.

Both are of short focal length. Focal length of eyepiece is slightly greater than that of the objective.

Angular Magnification or Magnifying Power (M):

Angular magnification or magnifying power of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when both are placed at the least distance of distinct vision.

$$M = \frac{\beta}{\alpha}$$

Since angles are small,
α = tan α and β = tan β

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{A''B''}{D} \times \frac{D}{A''A'''}$$

$$M = \frac{A''B''}{D} \times \frac{D}{AB}$$

$$M = \frac{A''B''}{AB}$$

$$M = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

$$M = M_e \times M_o$$

$$M_e = 1 - \frac{v_e}{f_e} \quad \text{or} \quad M_e = 1 + \frac{D}{f_e}$$

(v_e = - D = - 25 cm)

and $M_o = \dfrac{v_o}{-u_o}$

$$\therefore \quad M = \frac{v_o}{-u_o}\left(1 + \frac{D}{f_e}\right)$$

Since the object is placed very close to the principal focus of the objective and the image is formed very close to the eyepiece,
$u_o \approx f_o$ and $v_o \approx L$

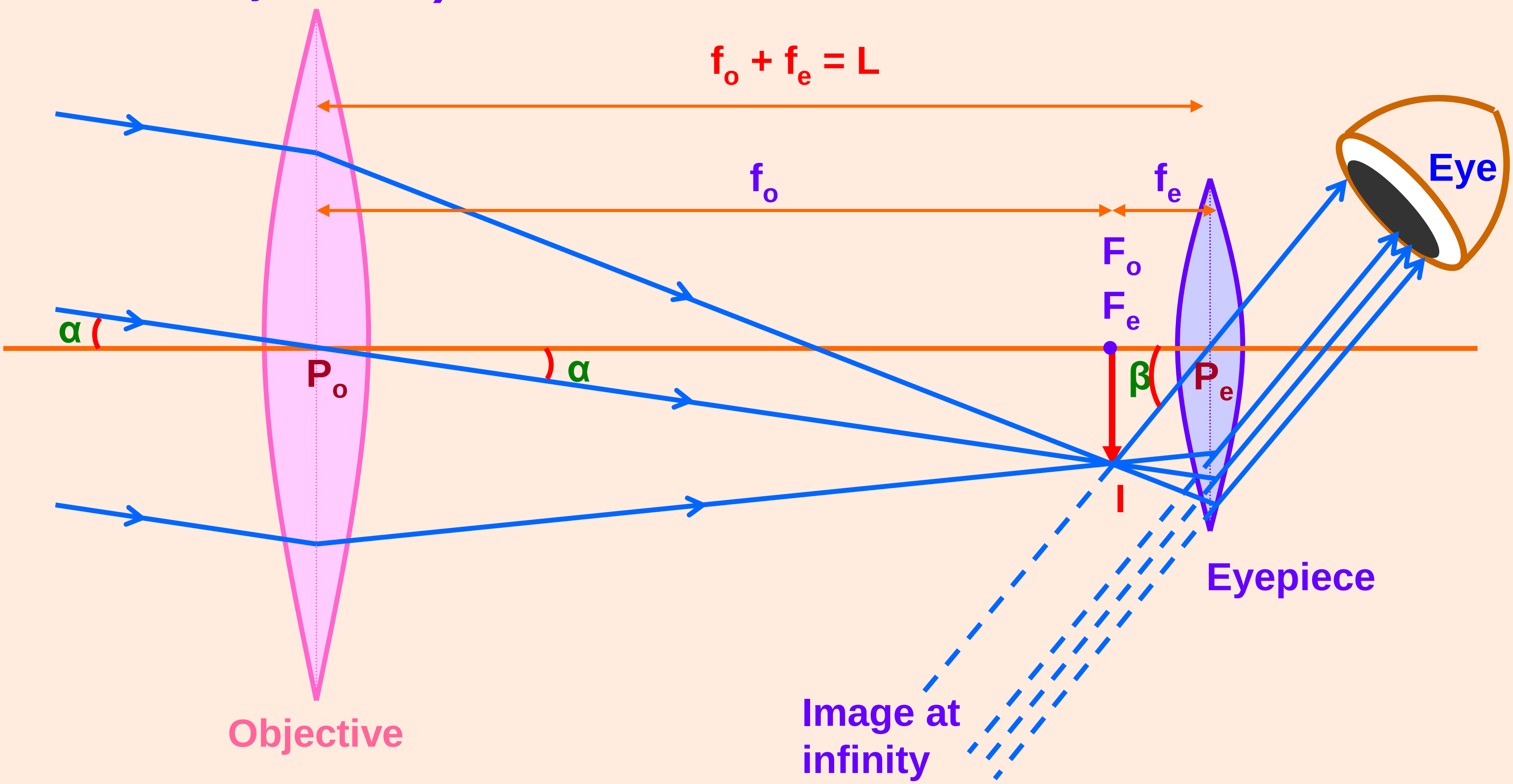
$$M = \frac{-L}{f_o}\left(1 + \frac{D}{f_e}\right)$$

or

$$M \approx \frac{-L}{f_o} \times \frac{D}{f_e}$$

(Normal adjustment i.e. image at infinity)

Astronomical Telescope: (Image formed at infinity – Normal Adjustment)



Focal length of the objective is much greater than that of the eyepiece.

Aperture of the objective is also large to allow more light to pass through it.

Angular magnification or Magnifying power of a telescope in normal adjustment is the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object as seen directly, when both the object and the image are at infinity.

$$M = \frac{\beta}{\alpha}$$

Since angles are small, $\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

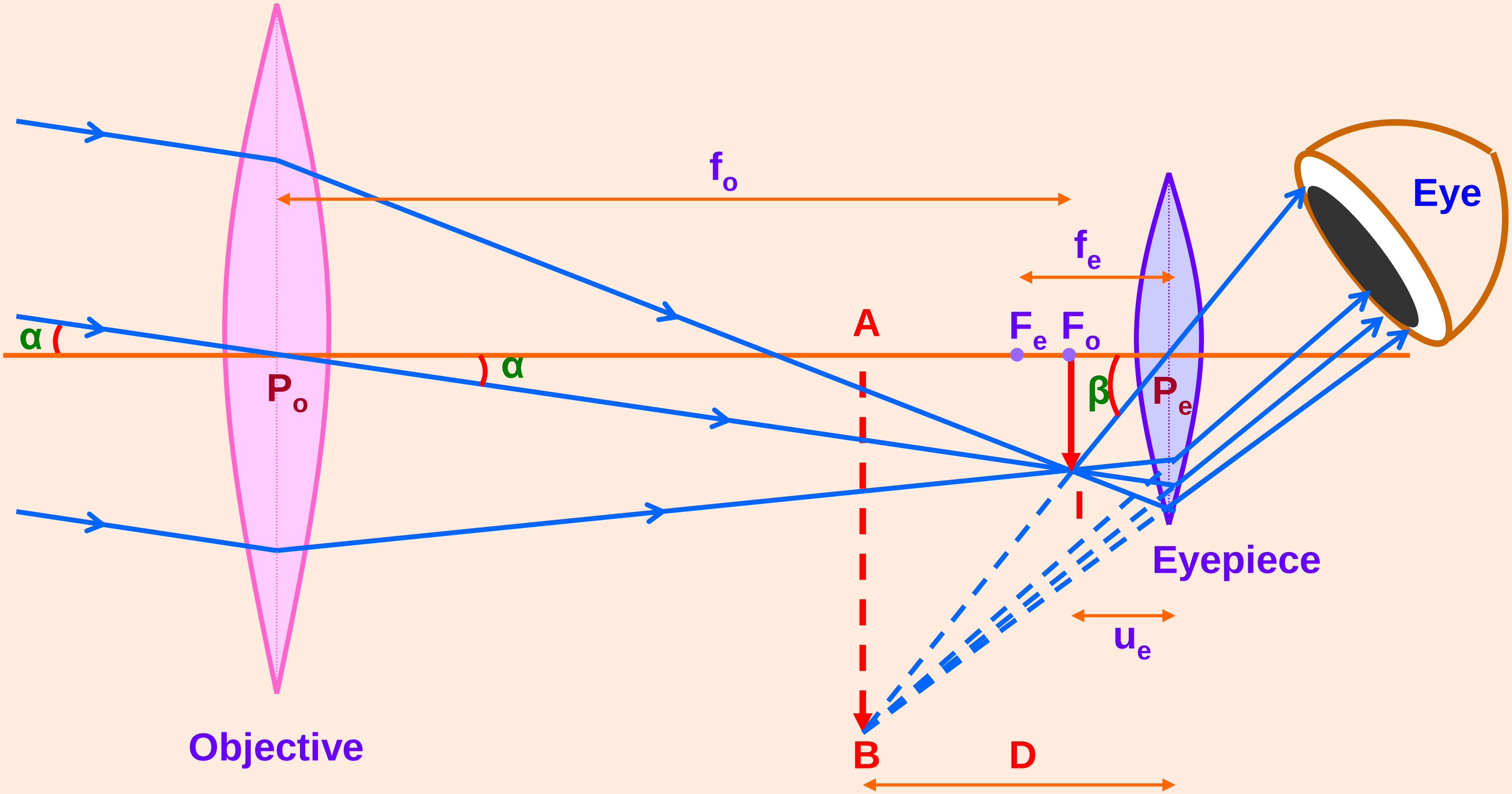
$$M = \frac{F_e I}{P_e F_e} \;/\; \frac{F_e I}{P_o F_e}$$

$$M = \frac{-I}{-f_e} \;/\; \frac{-I}{f_o}$$

$$M = \frac{-f_o}{f_e}$$

($f_o + f_e = L$ is called the length of the telescope in normal adjustment).

Astronomical Telescope: (Image formed at LDDV)



Angular magnification or magnifying power of a telescope in this case is defined as the ratio of the angle β subtended at the eye by the final image formed at the least distance of distinct vision to the angle α subtended at the eye by the object lying at infinity when seen directly.

$$M = \frac{\beta}{\alpha}$$

Since angles are small,
$\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{F_o I}{P_e F_o} \Big/ \frac{F_o I}{P_o F_o}$$

$$M = \frac{P_o F_o}{P_e F_o} \quad \text{or} \quad M = \frac{+ f_o}{- u_e}$$

Lens Equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{becomes}$$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\text{or} \quad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

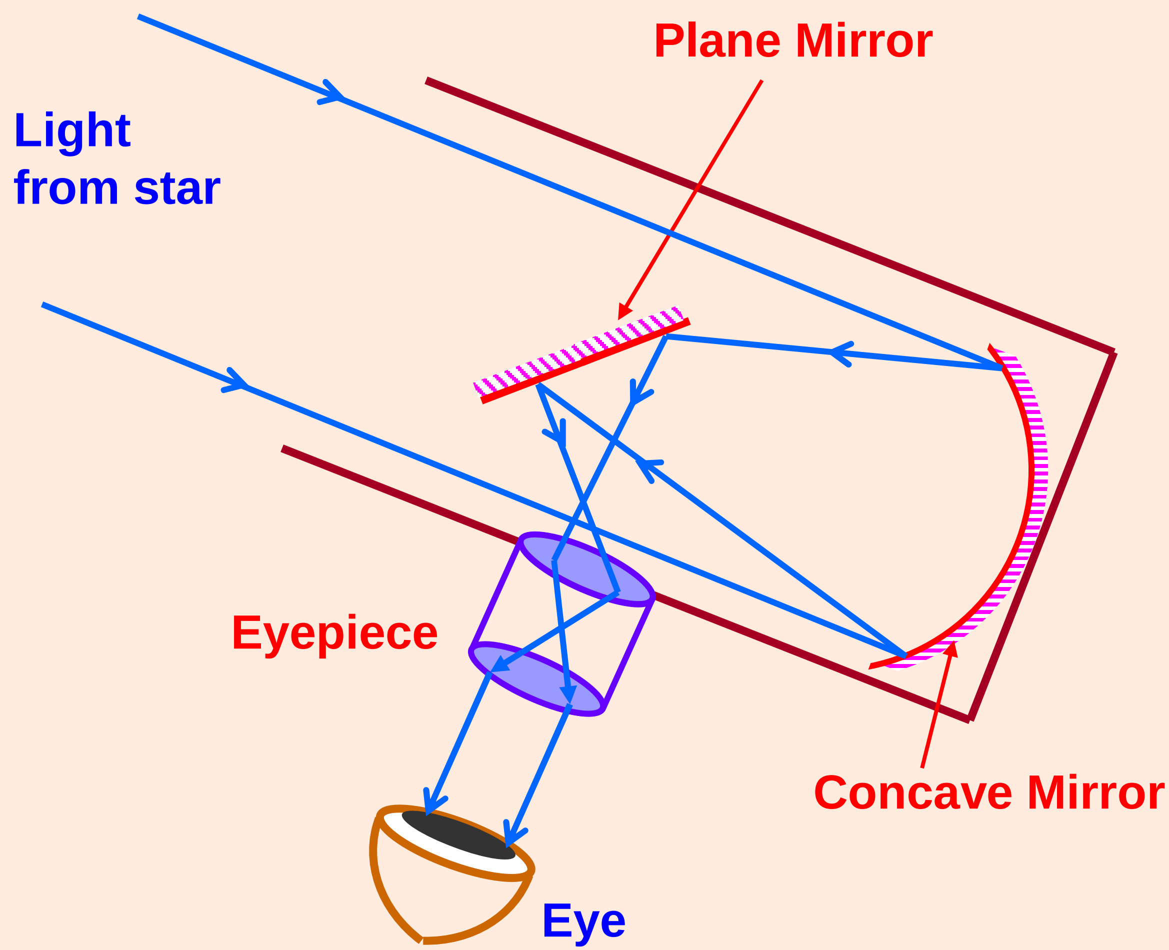
Multiplying by f_o on both sides and rearranging, we get

$$M = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Clearly focal length of objective must be greater than that of the eyepiece for larger magnifying power.

Also, it is to be noted that in this case M is larger than that in normal adjustment position.

Newtonian Telescope: (Reflecting Type)



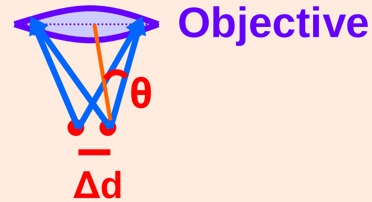
Magnifying Power:

$$M = \frac{f_o}{f_e}$$

Resolving Power of a Microscope:

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope.

$$\text{Resolving Power} = \frac{1}{\Delta d} = \boxed{\frac{2\,\mu \sin\theta}{\lambda}}$$

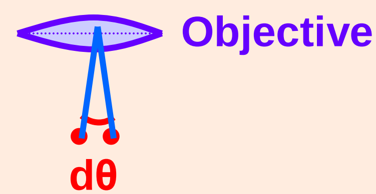


Resolving power depends on i) wavelength λ, ii) refractive index of the medium between the object and the objective and iii) half angle of the cone of light from one of the objects θ.

Resolving Power of a Telescope:

The resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects whose images are seen separately.

$$\text{Resolving Power} = \frac{1}{d\theta} = \boxed{\frac{a}{1.22\,\lambda}}$$

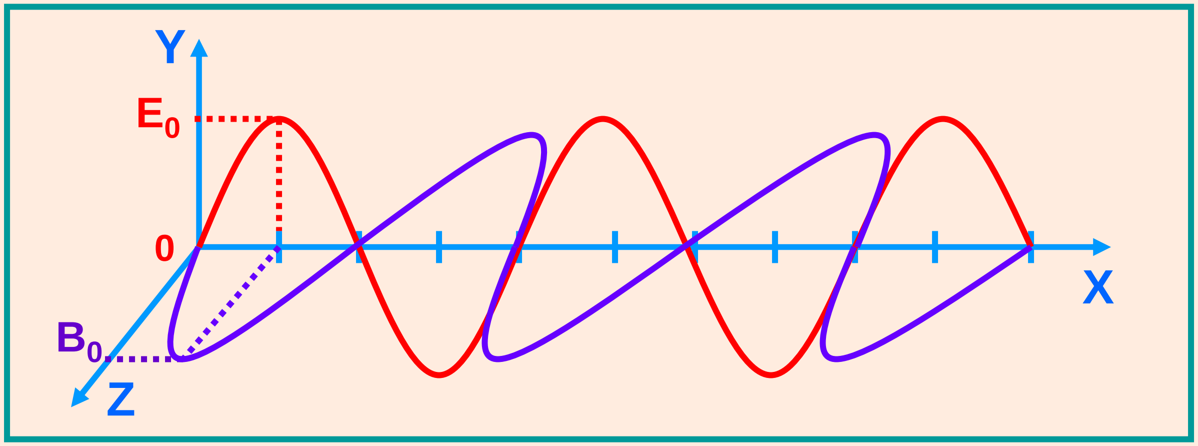


Resolving power depends on i) wavelength λ, ii) diameter of the objective a.

WAVE OPTICS - I

1. Electromagnetic Wave
2. Wavefront
3. Huygens' Principle
4. Reflection of Light based on Huygens' Principle
5. Refraction of Light based on Huygens' Principle
6. Behaviour of Wavefront in a Mirror, Lens and Prism
7. Coherent Sources
8. Interference
9. Young's Double Slit Experiment
10. Colours in Thin Films

Electromagnetic Wave:



1. **Variations in both electric and magnetic fields occur simultaneously. Therefore, they attain their maxima and minima at the same place and at the same time.**
2. **The direction of electric and magnetic fields are mutually perpendicular to each other and as well as to the direction of propagation of wave.**
3. **The speed of electromagnetic wave depends entirely on the electric and magnetic properties of the medium, in which the wave travels and not on the amplitudes of their variations.**

Wave is propagating along X – axis with speed $c = 1 / \sqrt{\mu_0 \varepsilon_0}$

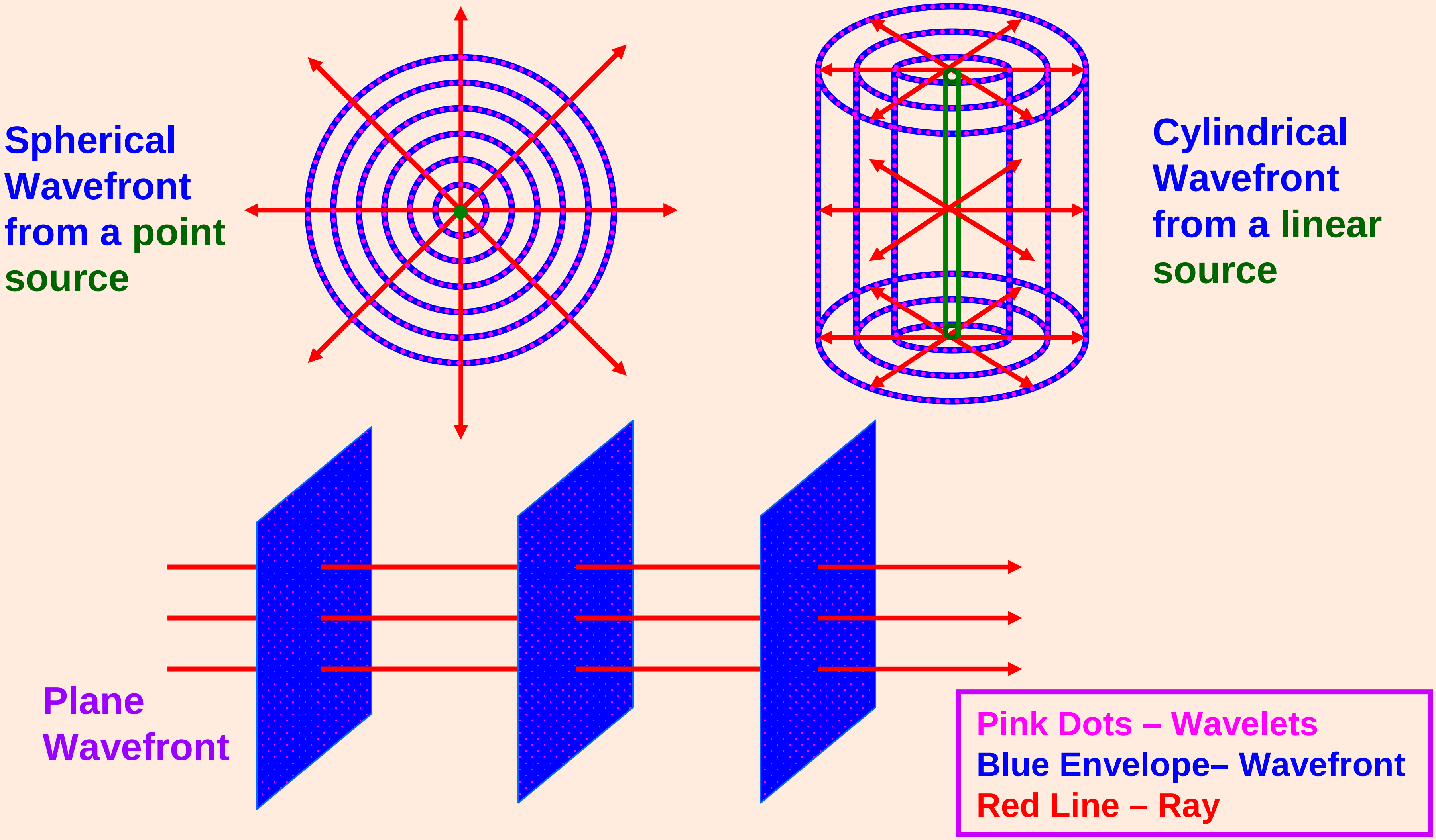
For discussion of optical property of EM wave, more significance is given to Electric Field, E. Therefore, Electric Field is called 'light vector'.

Wavefront:

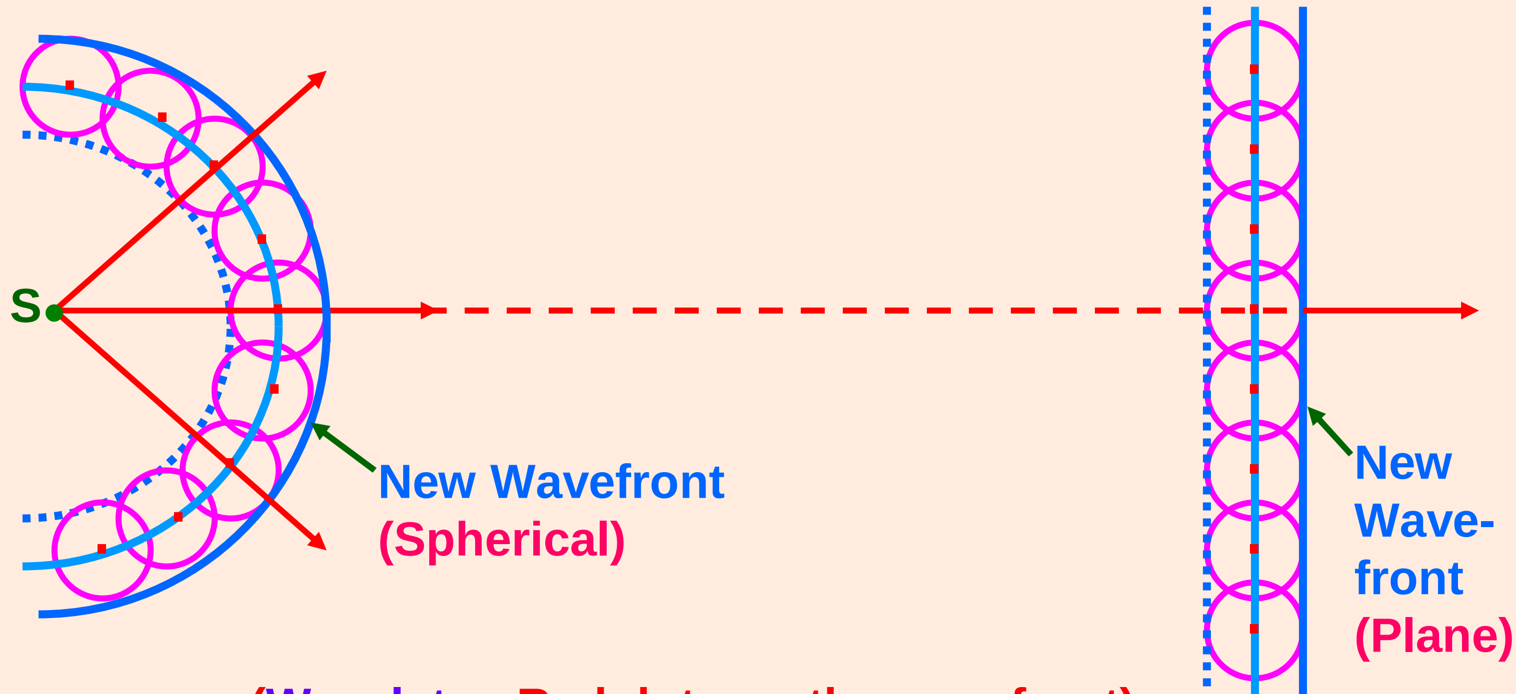
A wavelet is the point of disturbance due to propagation of light.

A wavefront is the locus of points (wavelets) having the same phase of oscillations.

A line perpendicular to a wavefront is called a ‘ray’.



Huygens' Construction or Huygens' Principle of Secondary Wavelets:



(Wavelets - Red dots on the wavefront)

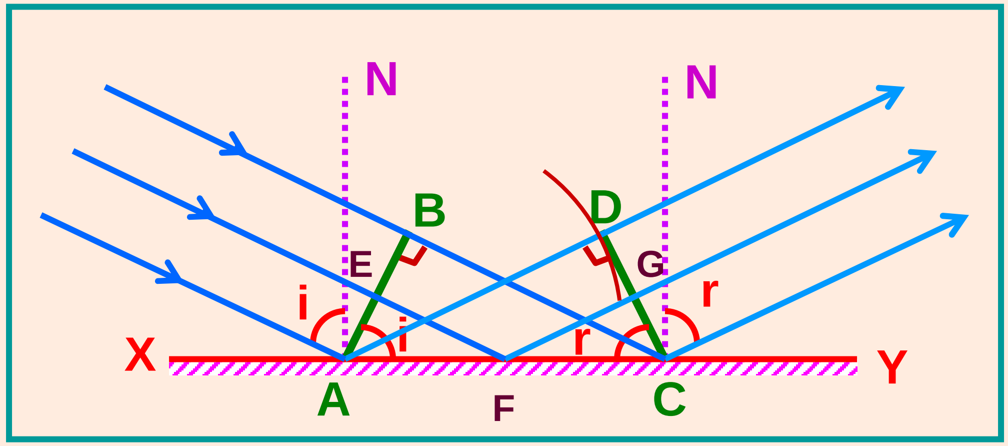
1. Each point on a wavefront acts as a fresh source of disturbance of light.
2. The new wavefront at any time later is obtained by taking the forward envelope of all the secondary wavelets at that time.

Note: Backward wavefront is rejected. Why?

Amplitude of secondary wavelet is proportional to ½ $(1+\cos\theta)$. Obviously, for the backward wavelet $\theta = 180°$ and $(1+\cos\theta)$ is 0.

Laws of Reflection at a Plane Surface (On Huygens' Principle):

If c be the speed of light, t be the time taken by light to go from B to C or A to D or E to G through F, then



$$t = \frac{EF}{c} + \frac{FG}{c}$$

$$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$$

$$t = \frac{AC \sin r + AF (\sin i - \sin r)}{c}$$

AB – Incident wavefront

CD – Reflected wavefront

XY – Reflecting surface

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only if $\sin i - \sin r = 0$.

i.e. $\sin i = \sin r$ or $i = r$

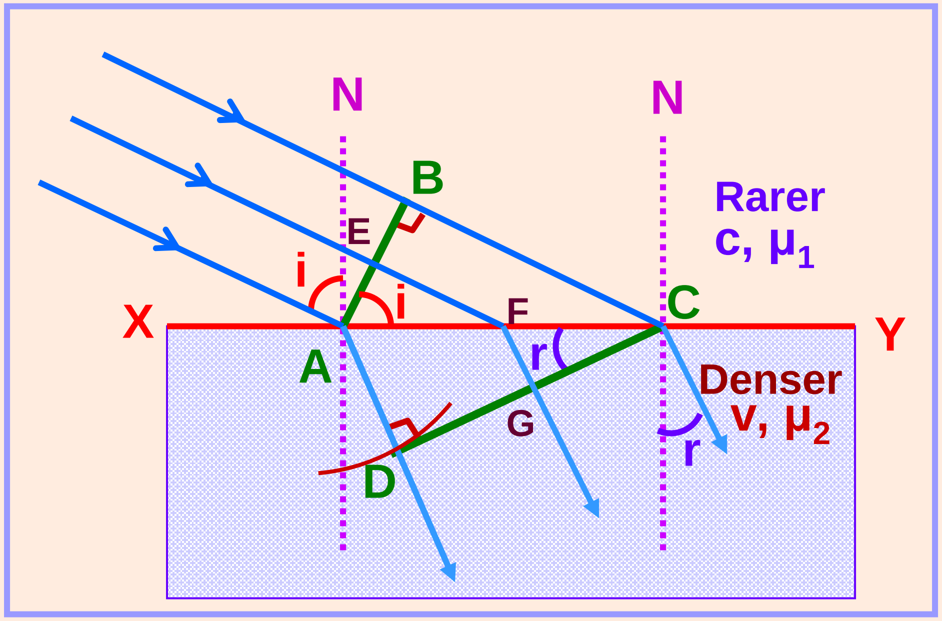
Laws of Refraction at a Plane Surface (On Huygens' Principle):

If c be the speed of light, t be the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{v}$$

$$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{v}$$

$$t = \frac{AC \sin r}{v} + AF\left(\frac{\sin i}{c} - \frac{\sin r}{v}\right)$$



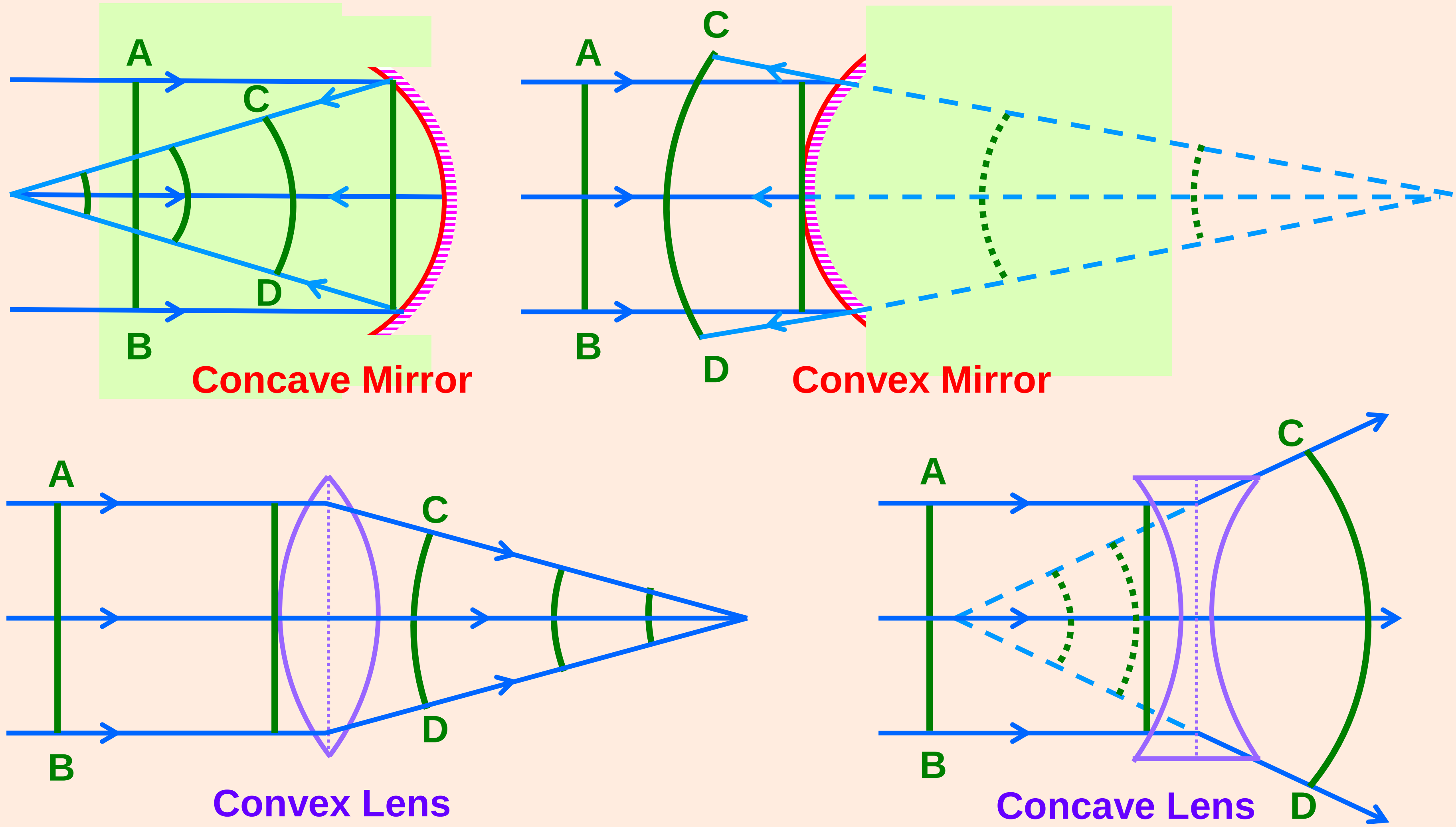
AB – Incident wavefront
CD – Refracted wavefront
XY – Refracting surface

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront.

So, t should not depend upon AF. This is possible only

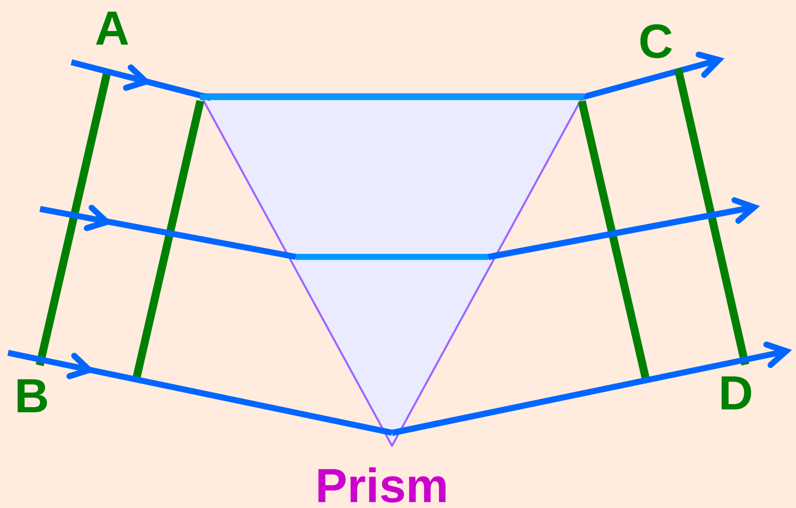
if $\frac{\sin i}{c} - \frac{\sin r}{v} = 0$ or $\frac{\sin i}{c} = \frac{\sin r}{v}$ or $\frac{\sin i}{\sin r} = \frac{c}{v} = \mu$

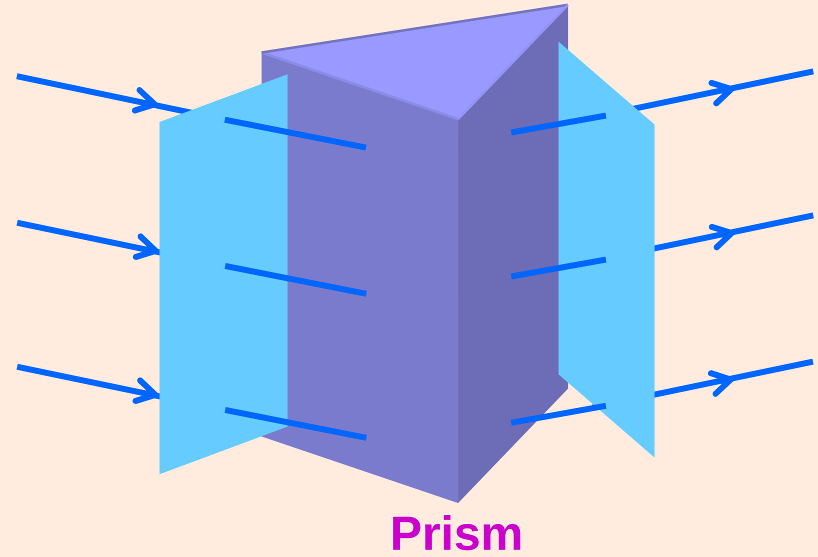
Behaviour of a Plane Wavefront in a Concave Mirror, Convex Mirror, Convex Lens, Concave Lens and Prism:



AB – Incident wavefront

CD – Reflected / Refracted wavefront





AB – Incident wavefront CD –Refracted wavefront

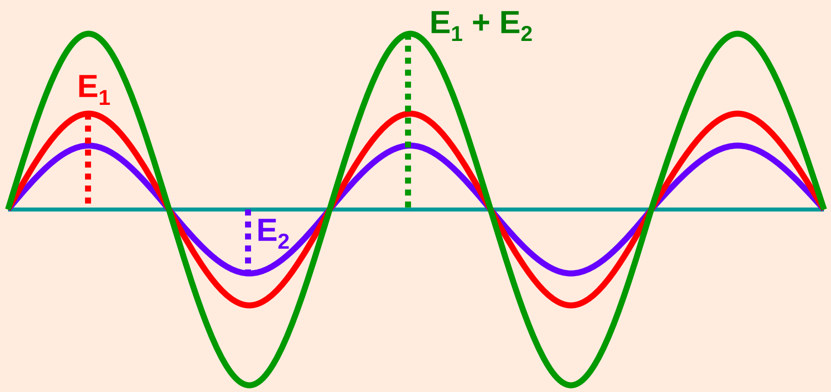
Coherent Sources:

Coherent Sources of light are those sources of light which emit light waves of same wavelength, same frequency and in same phase or having constant phase difference.

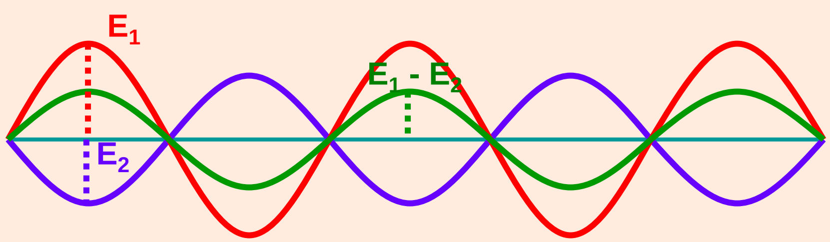
Coherent sources can be produced by two methods:

1. By division of wavefront (Young’s Double Slit Experiment, Fresnel’s Biprism and Lloyd’s Mirror)
2. By division of amplitude (Partial reflection or refraction)

Interference of Waves:

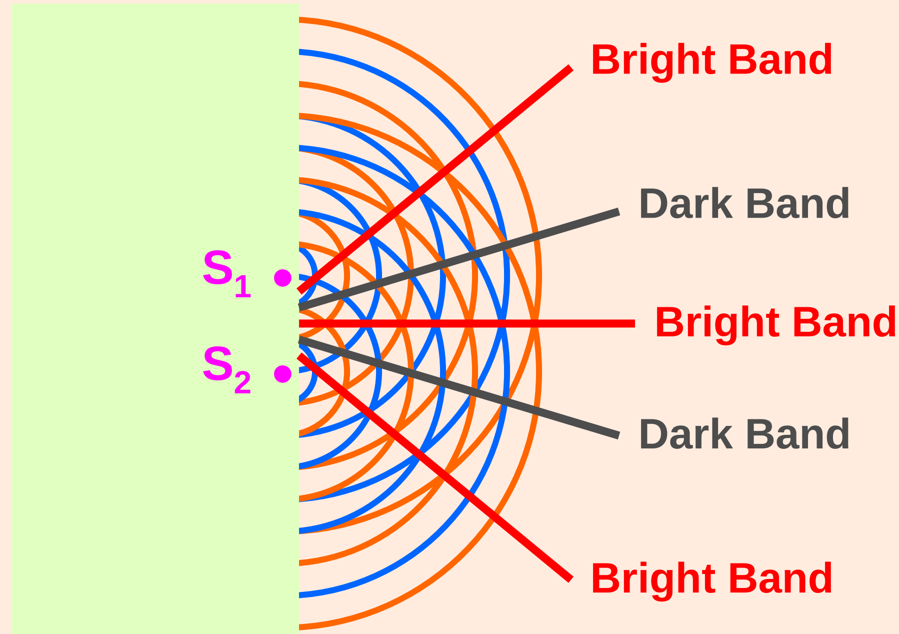


Constructive Interference $E = E_1 + E_2$



Destructive Interference $E = E_1 - E_2$

- 1st Wave (E_1)
- 2nd Wave (E_2)
- Resultant Wave
- Reference Line



- Crest
- Trough
- Bright Band
- Dark Band

The phenomenon of one wave interfering with another and the resulting redistribution of energy in the space around the two sources of disturbance is called interference of waves.

Theory of Interference of Waves:

$E_1 = a \sin \omega t$

$E_2 = b \sin (\omega t + \Phi)$

The waves are with same speed, wavelength, frequency, time period, nearly equal amplitudes, travelling in the same direction with constant phase difference of Φ.

ω is the angular frequency of the waves, a,b are the amplitudes and E_1, E_2 are the instantaneous values of Electric displacement.

Applying superposition principle, the magnitude of the resultant displacement of the waves is

$$E = E_1 + E_2$$

$$E = a \sin \omega t + b \sin (\omega t + \Phi)$$

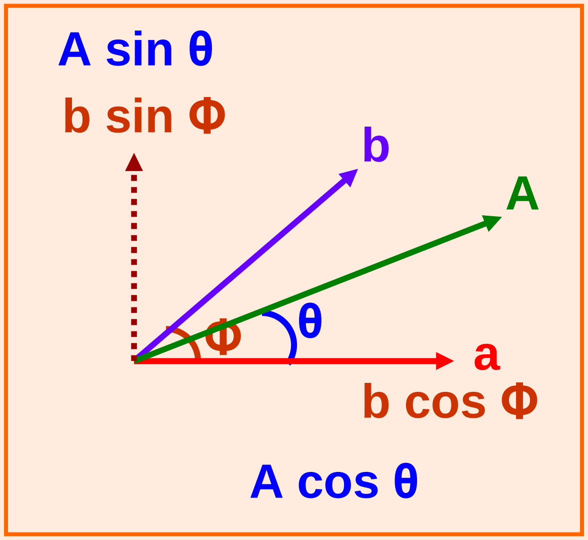
$$E = (a + b \cos \Phi) \sin \omega t + b \sin \Phi \cos \omega t$$

Putting $a + b \cos \Phi = A \cos \theta$

$b \sin \Phi = A \sin \theta$

We get $E = A \sin (\omega t + \theta)$

(where E is the resultant displacement, A is the resultant amplitude and θ is the resultant phase difference)



$$A = \sqrt{(a^2 + b^2 + 2ab \cos \Phi)}$$

$$\tan \theta = \frac{b \sin \Phi}{a + b \cos \Phi}$$

$$A = \sqrt{(a^2 + b^2 + 2ab \cos \Phi)}$$

Intensity I is proportional to the square of the amplitude of the wave.

So, $I \propto A^2$ i.e. $I \propto (a^2 + b^2 + 2ab \cos \Phi)$

Condition for Constructive Interference of Waves:

For constructive interference, I should be maximum which is possible only if $\cos \Phi = +1$.

i.e. $\Phi = 2n\pi$ **where n = 0, 1, 2, 3, …….**

Corresponding path difference is $\Delta = (\lambda / 2\pi) \times 2n\pi$

$$\Delta = n\lambda$$

$$I_{max} \propto (a + b)^2$$

Condition for Destructive Interference of Waves:

For destructive interference, I should be minimum which is possible only if $\cos \Phi = -1$.

i.e. $\Phi = (2n + 1)\pi$ **where n = 0, 1, 2, 3, …….**

Corresponding path difference is $\Delta = (\lambda / 2\pi) \times (2n + 1)\pi$

$$\Delta = (2n + 1) \lambda / 2$$

$$I_{min} \propto (a - b)^2$$

Comparison of intensities of maxima and minima:

$$I_{max} \ \alpha \ (a + b)^2$$

$$I_{min} \ \alpha \ (a - b)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a + b)^2}{(a - b)^2} = \frac{(a/b + 1)^2}{(a/b - 1)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(r + 1)^2}{(r - 1)^2}$$

where $r = a / b$ (ratio of the amplitudes)

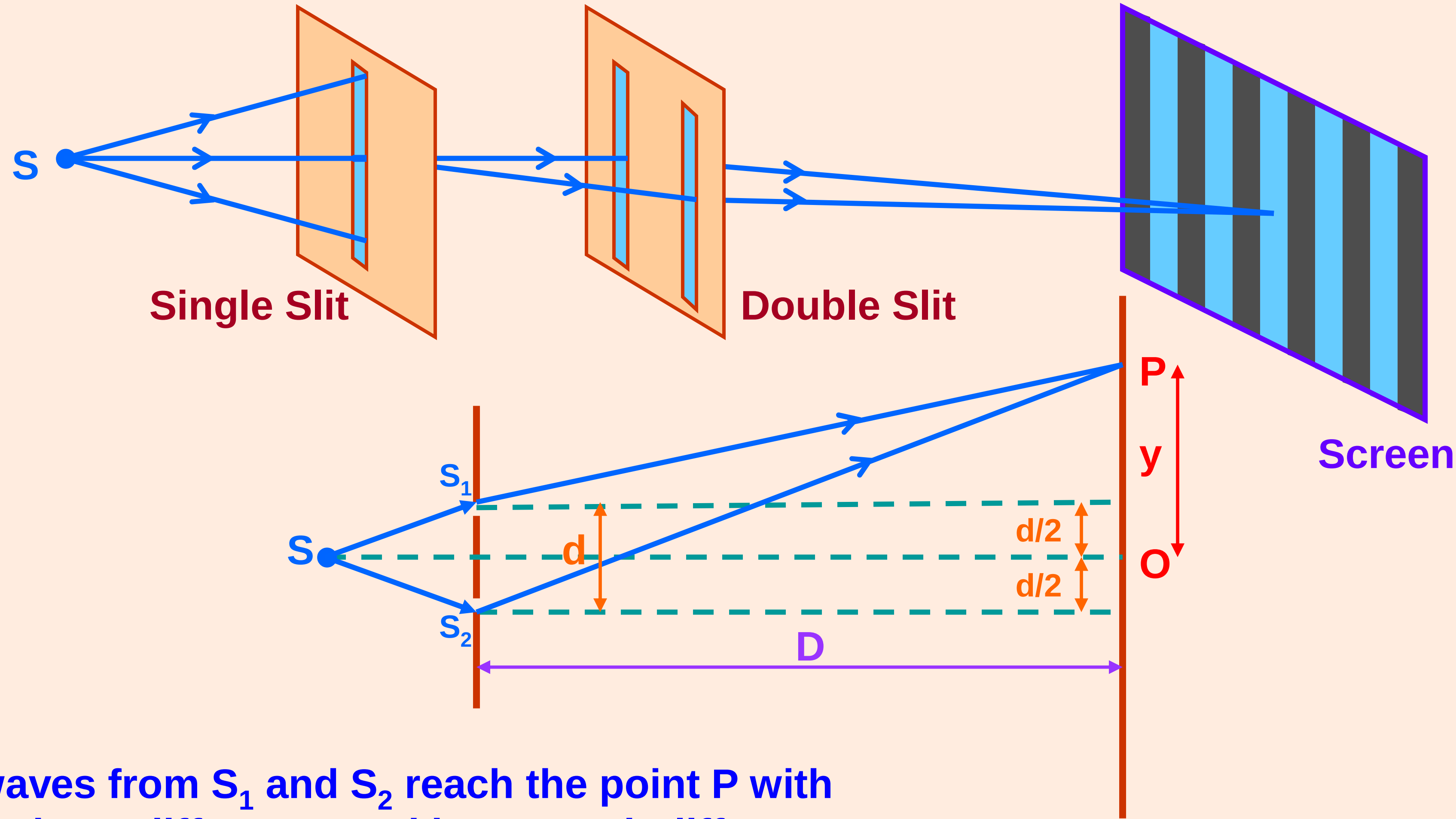
Relation between Intensity (I), Amplitude (a) of the wave and Width (w) of the slit:

$$I \ \alpha \ a^2$$

$$a \ \alpha \ \sqrt{w}$$

$$\frac{I_1}{I_2} = \frac{(a_1)^2}{(a_2)^2} = \frac{w_1}{w_2}$$

Young's Double Slit Experiment:



The waves from S_1 and S_2 reach the point P with some phase difference and hence path difference

$\Delta = S_2P - S_1P$

$S_2P^2 - S_1P^2 = [D^2 + \{y + (d/2)\}^2] - [D^2 + \{y - (d/2)\}^2]$

$(S_2P - S_1P)\ (S_2P + S_1P) = 2\ yd$

$\Delta\ (2D) = 2\ yd$

$\Delta = yd / D$

Positions of Bright Fringes:

For a bright fringe at P,

$\Delta = yd / D = n\lambda$

where n = 0, 1, 2, 3, …

$y = n D \lambda / d$

For n = 0, $y_0 = 0$

For n = 1, $y_1 = D \lambda / d$

For n = 2, $y_2 = 2 D \lambda / d$ ……

For n = n, $y_n = n D \lambda / d$

Positions of Dark Fringes:

For a dark fringe at P,

$\Delta = yd / D = (2n+1)\lambda/2$

where n = 0, 1, 2, 3, …

$y = (2n+1) D \lambda / 2d$

For n = 0, $y_0' = D \lambda / 2d$

For n = 1, $y_1' = 3D \lambda / 2d$

For n = 2, $y_2' = 5D \lambda / 2d$ …..

For n = n, $y_n' = (2n+1)D \lambda / 2d$

Expression for Dark Fringe Width:

$\beta_D = y_n - y_{n-1}$

$= n D \lambda / d - (n - 1) D \lambda / d$

$= D \lambda / d$

Expression for Bright Fringe Width:

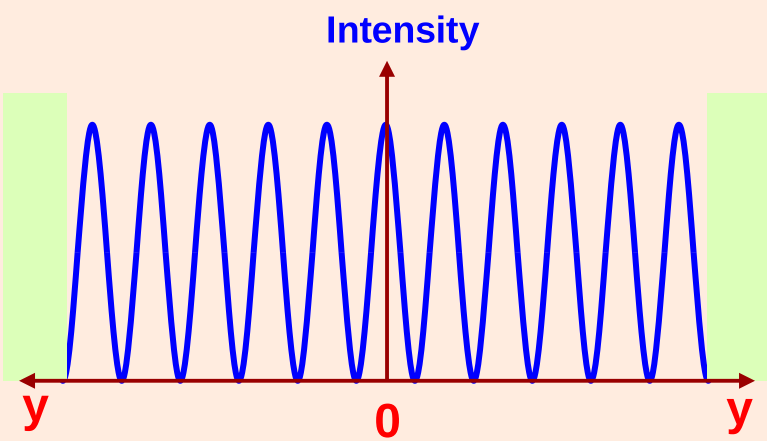
$\beta_B = y_n' - y_{n-1}'$

$= (2n+1) D \lambda / 2d - \{2(n-1)+1\} D \lambda / 2d$

$= D \lambda / d$

The expressions for fringe width show that the fringes are equally spaced on the screen.

Distribution of Intensity:



Suppose the two interfering waves have same amplitude say 'a', then

$I_{max} \propto (a+a)^2$ i.e. $I_{max} \propto 4a^2$

All the bright fringes have this same intensity.

$I_{min} = 0$

All the dark fringes have zero intensity.

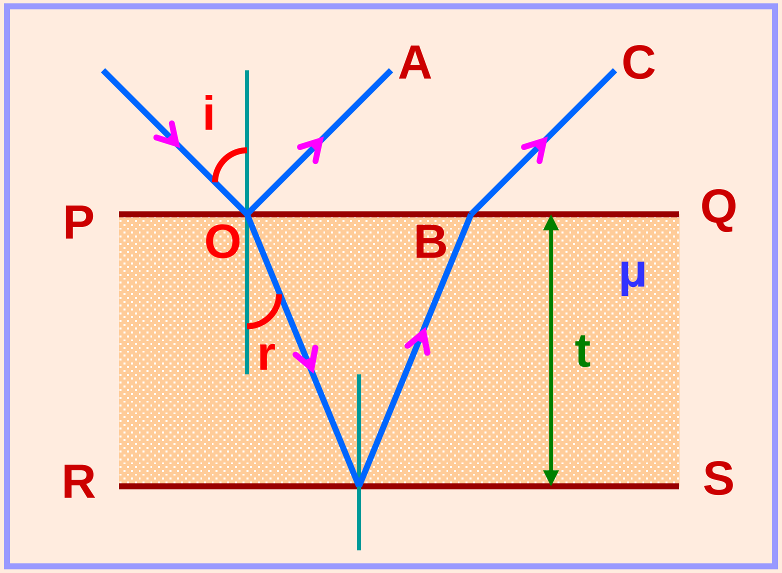
Conditions for sustained interference:

1. The two sources producing interference must be coherent.
2. The two interfering wave trains must have the same plane of polarisation.
3. The two sources must be very close to each other and the pattern must be observed at a larger distance to have sufficient width of the fringe. ($D \lambda / d$)
4. The sources must be monochromatic. Otherwise, the fringes of different colours will overlap.
5. The two waves must be having same amplitude for better contrast between bright and dark fringes.

Colours in Thin Films:

It can be proved that the path difference between the light partially reflected from PQ and that from partially transmitted and then reflected from RS is

$$\Delta = 2\mu t \cos r$$



Since there is a reflection at O, the ray OA suffers an additional phase difference of π and hence the corresponding path difference of λ/2.

For the rays OA and BC to interfere constructively (Bright fringe), the path difference must be $(n + ½) \lambda$

So, $2\mu t \cos r = (n + ½) \lambda$

For the rays OA and BC to interfere destructively (Dark fringe), the path difference must be $n\lambda$

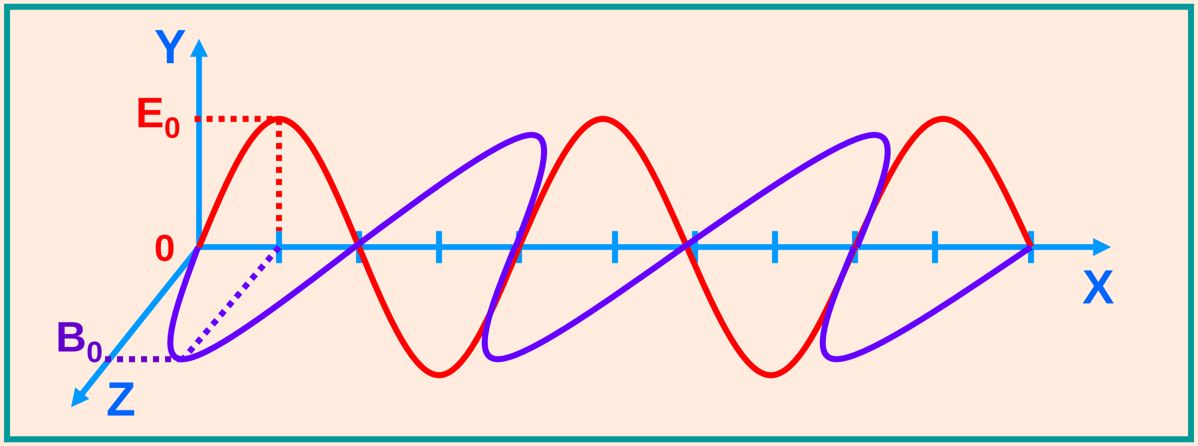
So, $2\mu t \cos r = n \lambda$

When white light from the sun falls on thin layer of oil spread over water in the rainy season, beautiful rainbow colours are formed due to interference of light.

WAVE OPTICS - II

1. Electromagnetic Wave
2. Diffraction
3. Diffraction at a Single Slit
4. Theory of Diffraction
5. Width of Central Maximum and Fresnel's Distance
6. Difference between Interference and Diffraction
7. Polarisation of Mechanical Waves
8. Polarisation of Light
9. Malus' Law
10. Polarisation by Reflection – Brewster's Law
11. Polaroids and their uses

Electromagnetic Wave:



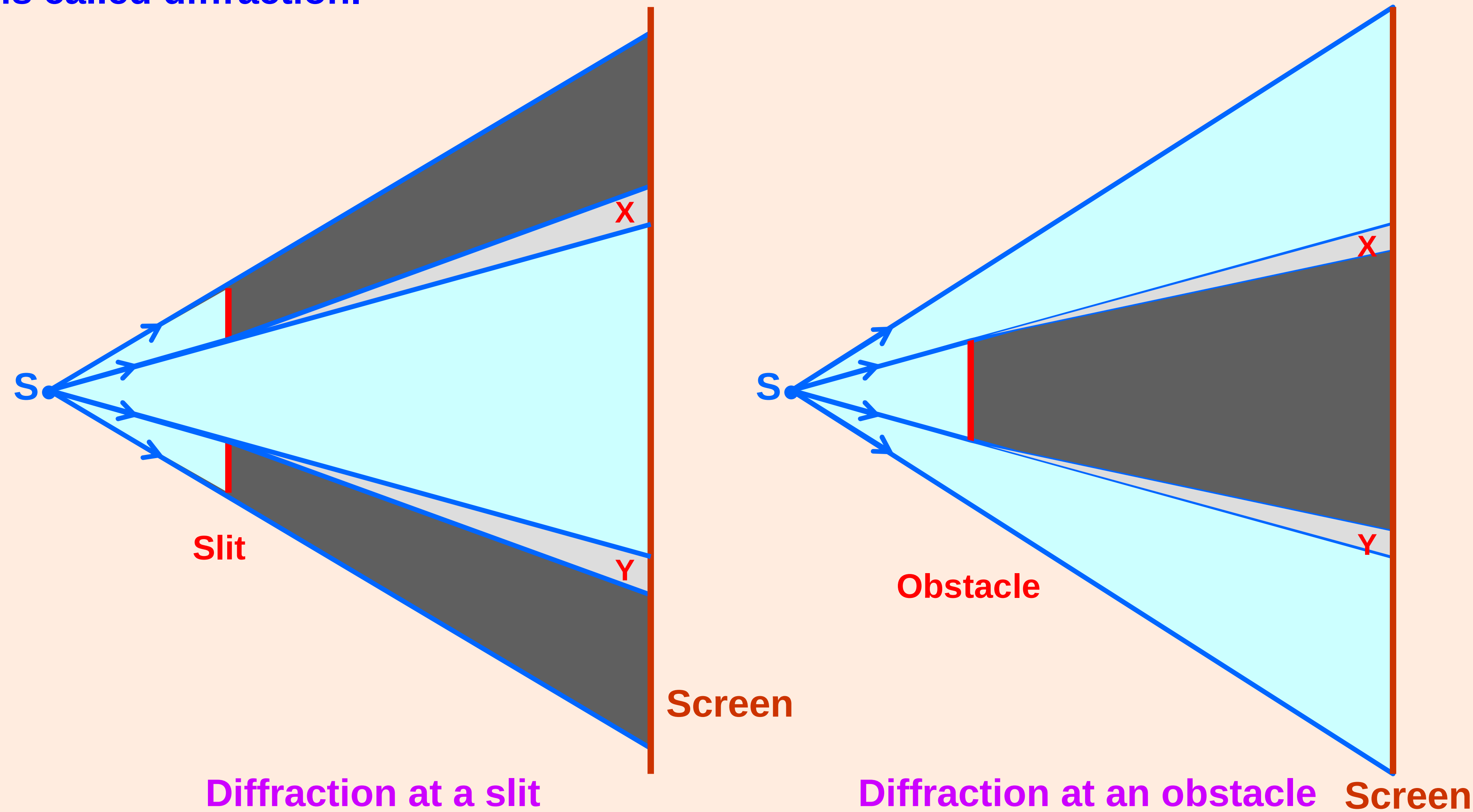
1. Variations in both electric and magnetic fields occur simultaneously. Therefore, they attain their maxima and minima at the same place and at the same time.
2. The direction of electric and magnetic fields are mutually perpendicular to each other and as well as to the direction of propagation of wave.
3. The speed of electromagnetic wave depends entirely on the electric and magnetic properties of the medium, in which the wave travels and not on the amplitudes of their variations.

Wave is propagating along X – axis with speed $c = 1 / \sqrt{\mu_0 \varepsilon_0}$

For discussion of EM wave, more significance is given to Electric Field, E.

Diffraction of light:

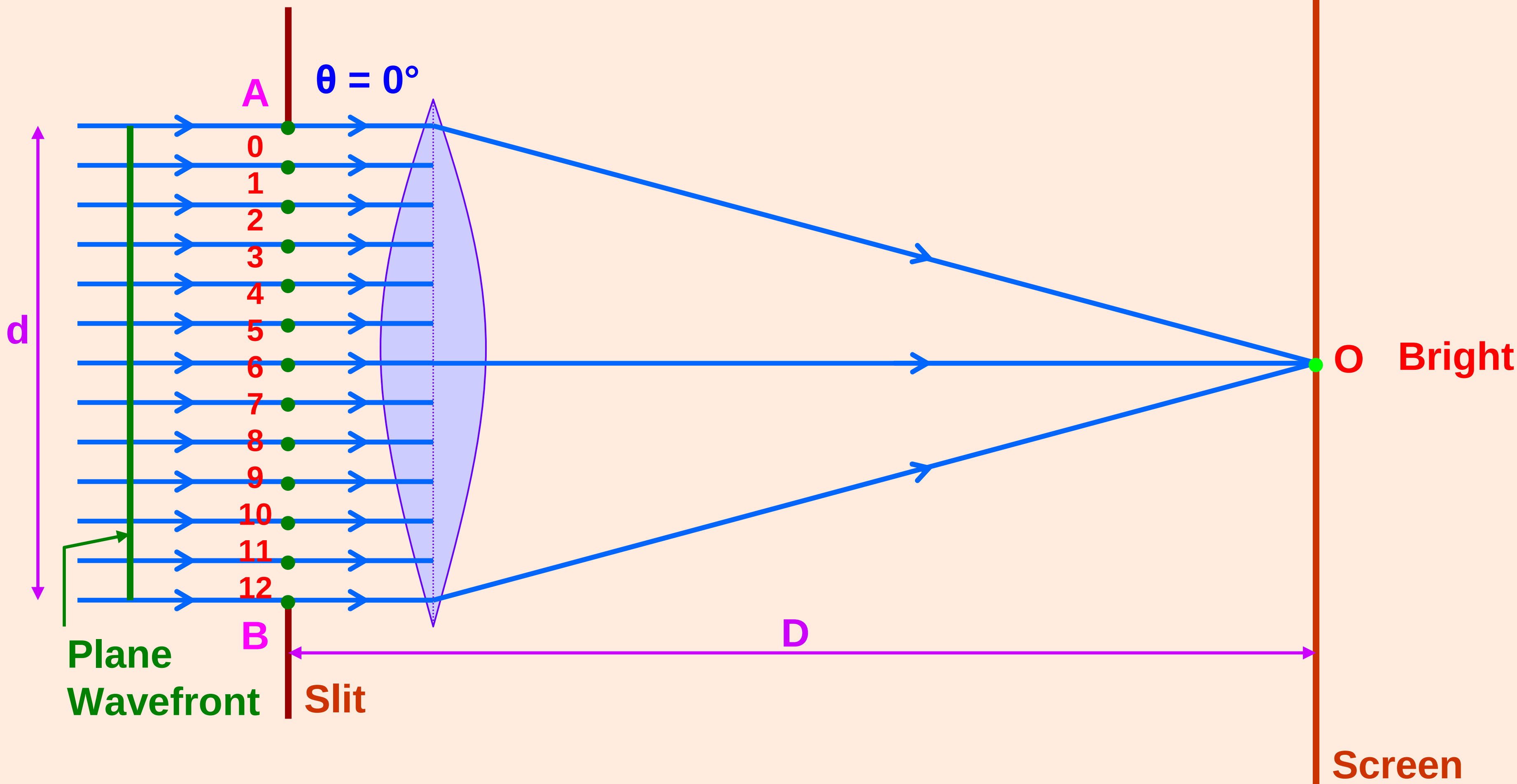
The phenomenon of bending of light around the corners and the encroachment of light within the geometrical shadow of the opaque obstacles is called diffraction.



X & Y – Region of diffraction

Diffraction of light at a single slit:

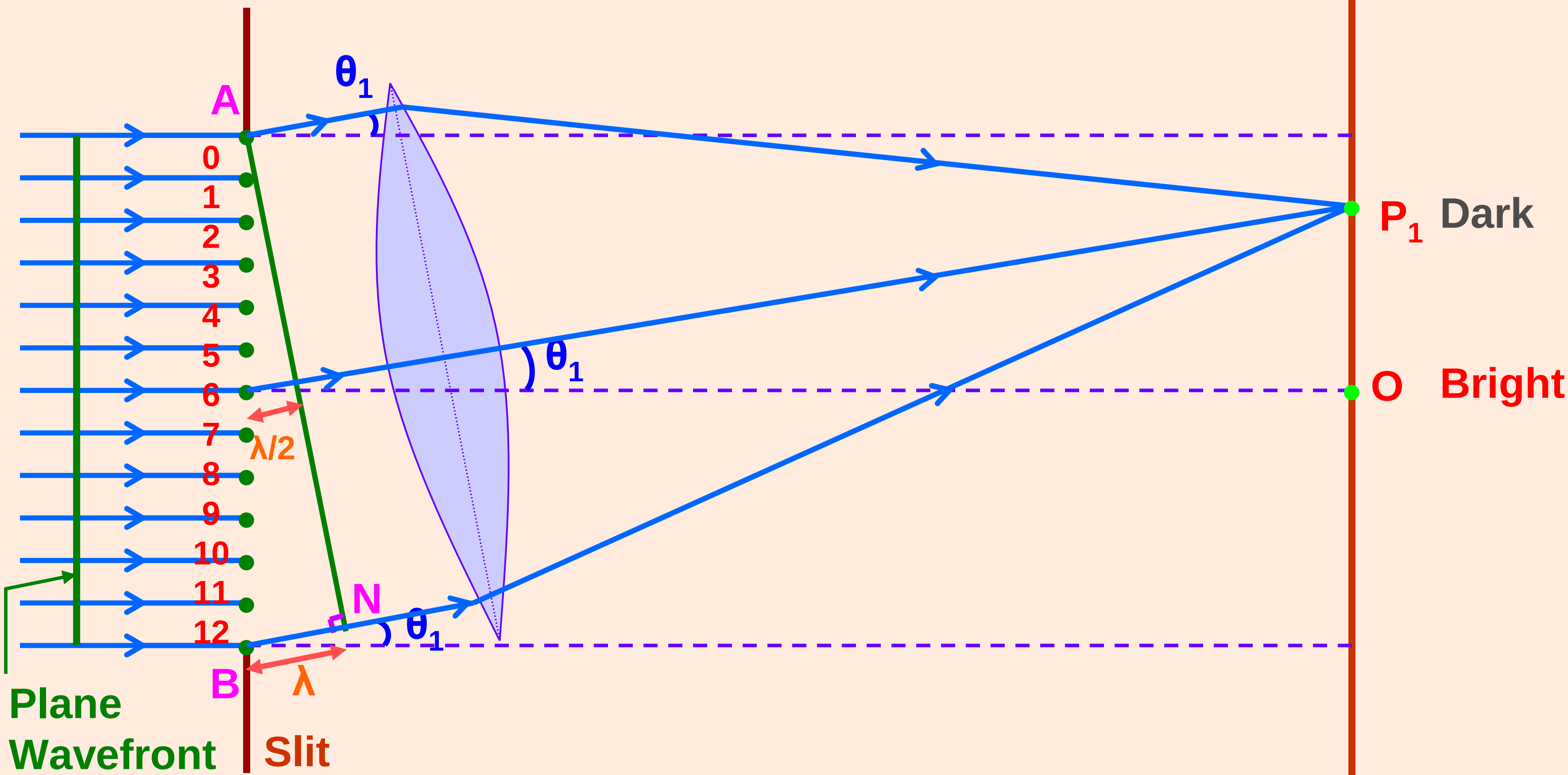
1) At an angle of diffraction θ = 0°:



The wavelets from the single wavefront reach the centre O on the screen in same phase and hence interfere constructively to give Central or Primary Maximum (Bright fringe).

2) At an angle of diffraction $\theta = \theta_1$:

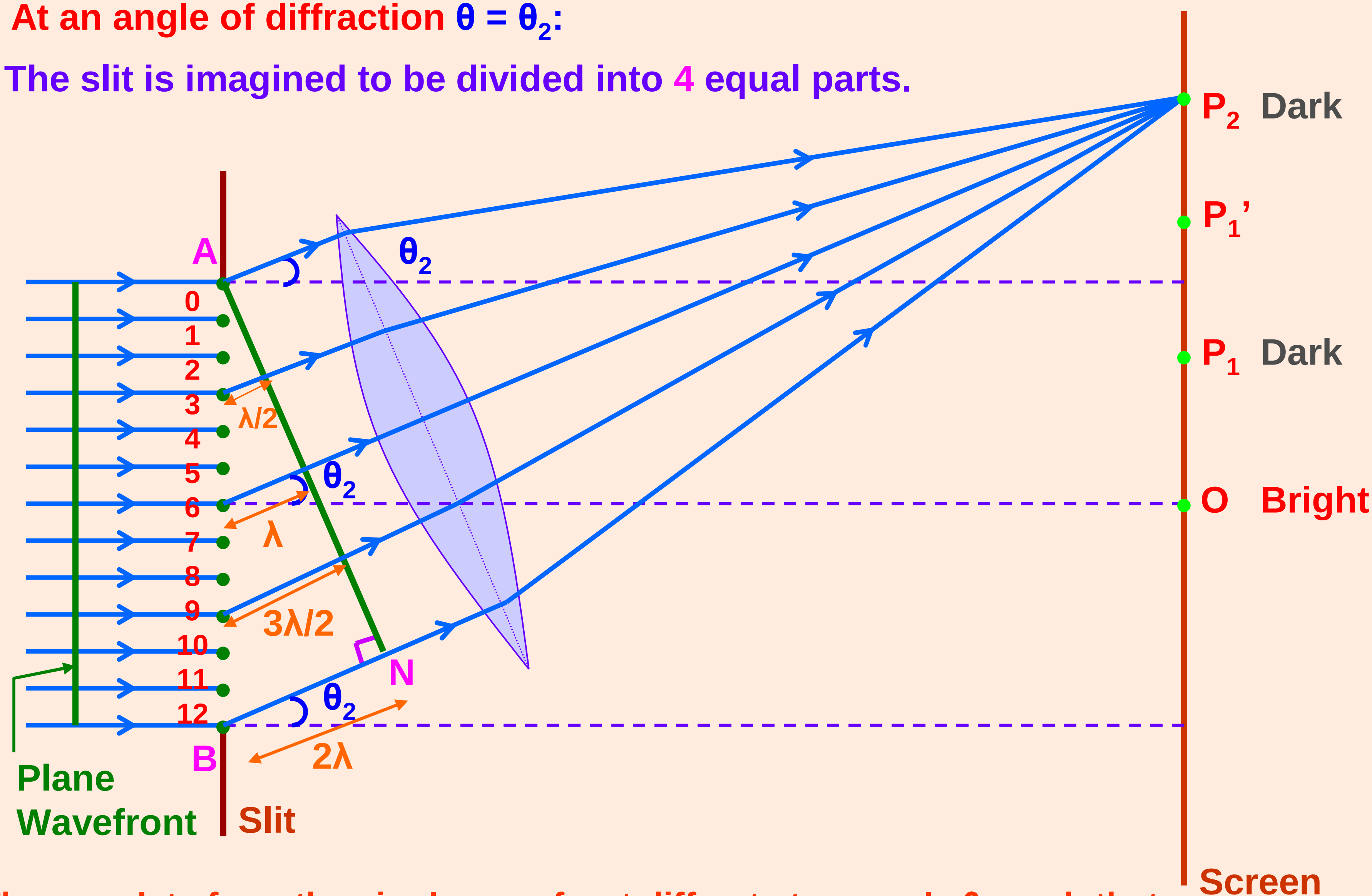
The slit is imagined to be divided into 2 equal halves.



The wavelets from the single wavefront diffract at an angle θ_1 such that BN is λ and reach the point P_1. The pairs (0,6), (1,7), (2,8), (3,9), (4,10), (5,11) and (6,12) interfere destructively with path difference λ/2 and give First Secondary Minimum (Dark fringe).

3) At an angle of diffraction $\theta = \theta_2$:

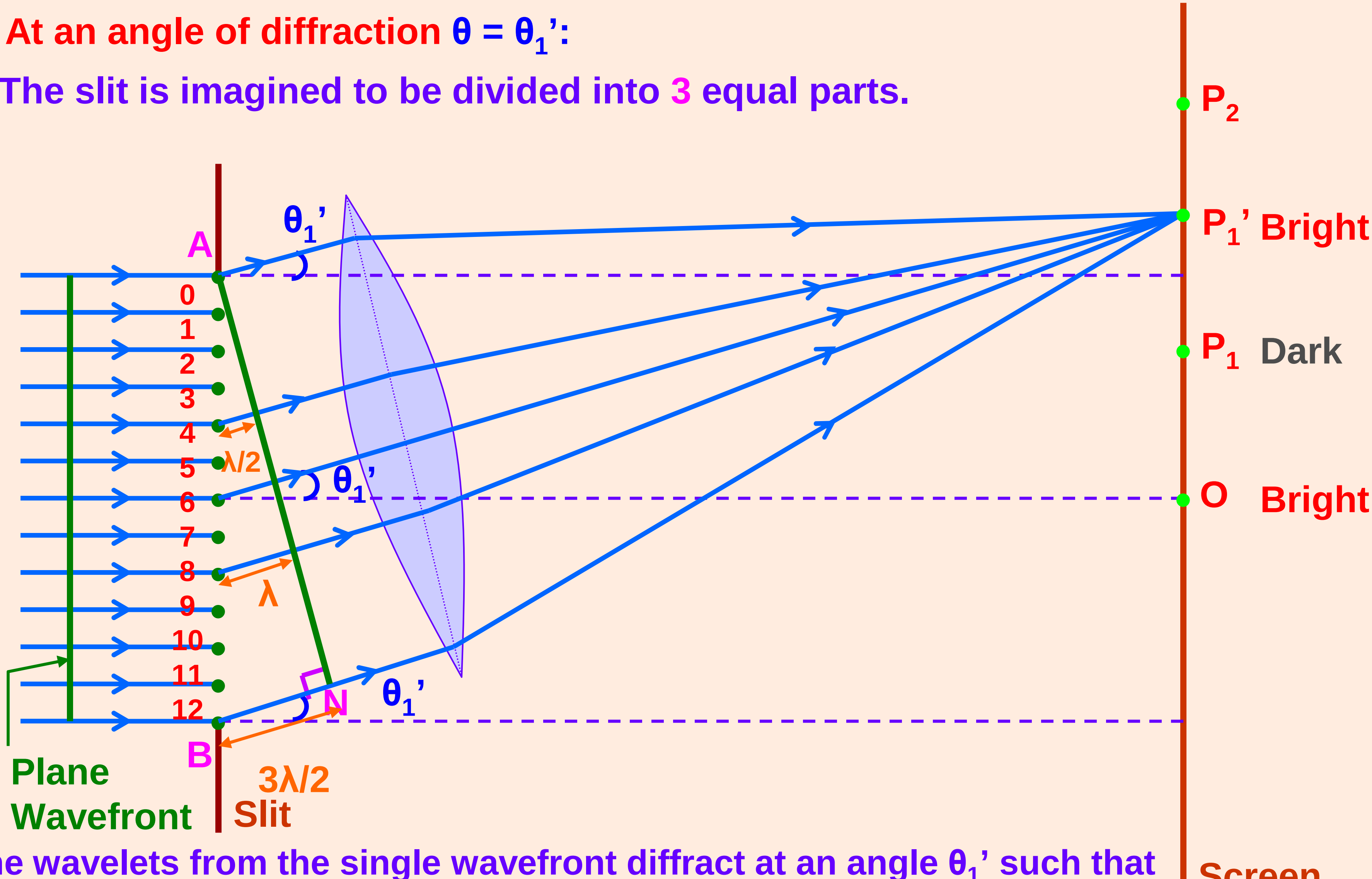
The slit is imagined to be divided into 4 equal parts.



The wavelets from the single wavefront diffract at an angle θ_2 such that BN is 2λ and reach the point P_2. The pairs (0,3), (1,4), (2,5), (3,6), (4,7), (5,8), (6,9), (7,10), (8,11) and (9,12) interfere destructively with path difference $\lambda/2$ and give Second Secondary Minimum (Dark fringe).

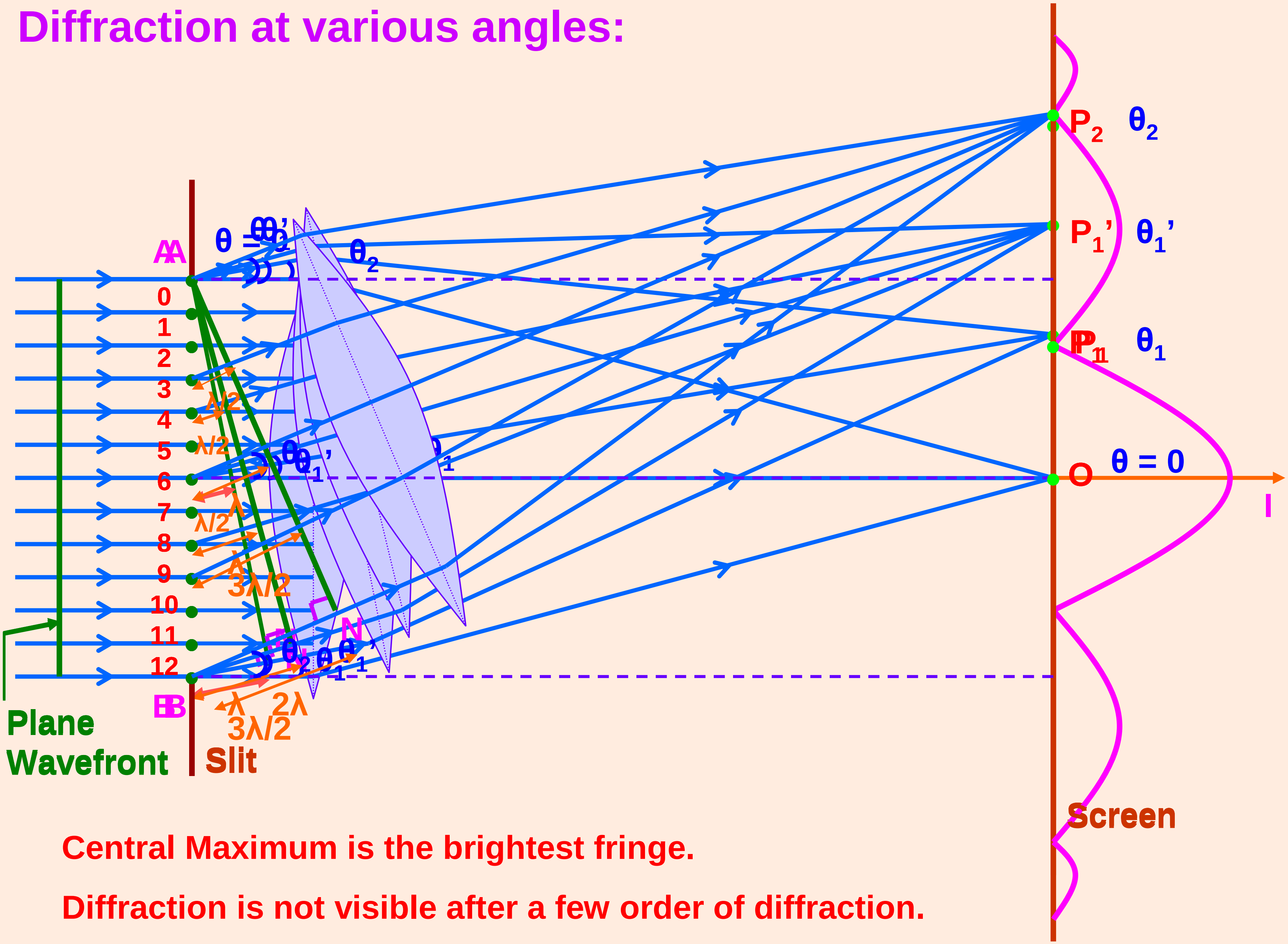
4) At an angle of diffraction $\theta = \theta_1'$:

The slit is imagined to be divided into 3 equal parts.



The wavelets from the single wavefront diffract at an angle θ_1' such that BN is $3\lambda/2$ and reach the point P_1'. The pairs (0,8), (1,9), (2,10), (3,11) and (4,12) interfere constructively with path difference λ and (0,4), (1,5), (2,6), …… and (8,12) interfere destructively with path difference λ/2. However due to a few wavelets interfering constructively First Secondary Maximum (Bright fringe) is formed.

Diffraction at various angles:



Central Maximum is the brightest fringe.

Diffraction is not visible after a few order of diffraction.

Theory:

The path difference between the 0th wavelet and 12th wavelet is BN.

If 'θ' is the angle of diffraction and 'd' is the slit width, then $BN = d \sin \theta$

To establish the condition for secondary minima, the slit is divided into 2, 4, 6, … equal parts such that corresponding wavelets from successive regions interfere with path difference of $\lambda/2$.

Or for nth secondary minimum, the slit can be divided into 2n equal parts.

For θ_1, $d \sin \theta_1 = \lambda$

For θ_2, $d \sin \theta_2 = 2\lambda$

For θ_n, $d \sin \theta_n = n\lambda$

Since θ_n is very small,

$d\, \theta_n = n\lambda$

$\theta_n = n\lambda / d$ (n = 1, 2, 3, ……)

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7, … equal parts such that corresponding wavelets from alternate regions interfere with path difference of λ.

Or for nth secondary minimum, the slit can be divided into (2n + 1) equal parts.

For θ_1', $d \sin \theta_1' = 3\lambda/2$

For θ_2', $d \sin \theta_2' = 5\lambda/2$

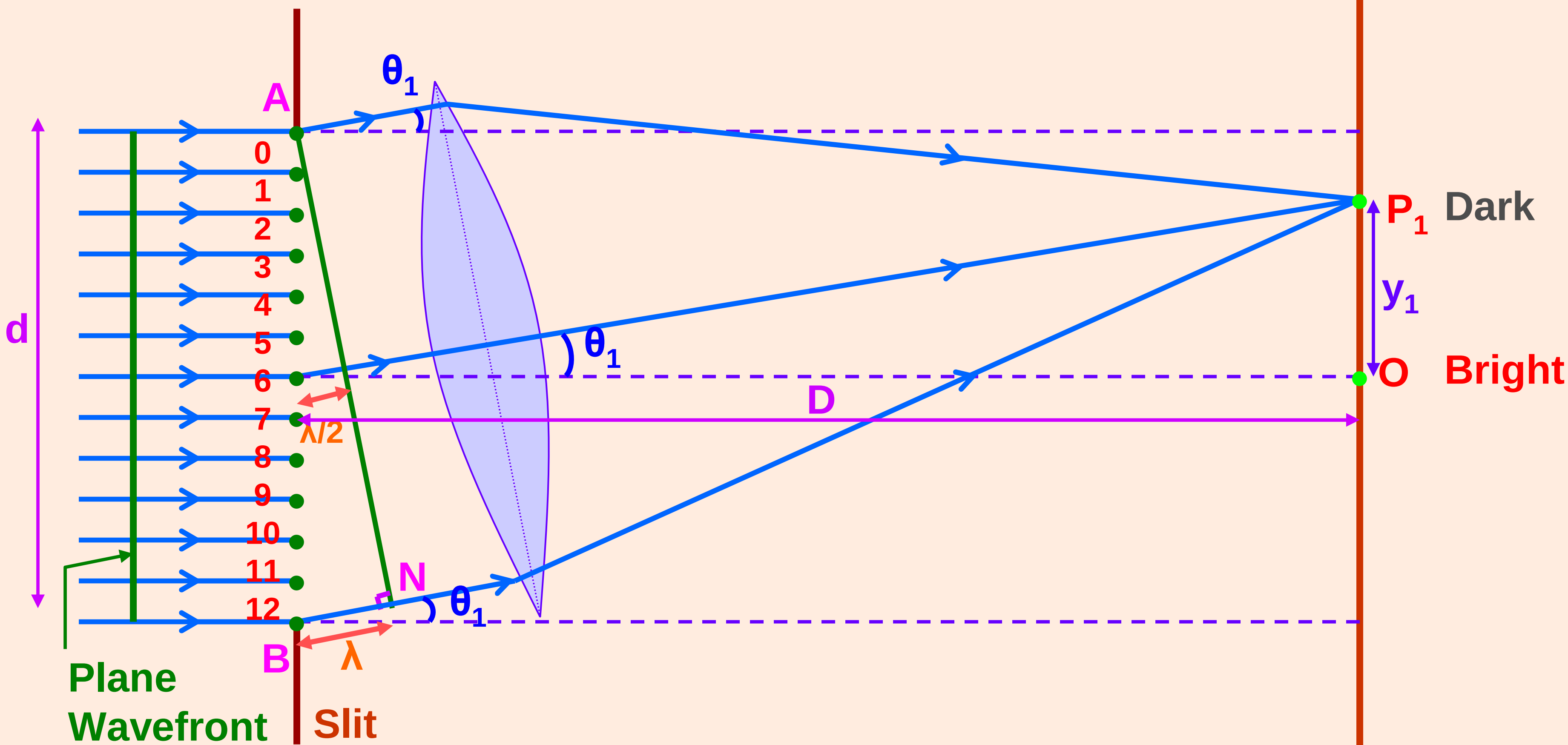
For θ_n', $d \sin \theta_n' = (2n + 1)\lambda/2$

Since θ_n' is very small,

$d\, \theta_n' = (2n + 1)\lambda / 2$

$\theta_n' = (2n + 1)\lambda / 2d$ (n = 1, 2, 3, ……)

Width of Central Maximum:



$\tan \theta_1 = y_1 / D$

or $\theta_1 = y_1 / D$ (since θ_1 is very small)

$d \sin \theta_1 = \lambda$

or $\theta_1 = \lambda / d$ (since θ_1 is very small)

$y_1 = D \lambda / d$

Since the Central Maximum is spread on either side of O, the width is

$\beta_0 = 2D \lambda / d$

Fresnel's Distance:

Fresnel's distance is that distance from the slit at which the spreading of light due to diffraction becomes equal to the size of the slit.

$y_1 = D \lambda / d$

At Fresnel's distance, $y_1 = d$ and $D = D_F$

So, $D_F \lambda / d = d$ or $D_F = d^2 / \lambda$

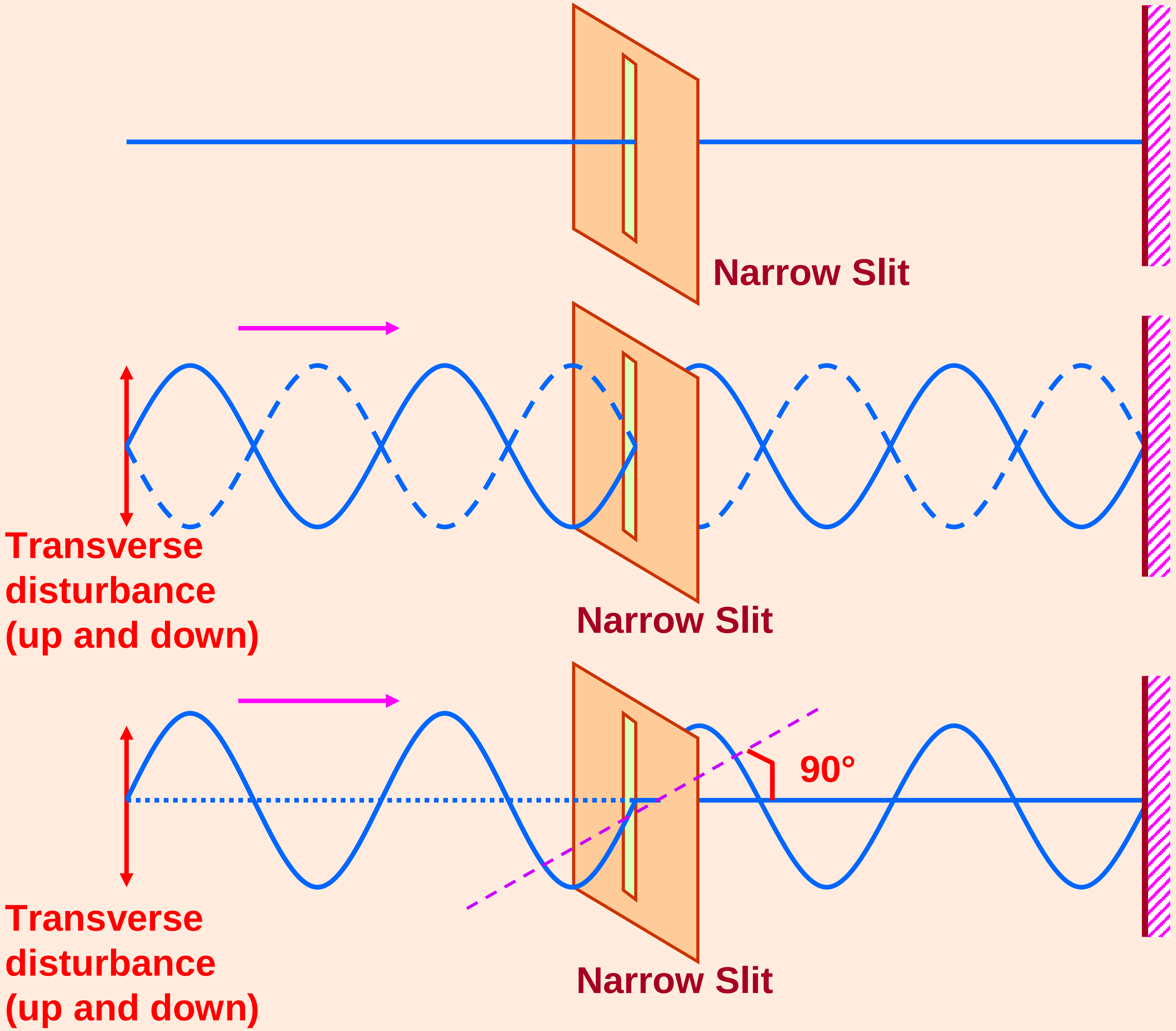
If the distance D between the slit and the screen is less than Fresnel's distance D_F, then the diffraction effects may be regarded as absent.

So, ray optics may be regarded as a limiting case of wave optics.

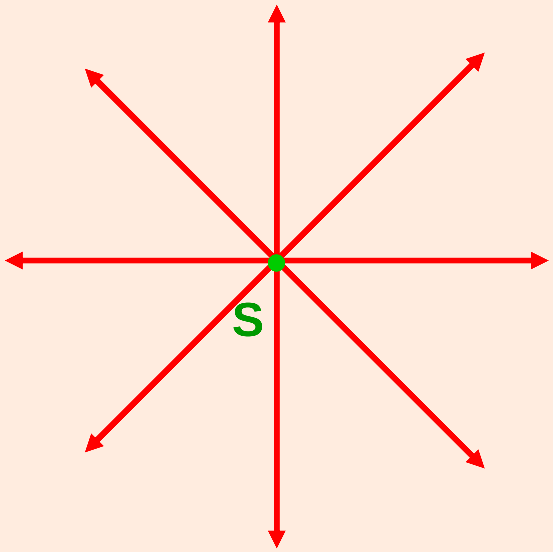
Difference between Interference and Diffraction:

Interference	Diffraction
1. Interference is due to the superposition of two different wave trains coming from coherent sources.	1. Diffraction is due to the superposition of secondary wavelets from the different parts of the same wavefront.
2. Fringe width is generally constant.	2. Fringes are of varying width.
3. All the maxima have the same intensity.	3. The maxima are of varying intensities.
4. There is a good contrast between the maxima and minima.	4. There is a poor contrast between the maxima and minima.

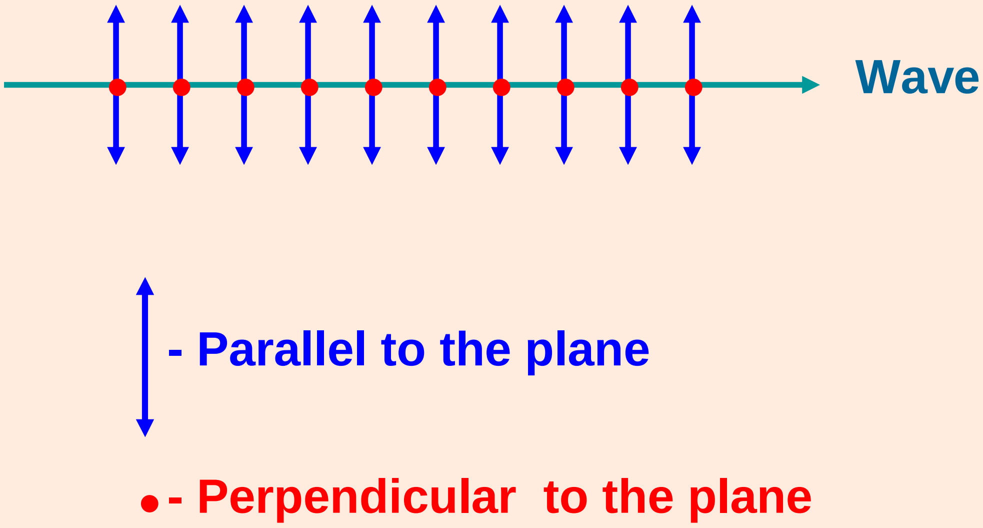
Polarisation of Transverse Mechanical Waves:



Polarisation of Light Waves:



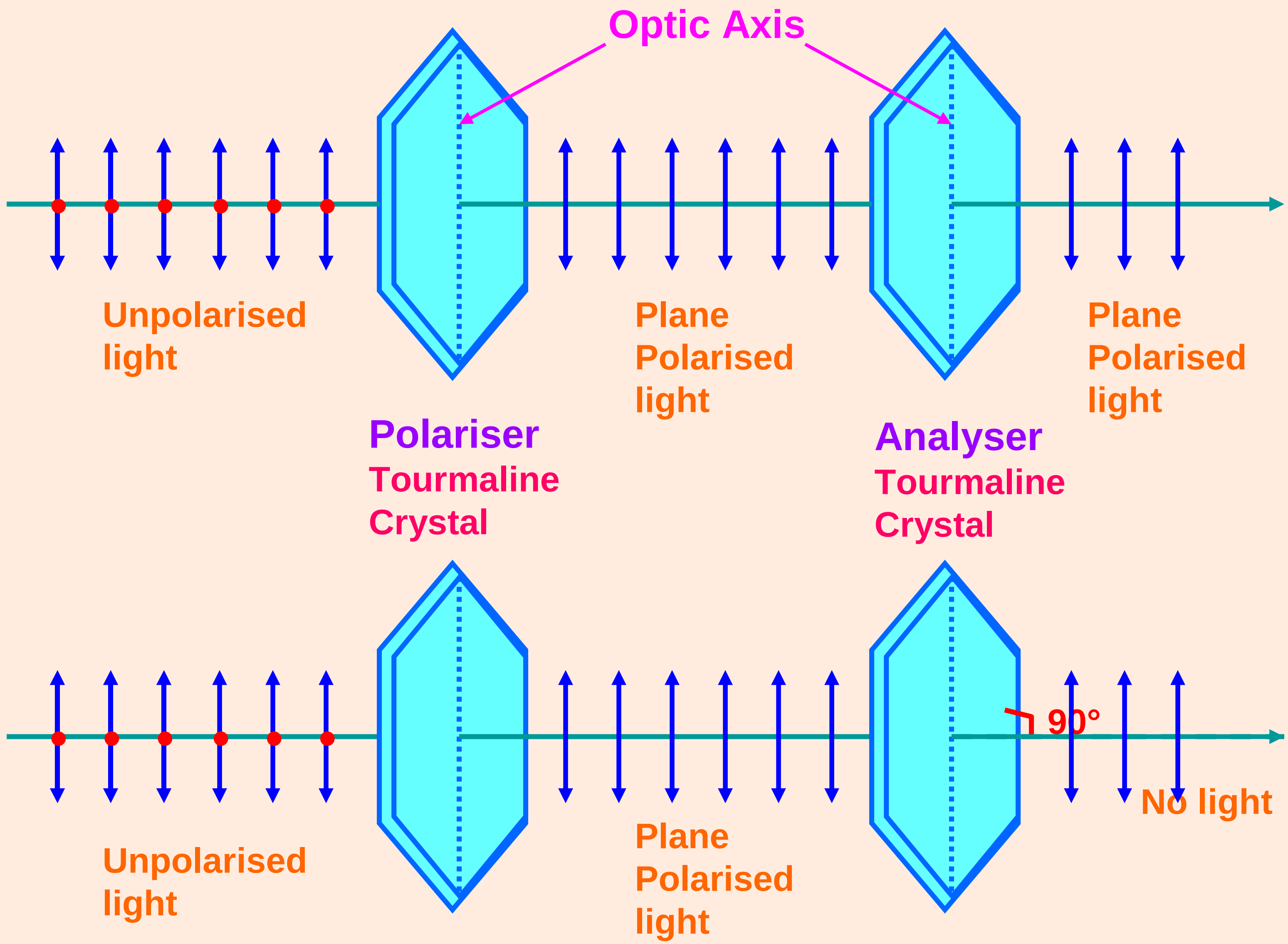
Natural Light

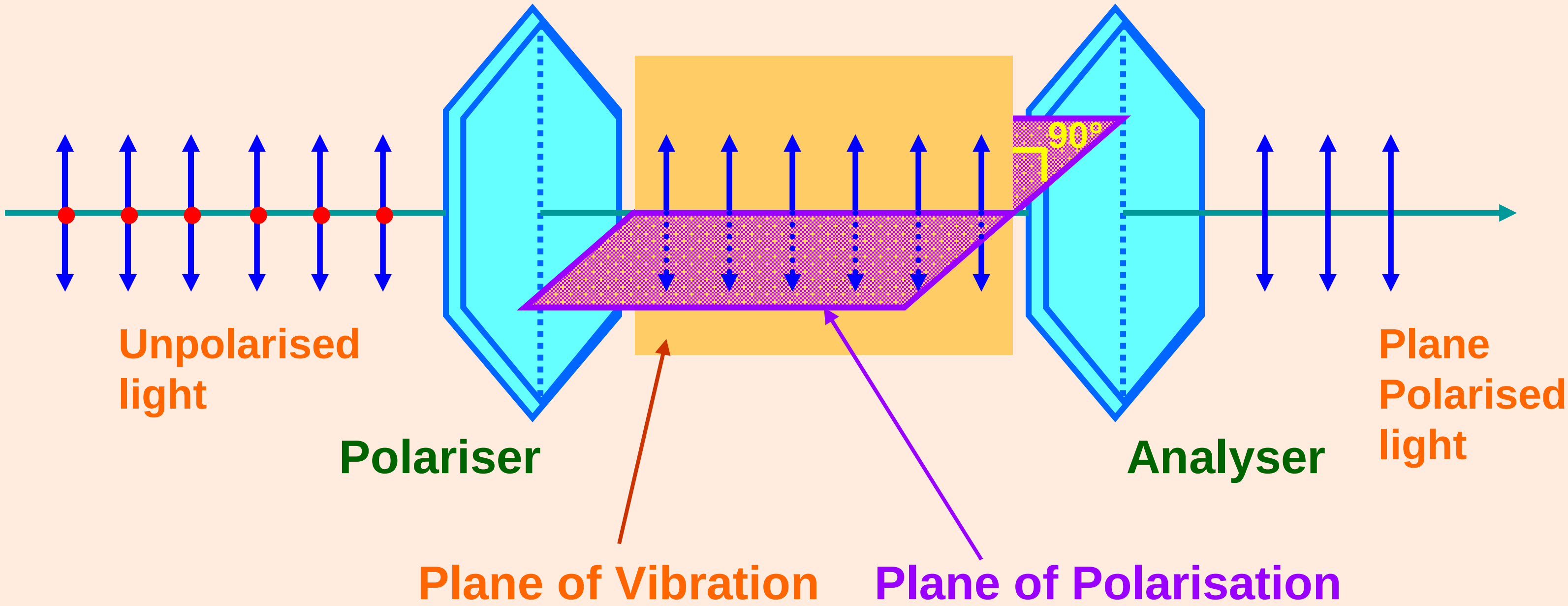


Representation of Natural Light

In natural light, millions of transverse vibrations occur in all the directions perpendicular to the direction of propagation of wave. But for convenience, we can assume the rectangular components of the vibrations with one component lying on the plane of the diagram and the other perpendicular to the plane of the diagram.

Light waves are electromagnetic waves with electric and magnetic fields oscillating at right angles to each other and also to the direction of propagation of wave. Therefore, the light waves can be polarised.





When unpolarised light is incident on the polariser, the vibrations parallel to the crystallographic axis are transmitted and those perpendicular to the axis are absorbed. Therefore the transmitted light is plane (linearly) polarised.

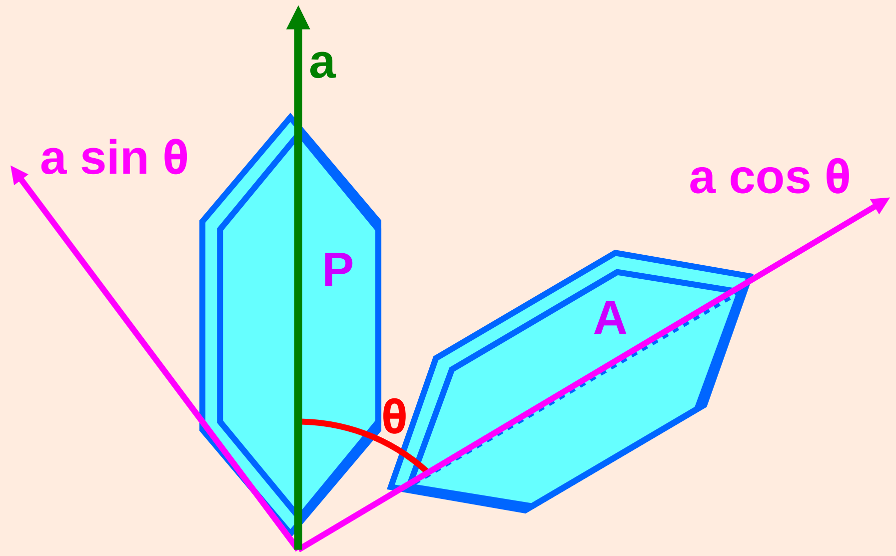
The plane which contains the crystallographic axis and vibrations transmitted from the polariser is called plane of vibration.

The plane which is perpendicular to the plane of vibration is called plane of polarisation.

Malus' Law:

When a beam of plane polarised light is incident on an analyser, the intensity I of light transmitted from the analyser varies directly as the square of the cosine of the angle θ between the planes of transmission of analyser and polariser.

$$I \propto \cos^2 \theta$$



If a be the amplitude of the electric vector transmitted by the polariser, then only the component $a \cos \theta$ will be transmitted by the analyser.

Intensity of transmitted light from the analyser is

$$I = k (a \cos \theta)^2$$

or $$I = k a^2 \cos^2 \theta$$

$$I = I_0 \cos^2 \theta$$

(where $I_0 = k a^2$ is the intensity of light transmitted from the polariser)

Case I : When $\theta = 0°$ or $180°$, $I = I_0$

Case II : When $\theta = 90°$, $I = 0$

Case III: When unpolarised light is incident on the analyser the intensity of the transmitted light is one-half of the intensity of incident light. (Since average value of $\cos^2\theta$ is ½)

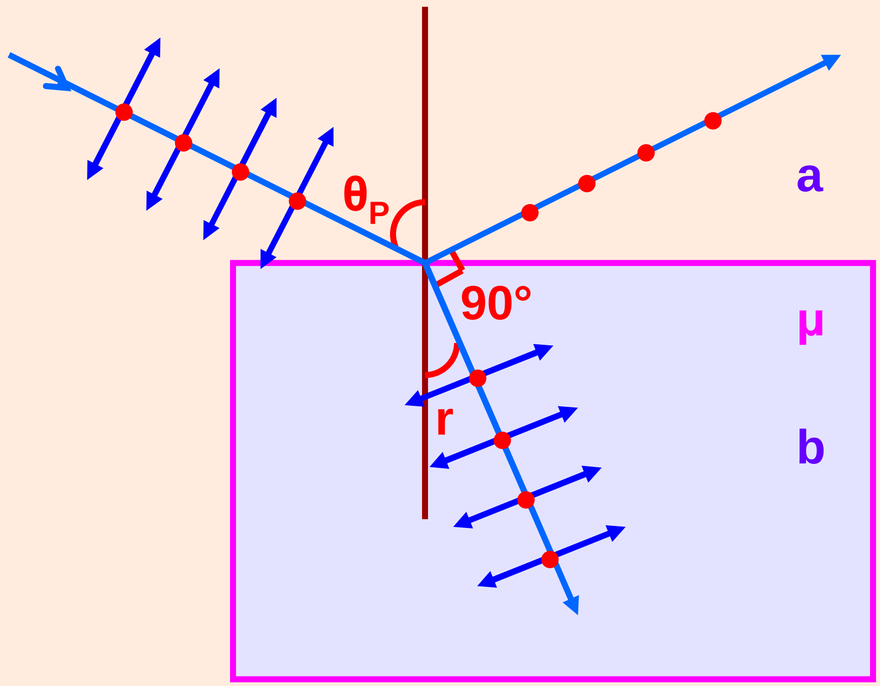
Polarisation by Reflection and Brewster's Law:

The incident light wave is made of parallel vibrations (π – components) on the plane of incidence and perpendicular vibrations (σ – components : perpendicular to plane of incidence).

At a particular angle θ_P, the parallel components completely refracted whereas the perpendicular components partially get refracted and partially get reflected.

i.e. the reflected components are all in perpendicular plane of vibration and hence plane polarised.

The intensity of transmitted light through the medium is greater than that of plane polarised (reflected) light.



$$\theta_P + r = 90° \quad \text{or} \quad r = 90° - \theta_P$$

$$_a\mu_b = \frac{\sin \theta_P}{\sin r}$$

$$_a\mu_b = \frac{\sin \theta_P}{\sin (90° - \theta_P)}$$

$$_a\mu_b = \tan \theta_P$$

Polaroids:

H – Polaroid is prepared by taking a sheet of polyvinyl alcohol (long chain polymer molecules) and subjecting to a large strain. The molecules are oriented parallel to the strain and the material becomes doubly refracting. When strained with iodine, the material behaves like a dichroic crystal.

K – Polaroid is prepared by heating a stretched polyvinyl alcohol film in the presence of HCl (an active dehydrating catalyst). When the film becomes slightly darkened, it behaves like a strong dichroic crystal.

Uses of Polaroids:

1) **Polaroid Sun Glasses**
2) **Polaroid Filters**
3) **For Laboratory Purpose**
4) **In Head-light of Automobiles**
5) **In Three – Dimensional Motion Picutres**
6) **In Window Panes**
7) **In Wind Shield in Automobiles**

PHOTOELECTRIC EFFECT AND
DUAL NATURE OF MATTER AND RADIATIONS

1. Photons
2. Photoelectric Effect
3. Experimental Set-up to study Photoelectric Effect
4. Effect of Intensity, Frequency, Potential on P.E. Current
5. Graphical representation of variation of P.E. Current
6. Laws of Photoelectric Effect
7. Einstein's Photoelectric Equation
8. Verification of Laws of Photoelectric Effect based on Einstein's Photoelectric Equation
9. Application of Photoelectric Effect
10. Matter Waves and de Broglie wavelength
11. Davission & Germer Experiment

Photon:

A packet or bundle of energy is called a photon.

Energy of a photon is $E = h\nu = \frac{hc}{\lambda}$

where h is the Planck's constant, v is the frequency of the radiation or photon, c is the speed of light (e.m. wave) and λ is the wavelength.

Properties of photons:

i) A photon travels at a speed of light c in vacuum. (i.e. 3×10^{-8} m/s)

ii) It has zero rest mass. i.e. the photon can not exist at rest.

iii) The kinetic mass of a photon is, $m = \frac{E}{c^2} = \frac{h}{c\lambda}$

iv) The momentum of a photon is, $p = \frac{E}{c} = \frac{h}{\lambda}$

v) Photons travel in a straight line.

vi) Energy of a photon depends upon frequency of the photon; so the energy of the photon does not change when photon travels from one medium to another.

vii) Wavelength of the photon changes in different media; so, velocity of a photon is different in different media.

viii) Photons are electrically neutral.

ix) Photons may show diffraction under given conditions.

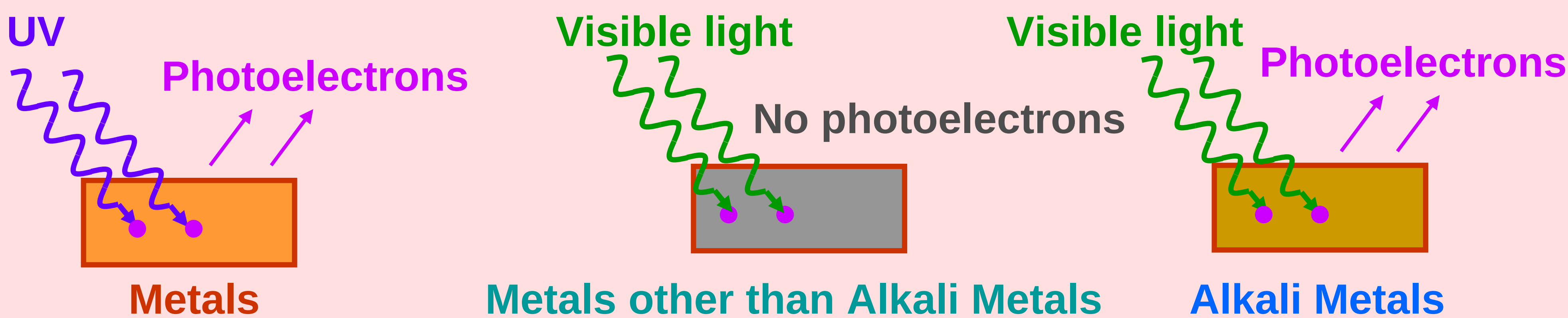
x) Photons are not deviated by magnetic and electric fields.

Photoelectric Effect:

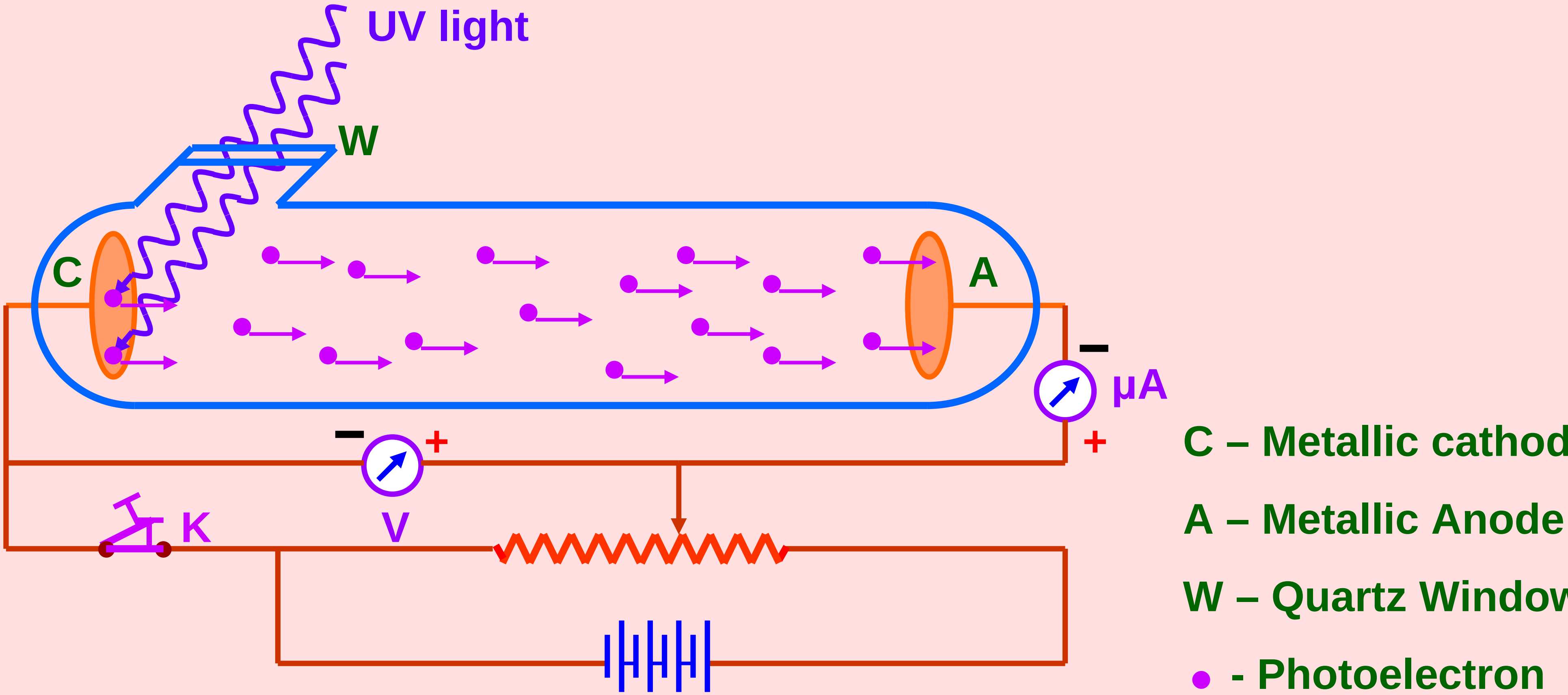
The phenomenon of emission of electrons from mainly metal surfaces exposed to light energy (X – rays, γ – rays, UV rays, Visible light and even Infra Red rays) of suitable frequency is known as photoelectric effect.

The electrons emitted by this effect are called photoelectrons.
The current constituted by photoelectrons is known as photoelectric current.

Note: Non metals also show photoelectric effect. Liquids and gases also show this effect but to limited extent.



Experimental Set-up to study Photoelectric Effect:



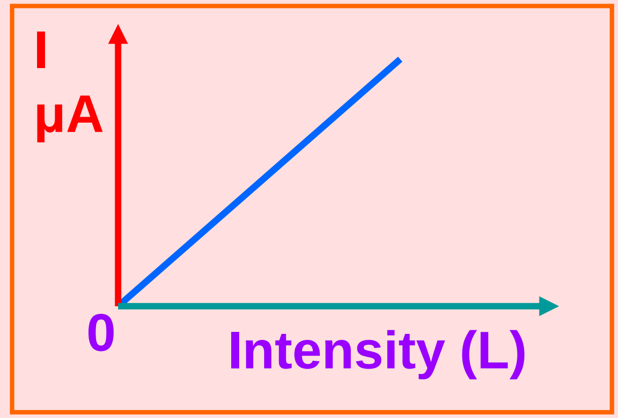
Glass transmits only visible and infra-red lights but not UV light.

Quartz transmits UV light.

When light of suitable frequency falls on the metallic cathode, photoelectrons are emitted. These photoelectrons are attracted towards the +ve anode and hence photoelectric current is constituted.

1) Effect of Intensity of Incident Light on Photoelectric Current:

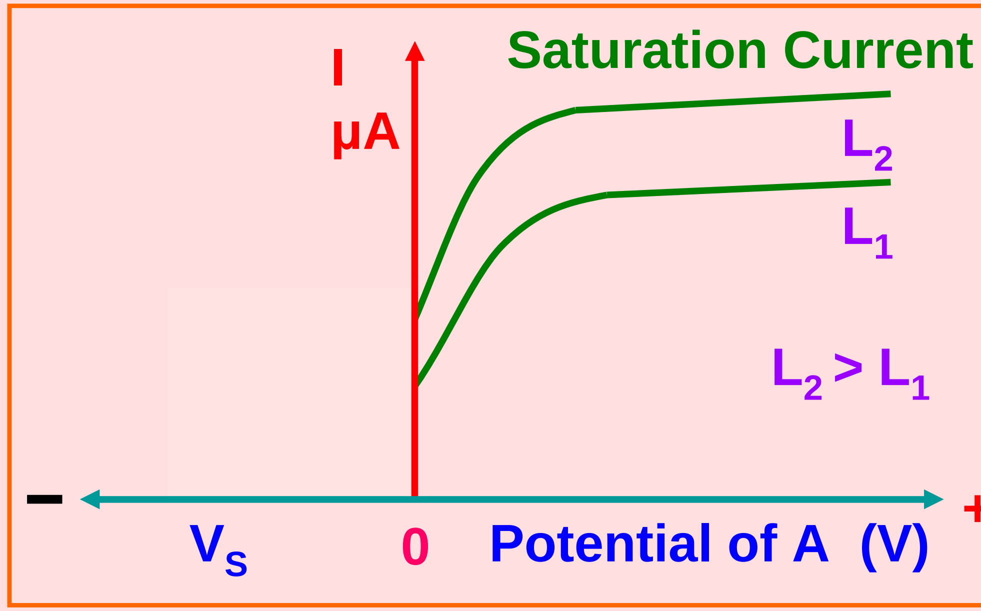
For a fixed frequency, the photoelectric current increases linearly with increase in intensity of incident light.



2) Effect of Potential on Photoelectric Current:

For a fixed frequency and intensity of incident light, the photoelectric current increases with increase in +ve potential applied to the anode.

When all the photoelectrons reach the plate A, current becomes maximum and is known as saturation current.



When the potential is decreased, the current decreases but does not become zero at zero potential.

This shows that even in the absence of accelerating potential, a few photoelectrons manage to reach the plate on their own due to their K.E.

When –ve potential is applied to the plate A w.r.t. C, photoelectric current becomes zero at a particular value of –ve potential called stopping potential or cut-off potential.

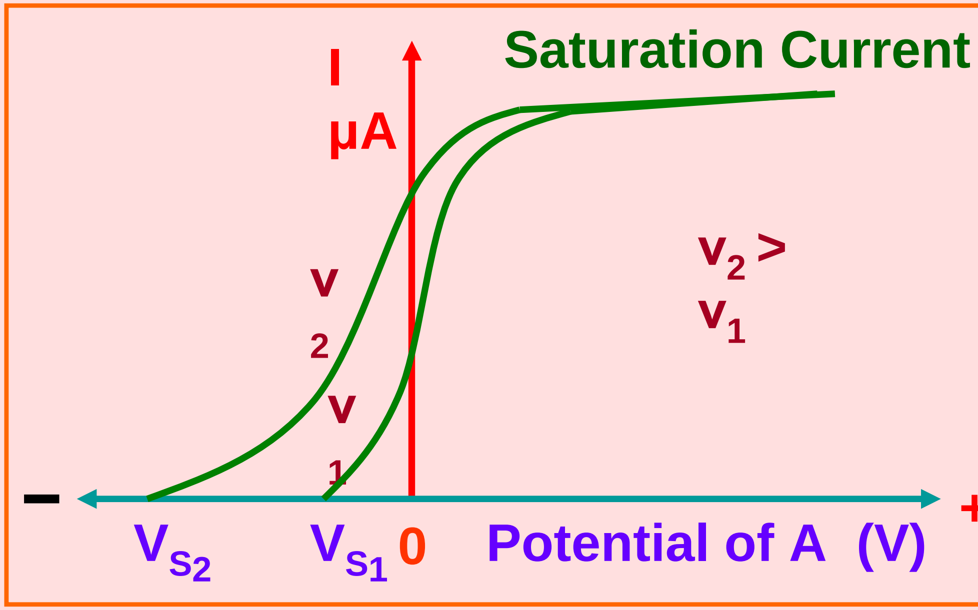
Intensity of incident light does not affect the stopping potential.

3) Effect of Frequency of Incident Light on Photoelectric Current:

For a fixed intensity of incident light, the photoelectric current does not depend on the frequency of the incident light. Because, the photoelectric current simply depends on the number of photoelectrons emitted and in turn on the number of photons incident and not on the energy of photons.

4) Effect of Frequency of Incident Light on Stopping Potential:

For a fixed intensity of incident light, the photoelectric current increases and is saturated with increase in +ve potential applied to the anode.



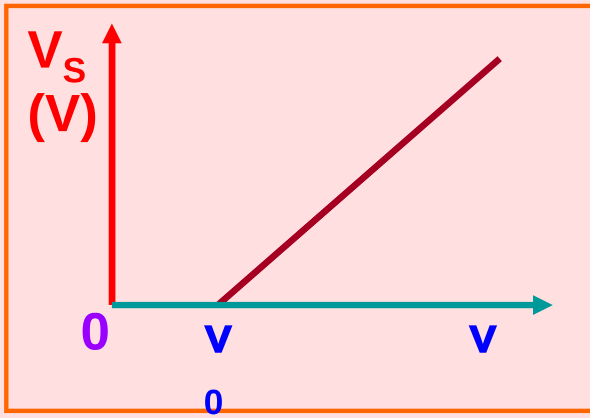
However, the saturation current is same for different frequencies of the incident lights.

When potential is decreased and taken below zero, photoelectric current decreases to zero but at different stopping potentials for different frequencies.

Higher the frequency, higher the stopping potential. i.e. $V_S \propto \nu$

5) Threshold Frequency:

The graph between stopping potential and frequency does not pass through the origin. It shows that there is a minimum value of frequency called threshold frequency below which photoelectric emission is not possible however high the intensity of incident light may be. It depends on the nature of the metal emitting photoelectrons.



Laws of Photoelectric Emission:

i) For a given substance, there is a minimum value of frequency of incident light called threshold frequency below which no photoelectric emission is possible, howsoever, the intensity of incident light may be.

ii) The number of photoelectrons emitted per second (i.e. photoelectric current) is directly proportional to the intensity of incident light provided the frequency is above the threshold frequency.

iii) The maximum kinetic energy of the photoelectrons is directly proportional to the frequency provided the frequency is above the threshold frequency.

iv) The maximum kinetic energy of the photoelectrons is independent of the intensity of the incident light.

v) The process of photoelectric emission is instantaneous. i.e. as soon as the photon of suitable frequency falls on the substance, it emits photoelectrons.

vi) The photoelectric emission is one-to-one. i.e. for every photon of suitable frequency one electron is emitted.

Einstein's Photoelectric Equation:

When a photon of energy $h\nu$ falls on a metal surface, the energy of the photon is absorbed by the electron and is used in two ways:

i) A part of energy is used to overcome the surface barrier and come out of the metal surface. This part of the energy is called '**work function**' ($\Phi = h\nu_0$).

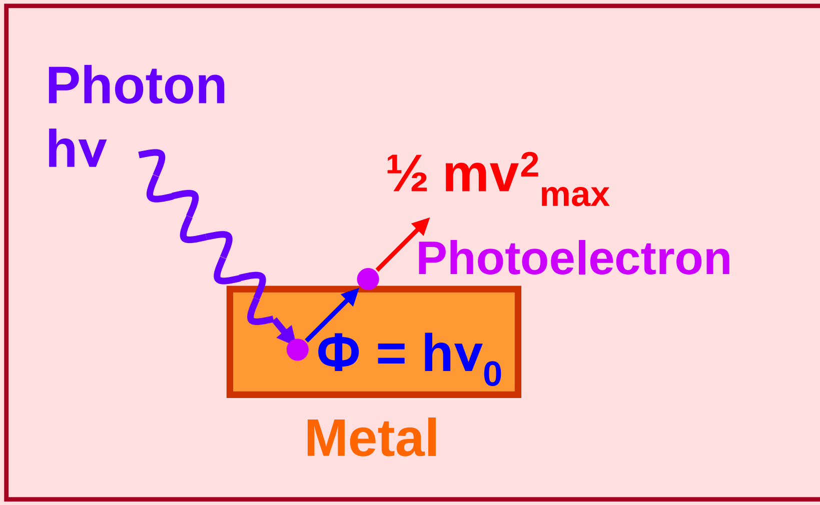
ii) The remaining part of the energy is used in giving a velocity 'v' to the emitted photoelectron. This is equal to the maximum kinetic energy of the photoelectrons ($\frac{1}{2} mv^2_{max}$) where 'm' is mass of the photoelectron.

According to law of conservation of energy,

$$h\nu = \Phi + \frac{1}{2} mv^2_{max}$$

$$= h\nu_0 + \frac{1}{2} mv^2_{max}$$

$$\frac{1}{2} mv^2_{max} = h(\nu - \nu_0)$$



Verification of Laws of Photoelectric Emission based on Einstein's Photoelectric Equation:

$$\frac{1}{2} mv^2_{max} = h (v - v_0)$$

i) If $v < v_0$, then $\frac{1}{2} mv^2_{max}$ is negative, which is not possible. Therefore, for photoelectric emission to take place $v > v_0$.

ii) Since one photon emits one electron, so the number photoelectrons emitted per second is directly proportional to the intensity of incident light.

iii) It is clear that $\frac{1}{2} mv^2_{max} \propto v$ as h and v_0 are constant. This shows that K.E. of the photoelectrons is directly proportional to the frequency of the incident light.

iv) Photoelectric emission is due to collision between a photon and an electron. As such there can not be any significant time lag between the incidence of photon and emission of photoelectron. i.e. the process is instantaneous. It is found that delay is only 10^{-8} seconds.

Application of Photoelectric Effect:

1. Automatic fire alarm
2. Automatic burglar alarm
3. Scanners in Television transmission
4. Reproduction of sound in cinema film
5. In paper industry to measure the thickness of paper
6. To locate flaws or holes in the finished goods
7. In astronomy
8. To determine opacity of solids and liquids
9. Automatic switching of street lights
10. To control the temperature of furnace
11. Photometry
12. Beauty meter – To measure the fair complexion of skin
13. Light meters used in cinema industry to check the light
14. Photoelectric sorting
15. Photo counting
16. Meteorology

Dual Nature of Radiation and Matter:

Wave theory of electromagnetic radiations explained the phenomenon of interference, diffraction and polarization.

On the other hand, quantum theory of e.m. radiations successfully explained the photoelectric effect, Compton effect, black body radiations, X- ray spectra, etc.

Thus, radiations have dual nature. i.e. wave and particle nature.

Louis de Broglie suggested that the particles like electrons, protons, neutrons, etc have also dual nature. i.e. they also can have particle as well as wave nature.

Note: In no experiment, matter exists both as a particle and as a wave simultaneously. It is either the one or the other aspect. i.e. The two aspects are complementary to each other.

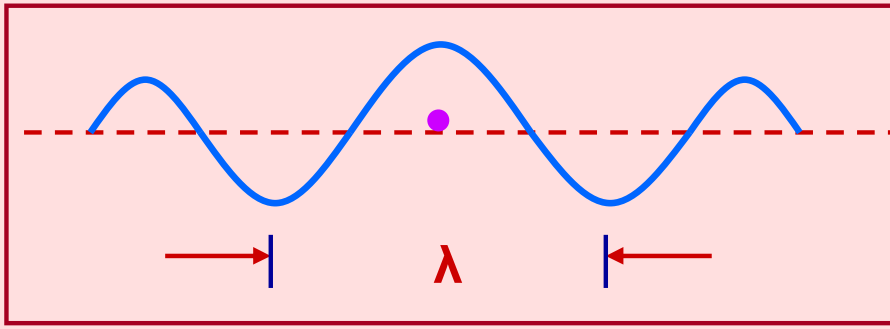
His suggestion was based on:

i) The nature loves symmetry.

ii) The universe is made of particles and radiations and both entities must be symmetrical.

de Broglie wave:

According to de Broglie, a moving material particle can be associated with a wave. i.e. a wave can guide the motion of the particle.



The waves associated with the moving material particles are known as de Broglie waves or matter waves.

Expression for de Broglie wave:

According to quantum theory, the energy of the photon is $E = h\nu = \frac{hc}{\lambda}$

According to Einstein's theory, the energy of the photon is $E = mc^2$

So, $\lambda = \frac{h}{mc}$ or $\lambda = \frac{h}{p}$ where $p = mc$ is momentum of a photon

If instead of a photon, we have a material particle of mass m moving with velocity v, then the equation becomes

$$\lambda = \frac{h}{mv}$$

which is the expression for de Broglie wavelength.

Conclusion:

$$\lambda = \frac{h}{mv}$$

i) de Broglie wavelength is inversely proportional to the velocity of the particle. If the particle moves faster, then the wavelength will be smaller and vice versa.

ii) If the particle is at rest, then the de Broglie wavelength is infinite. Such a wave can not be visualized.

iii) de Broglie wavelength is inversely proportional to the mass of the particle. The wavelength associated with a heavier particle is smaller than that with a lighter particle.

iv) de Broglie wavelength is independent of the charge of the particle.

Matter waves, like electromagnetic waves, can travel in vacuum and hence they are not mechanical waves.

Matter waves are not electromagnetic waves because they are not produced by accelerated charges.

Matter waves are probability waves, amplitude of which gives the probability of existence of the particle at the point.

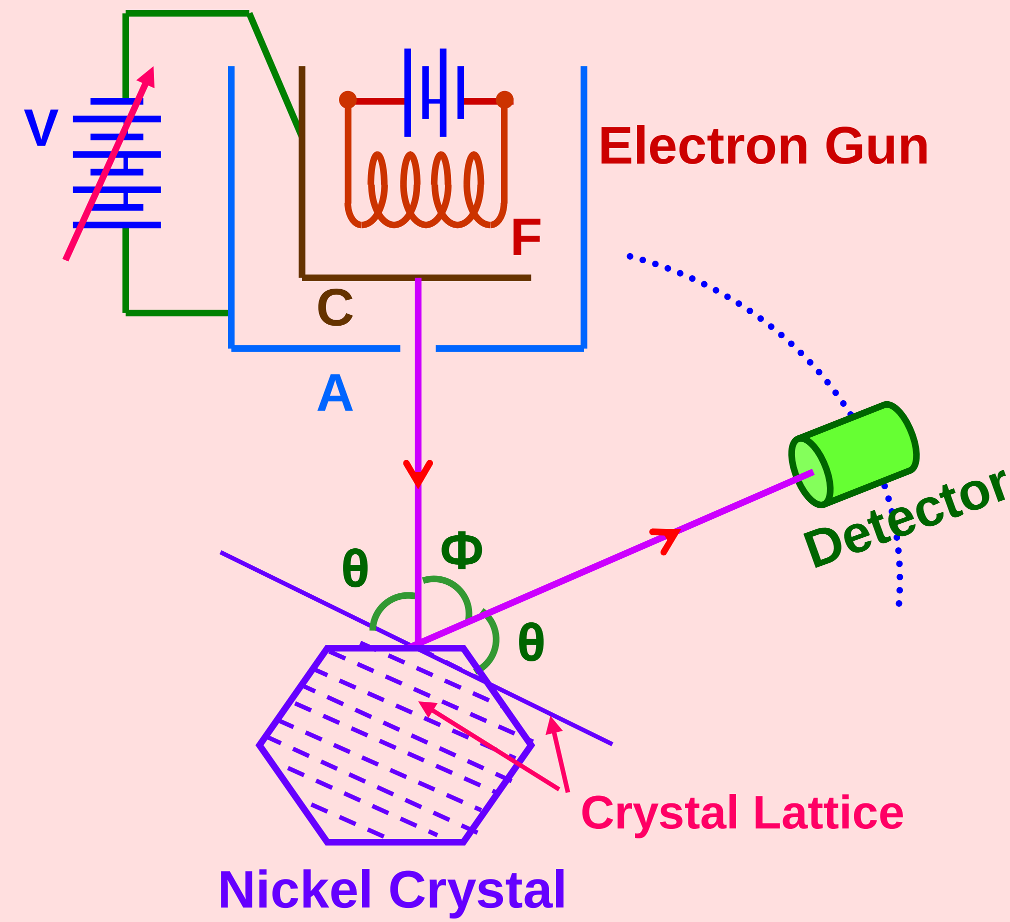
Davisson and Germer Experiment:

A beam of electrons emitted by the electron gun is made to fall on Nickel crystal cut along cubical axis at a particular angle.

The scattered beam of electrons is received by the detector which can be rotated at any angle.

The energy of the incident beam of electrons can be varied by changing the applied voltage to the electron gun.

Intensity of scattered beam of electrons is found to be maximum when angle of scattering is 50° and the accelerating potential is 54 V.



$\theta + 50° + \theta = 180°$ i.e. $\theta = 65°$

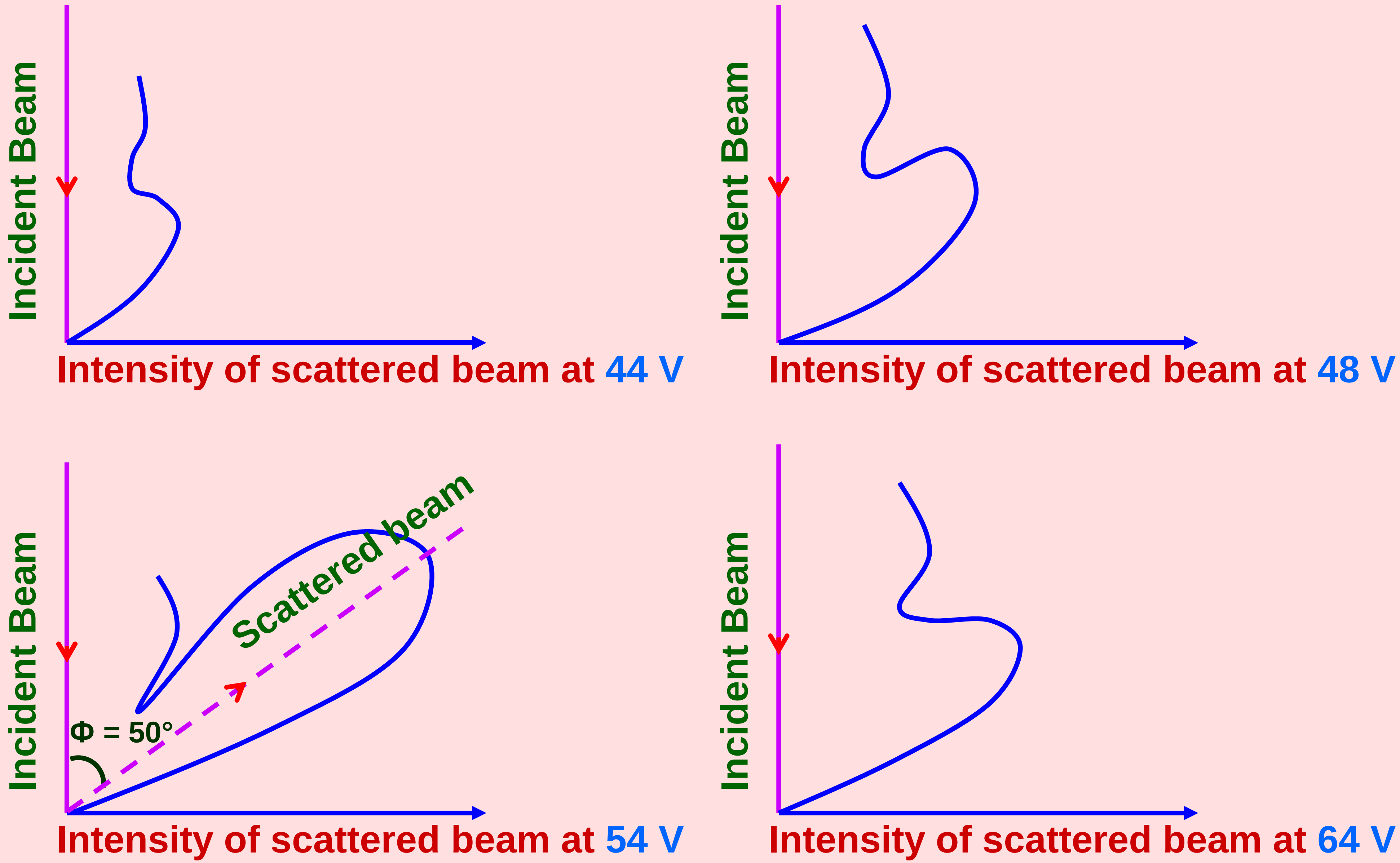
For Ni crystal, lattice spacing $d = 0.91$ Å

For first principal maximum, $n = 1$

Electron diffraction is similar to X-ray diffraction.

∴ Bragg's equation $2d\sin\theta = n\lambda$ gives

$\lambda = 1.65$ Å

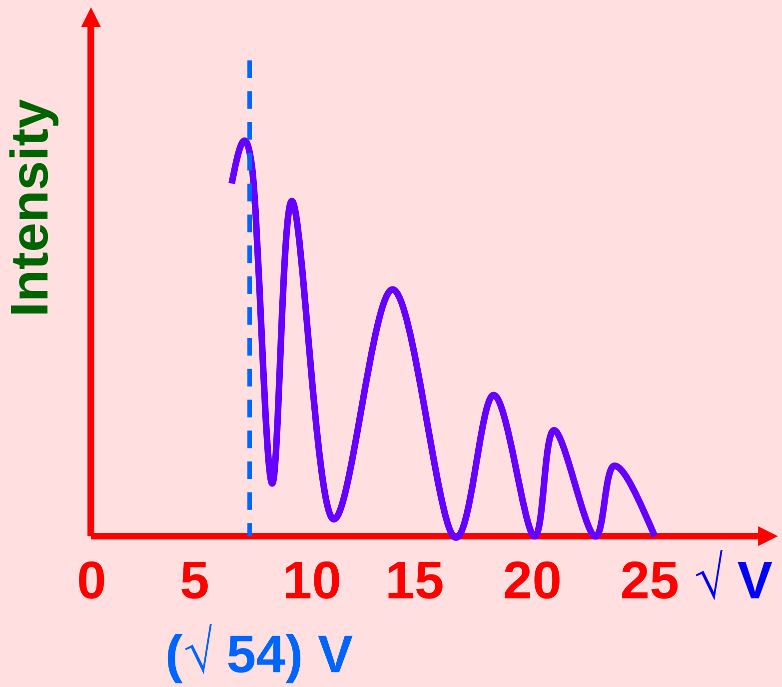


According to de Broglie's hypothesis,

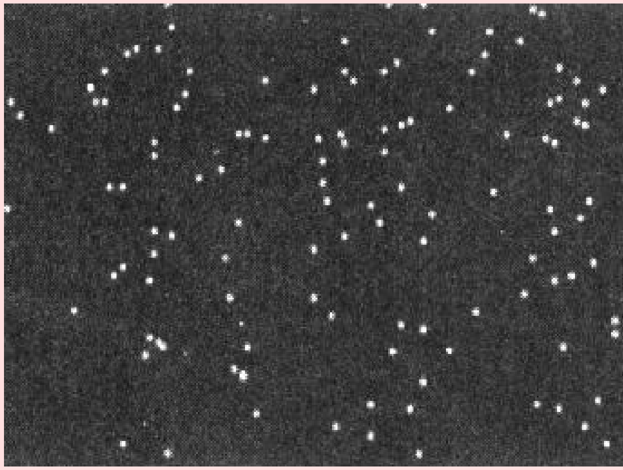
$$\lambda = \frac{h}{\sqrt{2meV}} \quad \text{or} \quad \lambda = \frac{12.27\ \text{Å}}{\sqrt{V}}$$

∴ de Broglie wavelength of moving electron at V = 54 Volt is 1.67 Å which is in close agreement with 1.65 Å.

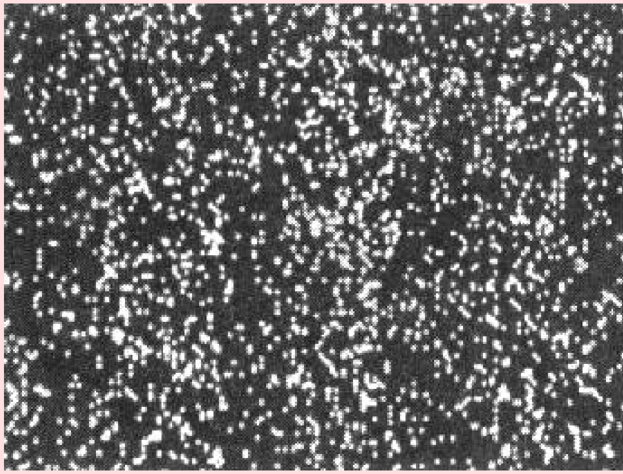
Intensity vs √ Anode Potential:



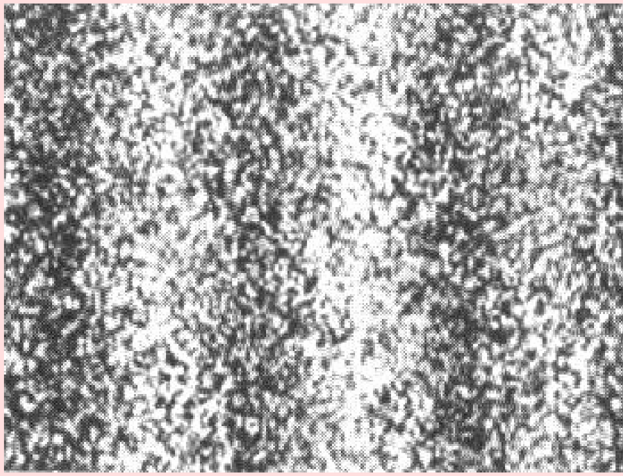
Diffraction pattern after 100 electrons



Diffraction pattern after 3000 electrons



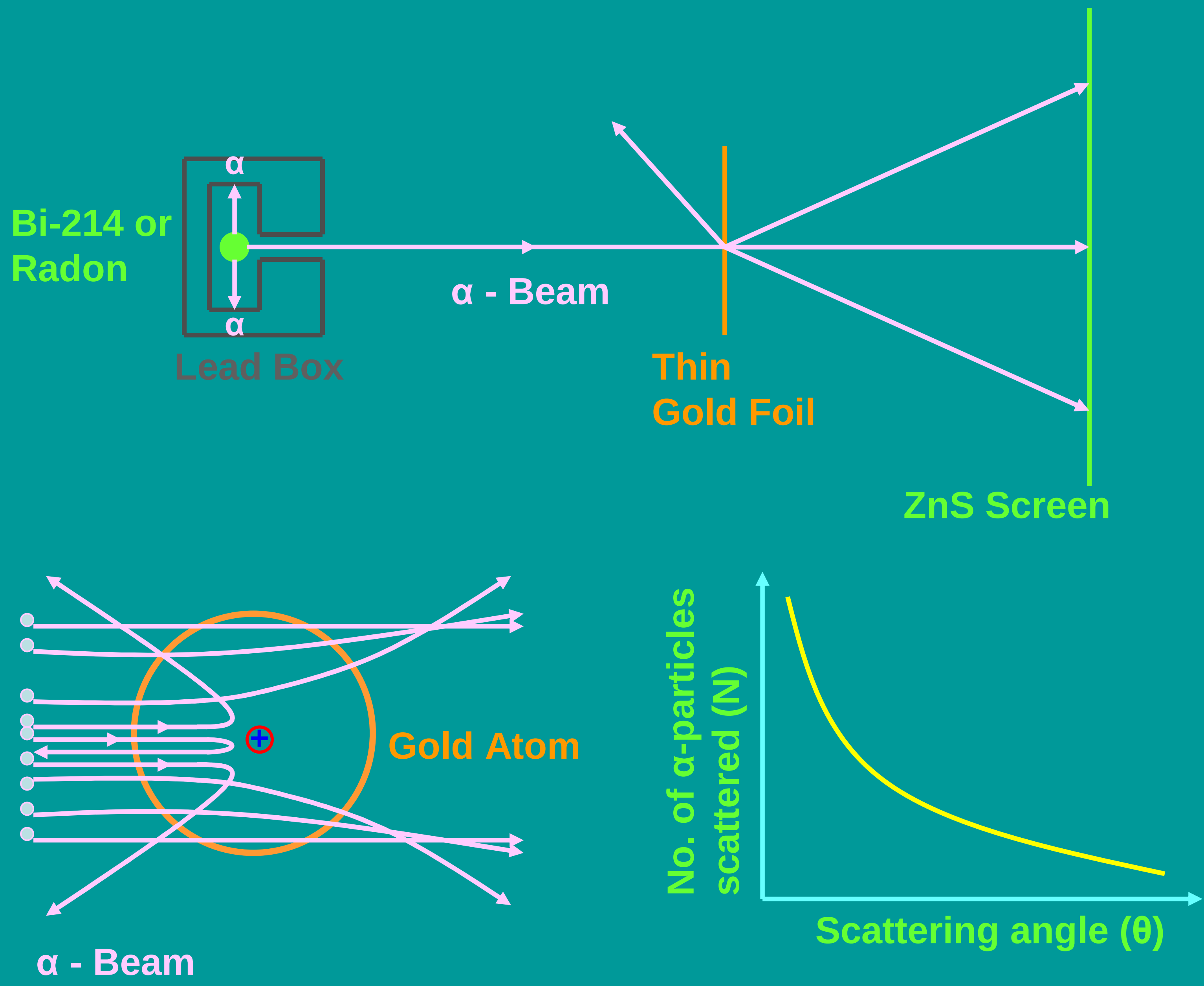
Diffraction pattern after 70000 electrons



ATOMS & NUCLEI

1. Rutherford's Alpha Scattering Experiment
2. Distance of Closest Approach (Nuclear Size)
3. Impact Parameter
4. Composition of Nucleus
5. Atomic Number, Mass Number and Atomic Mass Unit
6. Radius of the Nucleus and Nuclear Density
7. Mass Energy Relation and Mass Defect
8. Binding Energy and Binding Energy per Nucleon
9. Binding Energy Curve and Inferences
10. Nuclear Forces and Meson Theory
11. Radioactivity and Soddy's Displacement Law
12. Rutherford and Soddy's Laws of Radioactive Decay
13. Radioactive Disintegration Constant and Half-Life Period
14. Units of Radioactivity
15. Nuclear Fission and Fusion

Rutherford's Alpha Scattering Experiment



Alpha – particle is a nucleus of helium atom carrying a charge of '+2e' and mass equal to 4 times that of hydrogen atom. It travels with a speed nearly 10^4 m/s and is highly penetrating.

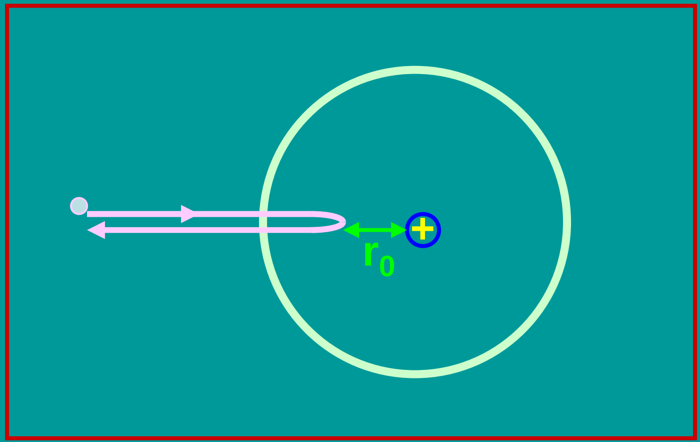
	Rutherford Experiment	Geiger & Marsden Experiment
Source of α-particle	Radon $_{86}Rn^{222}$	Bismuth $_{83}Bi^{214}$
Speed of α-particle	10^4 m/s	1.6×10^7 m/s
Thickness of Gold foil	10^{-6} m	2.1×10^{-7} m

S. No.	Observation	Conclusion
1	Most of the α-particles passed straight through the gold foil.	It indicates that most of the space in an atom is empty.
2	Some of the α-particles were scattered by only small angles, of the order of a few degrees.	α-particles being +vely charged and heavy compared to electron could only be deflected by heavy and positive region in an atom. It indicates that the positive charges and the most of the mass of the atom are concentrated at the centre called 'nucleus'.
3	A few α-particles (1 in 9000) were deflected through large angles (even greater than 90°). Some of them even retraced their path. i.e. angle of deflection was 180°.	α-particles which travel towards the nucleus directly get retarded due to Coulomb's force of repulsion and ultimately comes to rest and then fly off in the opposite direction.

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

Distance of Closest Approach (Nuclear size):

When the distance between α-particle and the nucleus is equal to the distance of the closest approach (r_0), the α-particle comes to rest.



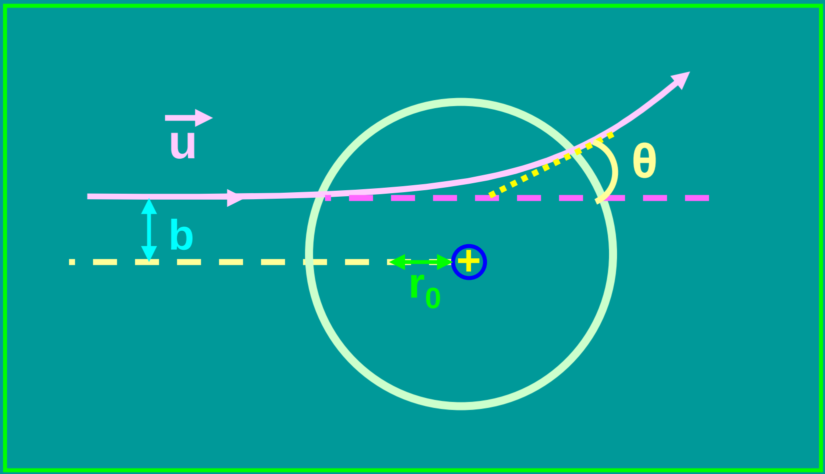
At this point or distance, the kinetic energy of α-particle is completely converted into electric potential energy of the system.

$$\frac{1}{2} m u^2 = \frac{1}{4\pi\varepsilon_0} \frac{2 Z e^2}{r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \frac{2 Z e^2}{\frac{1}{2} m u^2}$$

Impact Parameter (b):

The perpendicular distance of the velocity vector of the α-particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter.



$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0 (\frac{1}{2} mu^2)}$$

i) For large value of b, cot θ/2 is large and θ, the scattering angle is small.

i.e. α-particles travelling far away from the nucleus suffer small deflections.

ii) For small value of b, cot θ/2 is also small and θ, the scattering angle is large.
i.e. α-particles travelling close to the nucleus suffer large deflections.

iii) For b = 0 i.e. α-particles directed towards the centre of the nucleus,

cot θ/2 = 0 or θ/2 = 90° or θ = 180°

The α-particles retrace their path.

Composition of Nucleus:

Every atomic nucleus except that of Hydrogen has two types of particles – protons and neutrons. (Nucleus of Hydrogen contains only one proton)

Proton is a fundamental particle with positive charge 1.6×10^{-19} C and mass 1.67×10^{-27} kg (1836 times heavier than an electron).

Neutron is also a fundamental particle with no charge and mass 1.675×10^{-27} kg (1840 times heavier than an electron).

Atomic Number (Z):

The number of protons in a nucleus of an atom is called atomic number.

Atomic Mass Number (A):

The sum of number of protons and number of neutrons in a nucleus of an atom is called atomic mass number.

$$A = Z + N$$

Atomic Mass Unit (amu):

Atomic Mass Unit (amu) is (1 / 12)th of mass of 1 atom of carbon.

$$1 \text{ amu} = \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}} \text{ g} = 1.66 \times 10^{-27} \text{ kg}$$

Size of Nucleus:

Nucleus does not have a sharp or well-defined boundary.

However, the radius of nucleus can be given by

$R = R_0 A^{1/3}$ where $R_0 = 1.2 \times 10^{-5}$ m is a constant which is the same for all nuclei and A is the mass number of the nucleus.

Radius of nucleus ranges from 1 fm to 10 fm.

Nuclear Volume, $V = (4/3)\,\pi R^3 = (4/3)\,\pi R_0^3 A$

$$V \propto A$$

Nucleus Density:

Mass of nucleus, $M = A \text{ amu} = A \times 1.66 \times 10^{-27}$ kg

Nuclear Volume, $V = (4/3)\,\pi R^3 = (4/3)\,\pi R_0^3 A$

$$= \frac{4}{3} \times \frac{22}{7} \times (1.2 \times 10^{-15})^3 A \text{ m}^3$$

$$= 7.24 \times 10^{-45} A \text{ m}^3$$

Nucleus Density, $\rho = M / V = 2.29 \times 10^{17}$ kg / m^3

Discussion:

1. The nuclear density does not depend upon mass number. So, all the nuclei possess nearly the same density.
2. The nuclear density has extremely large value. Such high densities are found in white dwarf stars which contain mainly nuclear matter.
3. The nuclear density is not uniform throughout the nucleus. It has maximum value at the centre and decreases gradually as we move away from the centre of the nucleus.
4. The nuclear radius is the distance from the centre of the nucleus at which the density of nuclear matter decreases to one-half of its maximum value at the centre.

Mass – Energy Relation:

According to Newton's second law of motion, force acting on a body is defined as the rate of change of momentum.

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

If this force F displaces the body by a distance dx, its energy increases by

$$dK = F.dx = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$dK = m\frac{dx}{dt}dv + v\frac{dx}{dt}dm$$

$$dK = m\,v\,dv + v^2\,dm \qquad \ldots\ldots\ldots\ldots \text{(1)}$$

According to Einstein's relation of relativistic mass,

$$m = \frac{m_0}{[1 - (v^2 / c^2)]^{½}}$$

Squaring and manipulating, $m^2c^2 - m^2v^2 = m_0^2c^2$

Differentiating (with m_0 and c as constants)

$$c^2\, 2m\, dm - m^2\, 2v\, dv - v^2\, 2m\, dm = 0$$

or $$c^2\, dm - mv\, dv - v^2\, dm = 0$$

$$c^2\, dm = mv\, dv + v^2\, dm \qquad \text{.................(2)}$$

From (1) and (2), $dK = dm\, c^2$

If particle is accelerated from rest to a velocity v, let its mass m_0 increases to m.

Integrating,

$$\text{Total increase in K.E.} = \int_0^K dK = c^2 \int_{m_0}^{m} dm$$

$$\therefore \quad K = (m - m_0)\, c^2 \qquad \text{or} \qquad K + m_0\, c^2 = m\, c^2$$

Here m_0c^2 is the energy associated with the rest mass of the body and K is the kinetic energy.

Thus, the total energy of the body is given by $E = m\, c^2$

This is Einstein's mass - energy equivalence relation.

Mass Defect:

It is the difference between the rest mass of the nucleus and the sum of the masses of the nucleons composing a nucleus is known as mass defect.

$$\Delta m = [Zm_p + (A - Z) m_n] - M$$

Mass defect per nucleon is called packing fraction.

Binding Energy:

It is the energy required to break up a nucleus into its constituent parts and place them at an infinite distance from one another.

$$B.E = \Delta m c^2$$

Nuclear Forces:

They are the forces between p – p, p – n or n – n in the nucleus. They can be explained by Meson Theory.

There are three kinds of mesons – positive (π^+), negative (π^-) and neutral (π^0).

π^+ and π^- are 273 times heavier than an electron.
π^0 is 264 times heavier than an electron.

Nucleons (protons and neutrons) are surrounded by mesons.

Main points of Meson Theory:

1. **There is a continuous exchange of a meson between one nucleon and other. This gives rise to an exchange force between them and keep them bound.**

2. **Within the nucleus, a neutron is never permanently a neutron and a proton is never permanently a proton. They keep on changing into each other due to exchange of π-mesons.**

3. **The n – n forces arise due to exchange of π^0 – mesons between the neutrons.**

$n \rightarrow n + \pi^0$		(emission of π^0)
$n + \pi^0 \rightarrow n$		(absorption of π^0)

4. **The p – p forces arise due to exchange of π^0 – mesons between the protons.**

$p \rightarrow p + \pi^0$		(emission of π^0)
$p + \pi^0 \rightarrow p$		(absorption of π^0)

5. The n – p forces arise due to exchange of π^+ and π^- mesons between the nucleons.

$n \rightarrow p + \pi^-$ (emission of π^-)

$n + \pi^+ \rightarrow p$ (absorption of π^+)

$p \rightarrow n + \pi^+$ (emission of π^+)

$p + \pi^- \rightarrow n$ (absorption of π^-)

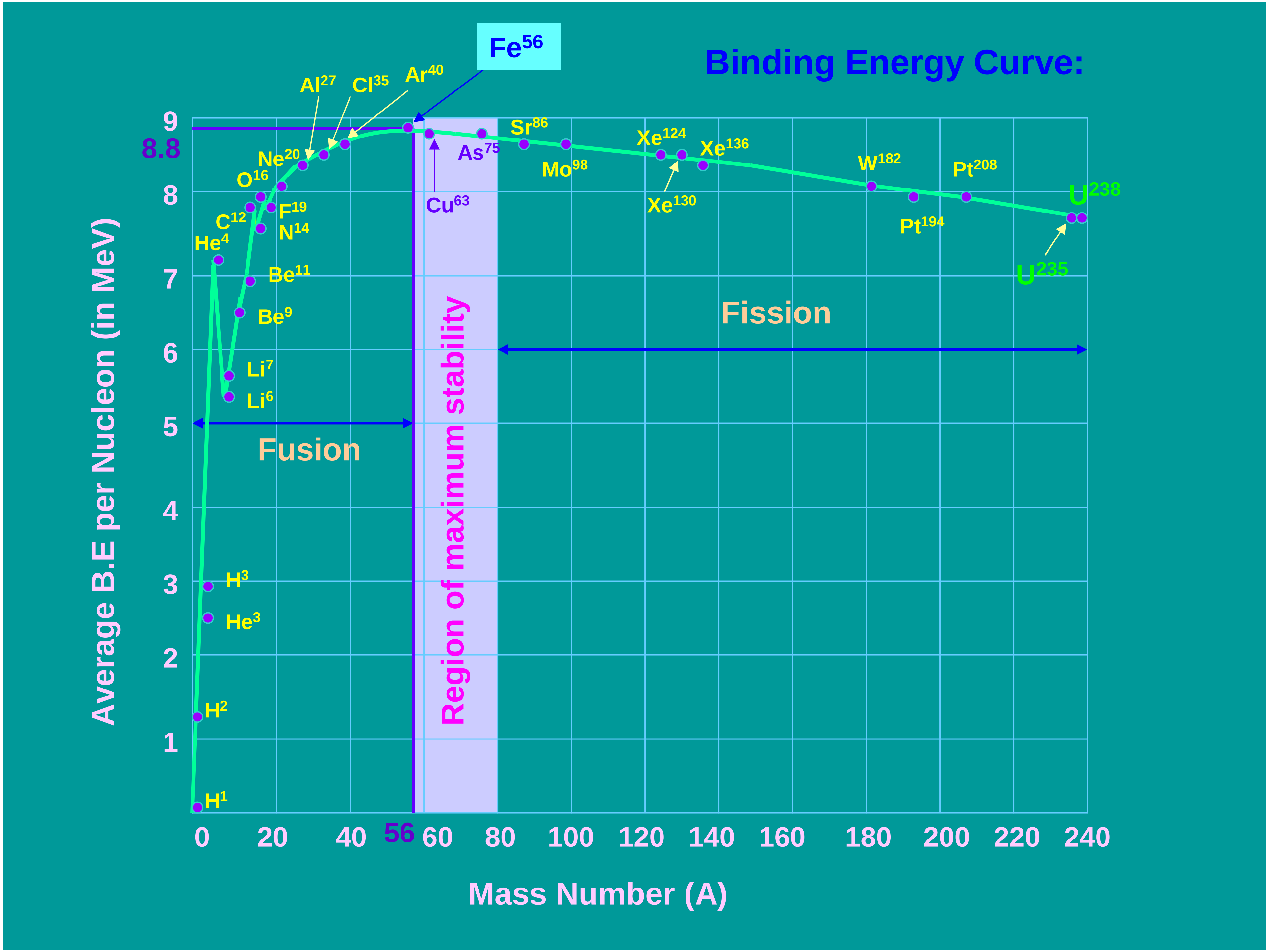
6. The time involved in such an exchange is so small that the free meson particles cannot be detected as such.

Binding Energy per Nucleon:

It is the binding energy divided by total number of nucleons.

It is denoted by $\bar{B}$

$\bar{B}$ = B.E / Nucleon = $\Delta m\, c^2 / A$

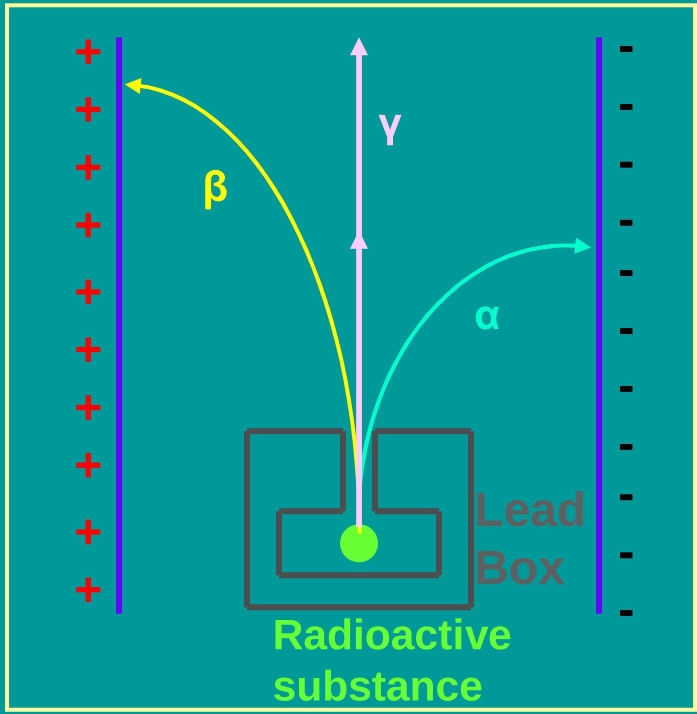

Binding Energy Curve:
Fe^{56}
Al^{27}
Cl^{35}
Ar^{40}
Ne^{20}
O^{16}
C^{12}
F^{19}
N^{14}
He^{4}
Be^{11}
Be^{9}
Li^{7}
Li^{6}
H^{3}
He^{3}
H^{2}
H^{1}
As^{75}
Cu^{63}
Sr^{86}
Mo^{98}
Xe^{124}
Xe^{136}
Xe^{130}
W^{182}
Pt^{208}
Pt^{194}
U^{238}
U^{235}
Fission
Fusion
Region of maximum stability
Average B.E per Nucleon (in MeV)
Mass Number (A)
9
8.8
8
7
6
5
4
3
2
1
0
20
40
56
60
80
100
120
140
160
180
200
220
240

Special Features of Binding Energy Curve

1. Binding energy per nucleon of very light nuclides such as $_1H^2$ is very small.
2. Initially, there is a rapid rise in the value of binding energy per nucleon.
3. Between mass numbers 4 and 20, the curve shows cyclic recurrence of peaks corresponding to $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$. This shows that the B.E. per nucleon of these nuclides is greater than those of their immediate neighbours. Each of these nuclei can be formed by adding an alpha particle to the preceding nucleus.
4. After A = 20, there is a gradual increase in B.E. per nucleon. The maximum value of 8.8 MeV is reached at A = 56. Therefore, Iron nucleus is the most stable.
5. Binding energy per nucleon of nuclides having mass numbers ranging from 40 to 120 are close to the maximum value. So, these elements are highly stable and non-radioactive.
6. Beyond A = 120, the value decreases and falls to 7.6 MeV for Uranium.
7. Beyond A = 128, the value shows a rapid decrease. This makes elements beyond Uranium (trans – uranium elements) quite unstable and radioactive.
8. The drooping of the curve at high mass number indicates that the nucleons are more tightly bound and they can undergo fission to become stable.
9. The drooping of the curve at low mass numbers indicates that the nucleons can undergo fusion to become stable.

Radioactivity:

Radioactivity is the phenomenon of emitting alpha, beta and gamma radiations spontaneously.



Soddy's Displacement Law:

1. $_{Z}Y^{A} \xrightarrow{\alpha} {}_{Z-2}Y^{A-4}$

2. $_{Z}Y^{A} \xrightarrow{\beta} {}_{Z+1}Y^{A}$

3. $_{Z}Y^{A} \xrightarrow{\gamma} {}_{Z}Y^{A}$ (Lower energy)

Rutherford and Soddy's Laws of Radioactive Decay:

1. The disintegration of radioactive material is purely a random process and it is merely a matter of chance. Which nucleus will suffer disintegration, or decay first can not be told.

2. The rate of decay is completely independent of the physical composition and chemical condition of the material.

3. The rate of decay is directly proportional to the quantity of material actually present at that instant. As the decay goes on, the original material goes on decreasing and the rate of decay consequently goes on decreasing.

If N is the number of radioactive atoms present at any instant, then the rate of decay is,

$$-\frac{dN}{dt} \propto N \quad \text{or} \quad -\frac{dN}{dt} = \lambda N$$

where λ is the decay constant or the disintegration constant.

Rearranging,

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating, $\log_e N = -\lambda t + C$ where C is the integration constant.

If at $t = 0$, we had N_0 atoms, then

$$\log_e N_0 = 0 + C$$

$$\therefore \quad \log_e N - \log_e N_0 = -\lambda t$$

or $\log_e (N / N_0) = -\lambda t$

or $\frac{N}{N_0} = e^{-\lambda t}$ or $N = N_0\, e^{-\lambda t}$

Radioactive Disintegration Constant (λ):

According to the laws of radioactive decay,

$$\frac{dN}{N} = -\lambda\, dt$$

If dt = 1 second, then

$$\frac{dN}{N} = -\lambda$$

Thus, λ may be defined as the relative number of atoms decaying per second.

Again, since $N = N_0\, e^{-\lambda t}$

And if, $t = 1 / \lambda$, then $N = N_0 / e$

or $$\frac{N}{N_0} = \frac{1}{e}$$

Thus, λ may also be defined as the reciprocal of the time when N / N_0 falls to 1 / e.

Half – Life Period:

Half life period is the time required for the disintegration of half of the amount of the radioactive substance originally present.

If T is the half – life period, then

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda T} \qquad \text{(since } N = N_0 / 2)$$

$$e^{\lambda T} = 2$$

$$\lambda T = \log_e 2 = 0.6931$$

$$T = \frac{0.6931}{\lambda} \quad \text{or} \quad \lambda = \frac{0.6931}{T}$$

Time t in which material changes from N_0 to N:

$$t = 3.323\ T \log_{10} (N_0 / N)$$

Number of Atoms left behind after n Half – Lives:

$$N = N_0 (1/2)^n \quad \text{or} \quad N = N_0 (1/2)^{t/T}$$

Units of Radioactivity:

1. **The curie (Ci): The activity of a radioactive substance is said to be one curie if it undergoes 3.7×10^{10} disintegrations per second.**

   1 curie = 3.7×10^{10} disintegrations / second

2. **The rutherford (Rd): The activity of a radioactive substance is said to be one rutherford if it undergoes 10^{6} disintegrations per second.**

   1 rutherford = 10^{6} disintegrations / second

3. **The becquerel (Bq): The activity of a radioactive substance is said to be one becquerel if it undergoes 1 disintegration per second.**

   1 becquerel = 1 disintegration / second

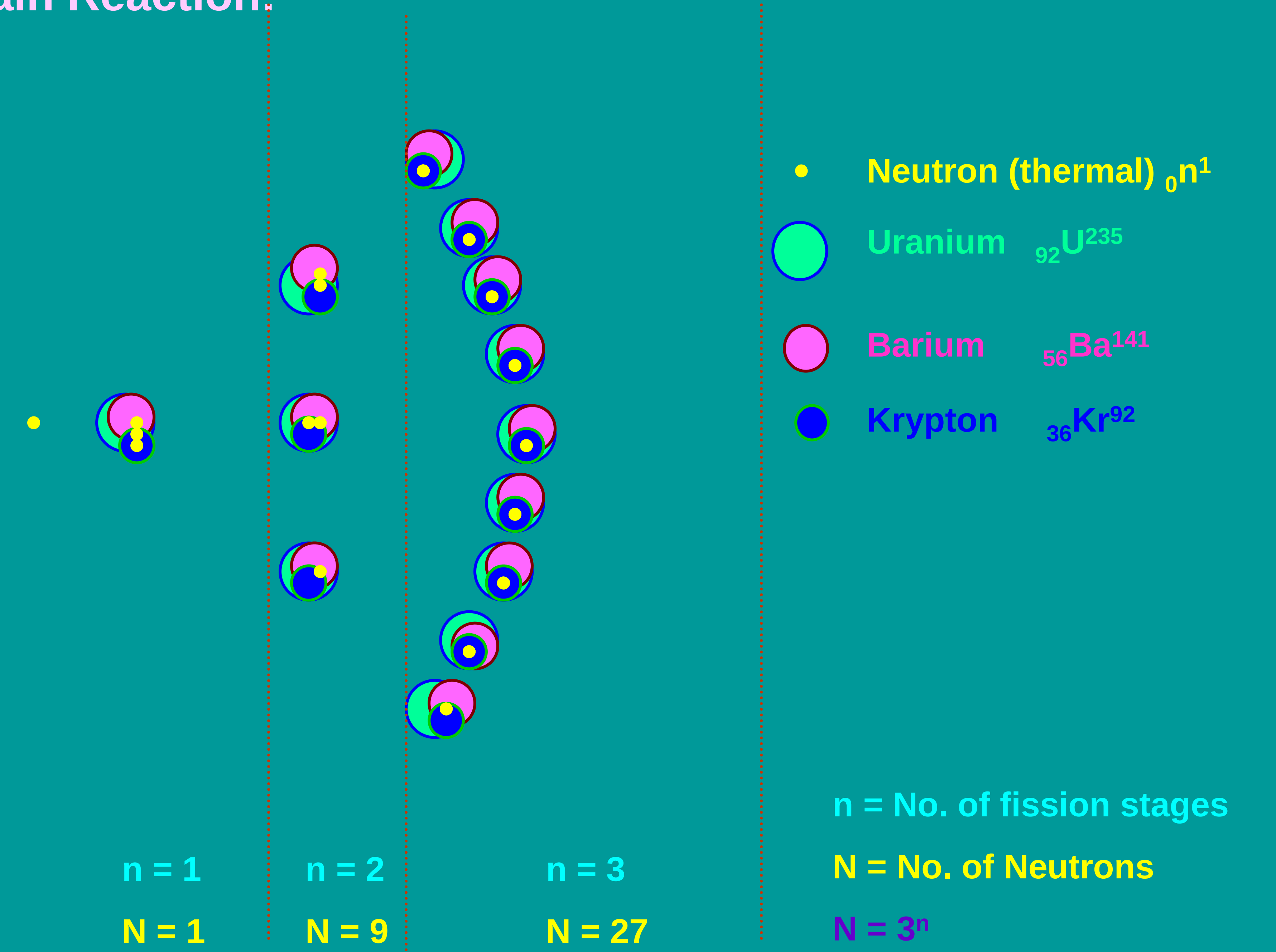
1 curie = 3.7×10^{4} rutherford = 3.7×10^{10} becquerel

Nuclear Fission:

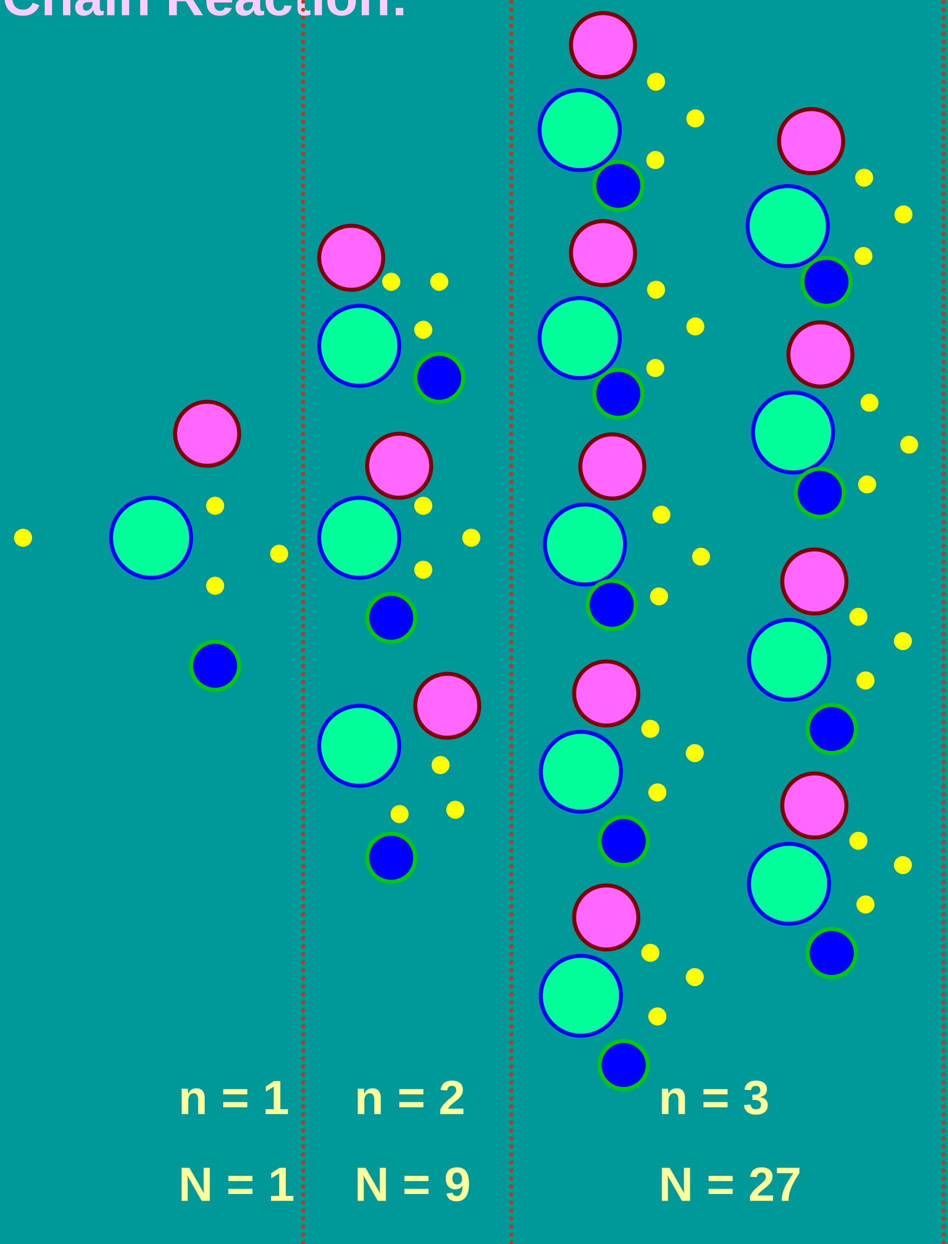
Nuclear fission is defined as a type of nuclear disintegration in which a heavy nucleus splits up into two nuclei of comparable size accompanied by a release of a large amount of energy.

$$_{0}n^{1} + {}_{92}U^{235} \rightarrow ({}_{92}U^{236}) \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + \gamma\ (200\ \text{MeV})$$

Chain Reaction:



Chain Reaction:



Critical Size:

For chain reaction to occur, the size of the fissionable material must be above the size called 'critical size'.

A released neutron must travel minimum through 10 cm so that it is properly slowed down (thermal neutron) to cause further fission.

If the size of the material is less than the critical size, then all the neutrons are lost.

If the size is equal to the critical size, then the no. of neutrons produced is equal to the no. of neutrons lost.

If the size is greater than the critical size, then the reproduction ratio of neutrons is greater than 1 and chain reaction can occur.

Nuclear Fusion:

Nuclear fusion is defined as a type of nuclear reaction in which two lighter nuclei merge into one another to form a heavier nucleus accompanied by a release of a large amount of energy.

Energy Source of Sun:

Proton – Proton Cycle:

$$_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + 0.4\ \text{MeV}$$

$$_1H^1 + {}_1H^2 \rightarrow {}_2He^3 + 5.5\ \text{MeV}$$

$$_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + 12.9\ \text{MeV}$$

Energy Source of Star:

Carbon - Nitrogen Cycle:

$$_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + \gamma \quad \text{(energy)}$$

$$_7N^{13} \rightarrow {}_6C^{13} + {}_1e^0 \quad \text{(positron)}$$

$$_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14} + \gamma \quad \text{(energy)}$$

$$_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} + \gamma \quad \text{(energy)}$$

$$_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 \quad \text{(positron)}$$

$$_7N^{15} + {}_1H^1 \rightarrow {}_6C^{12} + {}_2He^4 + \gamma \quad \text{(energy)}$$

End of Atomic Nucleus

ELECTRONIC DEVICES - I

1. Energy Bands in Solids
2. Energy Band Diagram
3. Metals, Semiconductors and Insulators
4. Intrinsic Semiconductor
5. Electrons and Holes
6. Doping of a Semiconductor
7. Extrinsic Semiconductor
8. N-type and P-type Semiconductor
9. Carrier Concentration in Semiconductors
10. Distinction between Intrinsic and Extrinsic Semiconductors
11. Distinction between Semiconductor and Metal
12. Conductivity of a Semiconductor

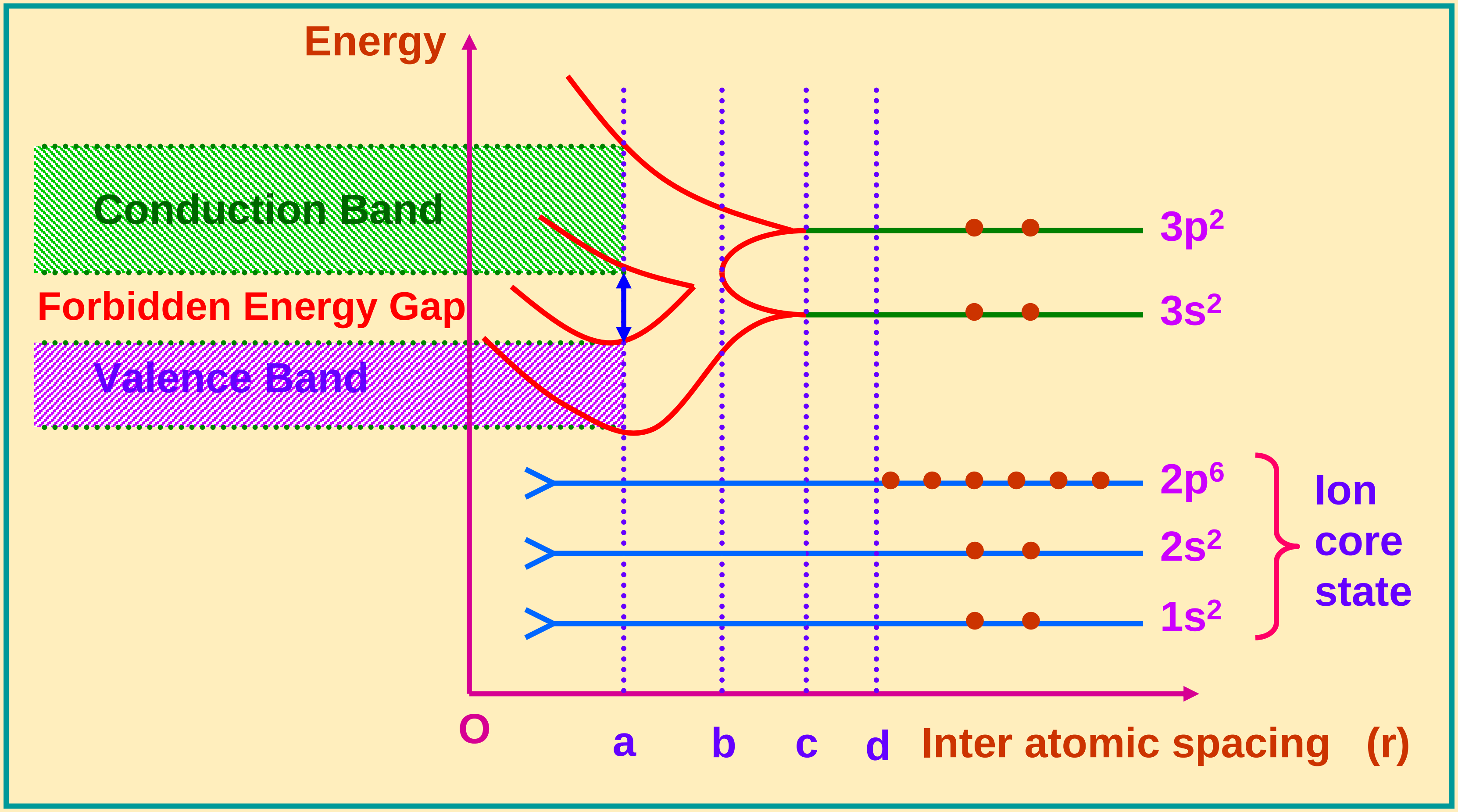
Energy Bands in Solids:

- **According to Quantum Mechanical Laws, the energies of electrons in a free atom can not have arbitrary values but only some definite (quantized) values.**
- **However, if an atom belongs to a crystal, then the energy levels are modified.**
- **This modification is not appreciable in the case of energy levels of electrons in the inner shells (completely filled).**
- **But in the outermost shells, modification is appreciable because the electrons are shared by many neighbouring atoms.**
- **Due to influence of high electric field between the core of the atoms and the shared electrons, energy levels are split-up or spread out forming energy bands.**

Consider a single crystal of silicon having N atoms. Each atom can be associated with a lattice site.

Electronic configuration of Si is $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^2$. (Atomic No. is 14)

Formation of Energy Bands in Solids:



(i) r = Od (>> Oa):

Each of N atoms has its own energy levels. The energy levels are identical, sharp, discrete and distinct.

The outer two sub-shells (3s and 3p of M shell or n = 3 shell) of silicon atom contain two s electrons and two p electrons. So, there are 2N electrons completely filling 2N possible s levels, all of which are at the same energy.

Of the 6N possible p levels, only 2N are filled and all the filled p levels have the same energy.

(ii) Oc < r < Od:

There is no visible splitting of energy levels but there develops a tendency for the splitting of energy levels.

(iii) r = Oc:

The interaction between the outermost shell electrons of neighbouring silicon atoms becomes appreciable and the splitting of the energy levels commences.

(iv) Ob < r < Oc:

The energy corresponding to the s and p levels of each atom gets slightly changed. Corresponding to a single s level of an isolated atom, we get 2N levels. Similarly, there are 6N levels for a single p level of an isolated atom.

Since N is a very large number (≈ 10^{29} atoms / m^3) and the energy of each level is of a few eV, therefore, the levels due to the spreading are very closely spaced. The spacing is ≈ 10^{-23} eV for a 1 cm^3 crystal.

The collection of very closely spaced energy levels is called an energy band.

(v) r = Ob:

The energy gap disappears completely. 8N levels are distributed continuously. We can only say that 4N levels are filled and 4N levels are empty.

(v) r = Oa:

The band of 4N filled energy levels is separated from the band of 4N unfilled energy levels by an energy gap called forbidden gap or energy gap or band gap.

The lower completely filled band (with valence electrons) is called the valence band and the upper unfilled band is called the conduction band.

Note:

1. **The exact energy band picture depends on the relative orientation of atoms in a crystal.**
2. **If the bands in a solid are completely filled, the electrons are not permitted to move about, because there are no vacant energy levels available.**

Metals:

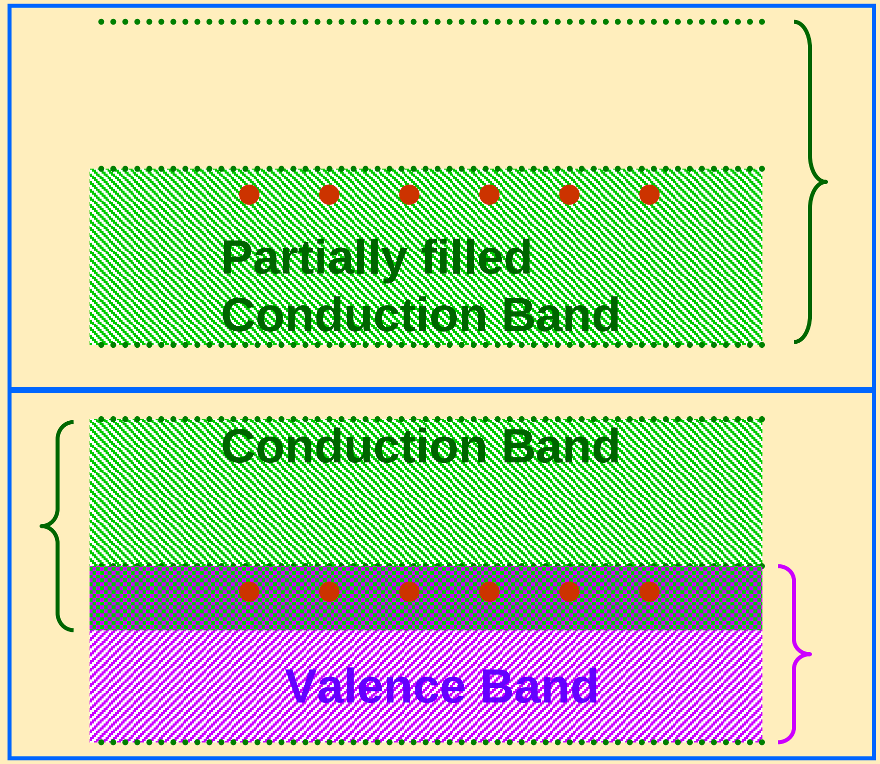
The first possible energy band diagram shows that the conduction band is only partially filled with electrons.

With a little extra energy the electrons can easily reach the empty energy levels above the filled ones and the conduction is possible.

The second possible energy band diagram shows that the conduction band is overlapping with the valence band.

This is because the lowest levels in the conduction band needs less energy than the highest levels in the valence band.

The electrons in valence band overflow into conduction band and are free to move about in the crystal for conduction.



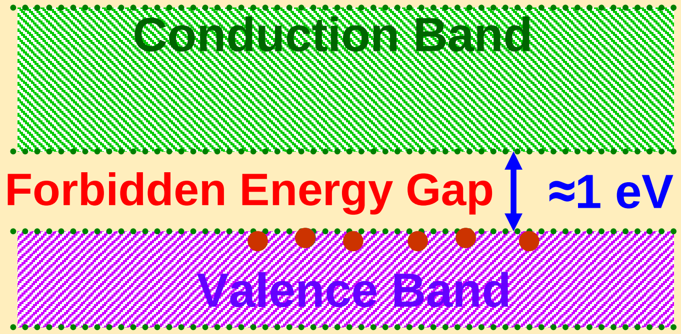
The highest energy level in the conduction band occupied by electrons in a crystal, at absolute 0 temperature, is called Fermi Level.

The energy corresponding to this energy level is called Fermi energy.

If the electrons get enough energy to go beyond this level, then conduction takes place.

Semiconductors:

At absolute zero temperature, no electron has energy to jump from valence band to conduction band and hence the crystal is an insulator.



At room temperature, some valence electrons gain energy more than the energy gap and move to conduction band to conduct even under the influence of a weak electric field.

$E_{g\text{-}Si}$ = 1.1 eV Eg_{Ge}= 0.74 eV

The fraction is $p \propto e^{-\frac{E_g}{k_B T}}$

Since E_g is small, therefore, the fraction is sizeable for semiconductors.

As an electron leaves the valence band, it leaves some energy level in band as unfilled.

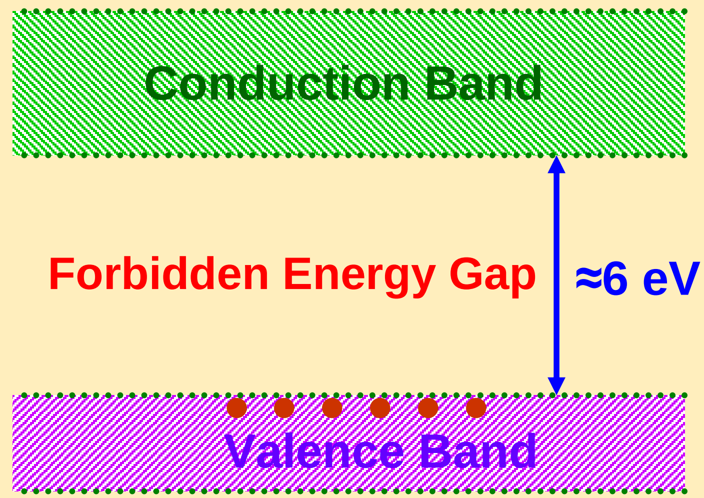
Such unfilled regions are termed as 'holes' in the valence band. They are mathematically taken as positive charge carriers.

Any movement of this region is referred to a positive hole moving from one position to another.

Insulators:

Electrons, however heated, can not practically jump to conduction band from valence band due to a large energy gap. Therefore, conduction is not possible in insulators.

$E_{g\text{-}Diamond} = 7\ eV$



Electrons and Holes:

On receiving an additional energy, one of the electrons from a covalent band breaks and is free to move in the crystal lattice.

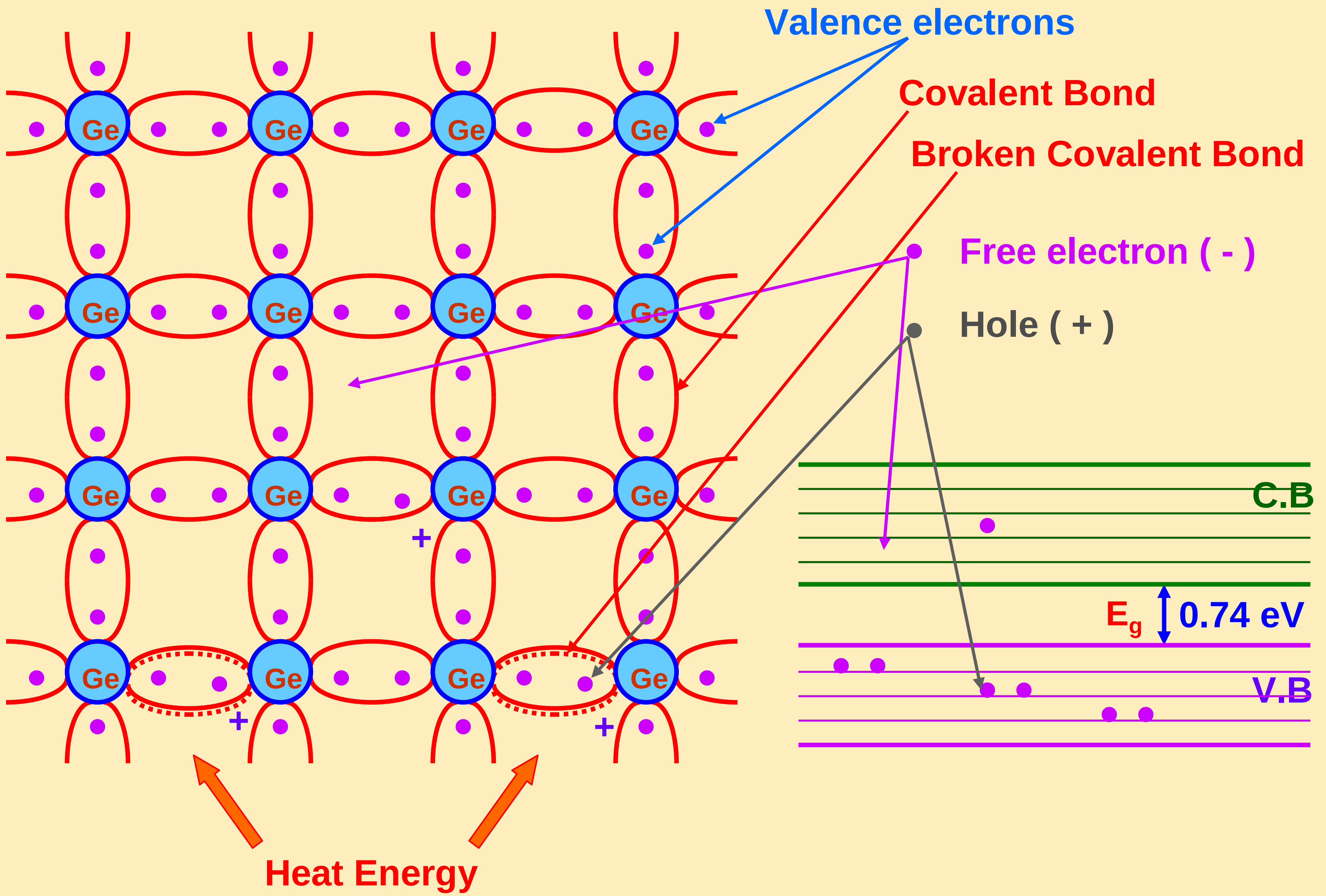
While coming out of the covalent bond, it leaves behind a vacancy named 'hole'.

An electron from the neighbouring atom can break away and can come to the place of the missing electron (or hole) completing the covalent bond and creating a hole at another place.

The holes move randomly in a crystal lattice.

The completion of a bond may not be necessarily due to an electron from a bond of a neighbouring atom. The bond may be completed by a conduction band electron. i.e., free electron and this is referred to as 'electron – hole recombination'.

Intrinsic or Pure Semiconductor:



Intrinsic Semiconductor is a pure semiconductor.

The energy gap in Si is 1.1 eV and in Ge is 0.74 eV.

Si: $1s^2, 2s^2, 2p^6, 3s^2, 3p^2$. (Atomic No. is 14)

Ge: $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^2, 4p^2$. (Atomic No. is 32)

In intrinsic semiconductor, the number of thermally generated electrons always equals the number of holes.

So, if n_i and p_i are the concentration of electrons and holes respectively, then $n_i = p_i$.

The quantity n_i or p_i is referred to as the 'intrinsic carrier concentration'.

Doping a Semiconductor:

Doping is the process of deliberate addition of a very small amount of impurity into an intrinsic semiconductor.

The impurity atoms are called 'dopants'.

The semiconductor containing impurity is known as 'impure or extrinsic semiconductor'.

Methods of doping:

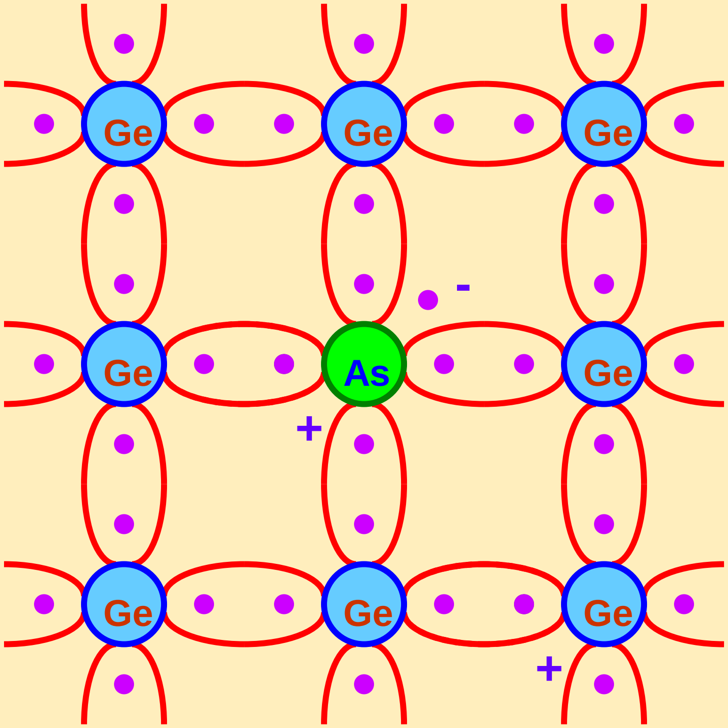
i) Heating the crystal in the presence of dopant atoms.

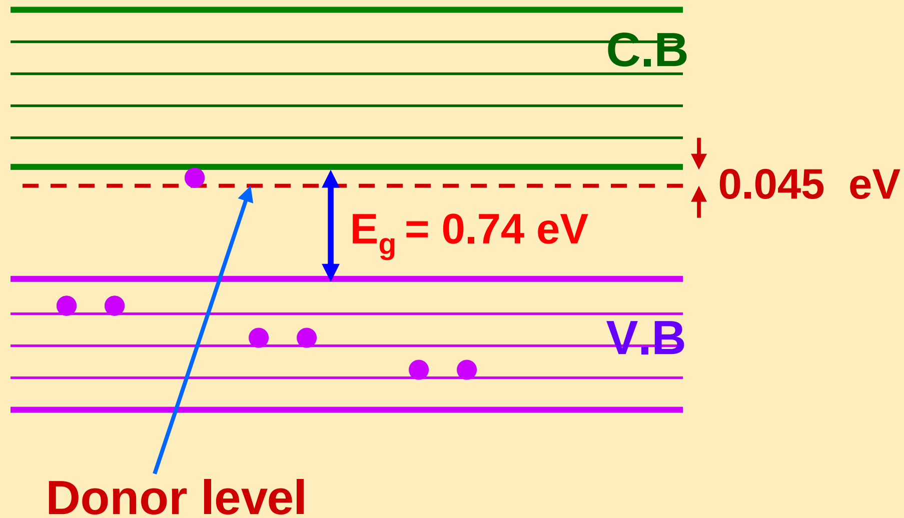
ii) Adding impurity atoms in the molten state of semiconductor.

iii) Bombarding semiconductor by ions of impurity atoms.

Extrinsic or Impure Semiconductor:

N - Type Semiconductors:





When a semiconductor of Group IV (tetra valent) such as Si or Ge is doped with a penta valent impurity (Group V elements such as P, As or Sb), N – type semiconductor is formed.

When germanium (Ge) is doped with arsenic (As), the four valence electrons of As form covalent bonds with four Ge atoms and the fifth electron of As atom is loosely bound.

The energy required to detach the fifth loosely bound electron is only of the order of 0.045 eV for germanium.

A small amount of energy provided due to thermal agitation is sufficient to detach this electron and it is ready to conduct current.

The force of attraction between this mobile electron and the positively charged (+ 5) impurity ion is weakened by the dielectric constant of the medium.

So, such electrons from impurity atoms will have energies slightly less than the energies of the electrons in the conduction band.

Therefore, the energy state corresponding to the fifth electron is in the forbidden gap and slightly below the lower level of the conduction band.

This energy level is called 'donor level'.

The impurity atom is called 'donor'.

N – type semiconductor is called 'donor – type semiconductor'.

Carrier Concentration in N - Type Semiconductors:

When intrinsic semiconductor is doped with donor impurities, not only does the number of electrons increase, but also the number of holes decreases below that which would be available in the intrinsic semiconductor.

The number of holes decreases because the larger number of electrons present causes the rate of recombination of electrons with holes to increase.

Consequently, in an N-type semiconductor, free electrons are the majority charge carriers and holes are the minority charge carriers.

If n and p represent the electron and hole concentrations respectively in N-type semiconductor, then

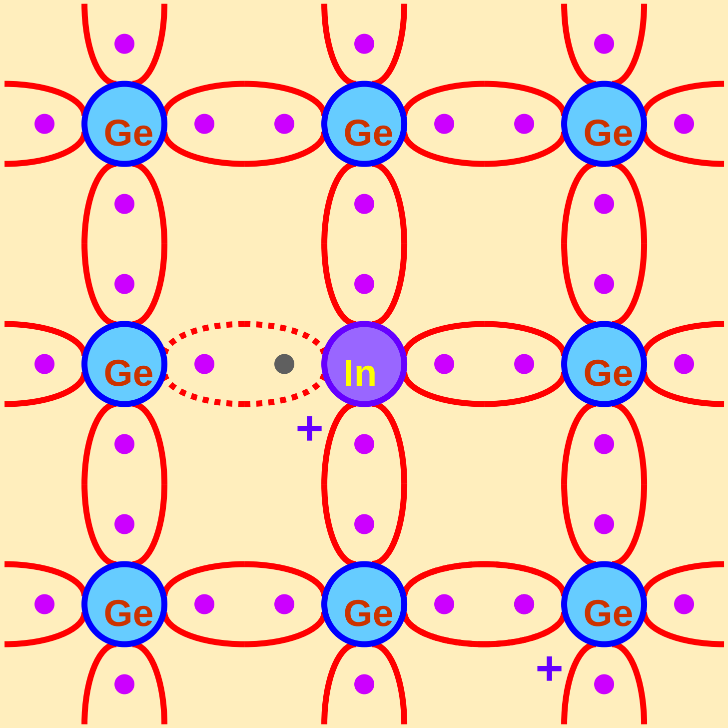
$$n\,p = n_i\,p_i = n_i^2$$

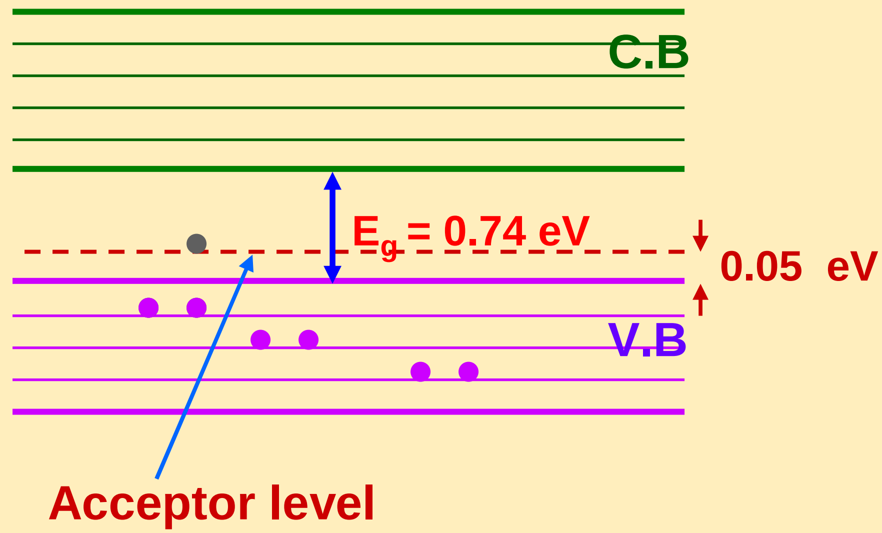
where n_i and p_i are the intrinsic carrier concentrations.

The rate of recombination of electrons and holes is proportional to n and p. Or, the rate of recombination is proportional to the product np. Since the rate of recombination is fixed at a given temperature, therefore, the product np must be a constant.

When the concentration of electrons is increased above the intrinsic value by the addition of donor impurities, the concentration of holes falls below its intrinsic value, making the product np a constant, equal to n_i^2.

P - Type Semiconductors:





When a semiconductor of Group IV (tetra valent) such as Si or Ge is doped with a tri valent impurity (Group III elements such as In, B or Ga), P – type semiconductor is formed.

When germanium (Ge) is doped with indium (In), the three valence electrons of In form three covalent bonds with three Ge atoms. The vacancy that exists with the fourth covalent bond with fourth Ge atom constitutes a hole.

The hole which is deliberately created may be filled with an electron from neighbouring atom, creating a hole in that position from where the electron jumped.

Therefore, the tri valent impurity atom is called 'acceptor'.

Since the hole is associated with a positive charge moving from one position to another, therefore, this type of semiconductor is called
P – type semiconductor.

The acceptor impurity produces an energy level just above the valence band.

This energy level is called 'acceptor level'.

The energy difference between the acceptor energy level and the top of the valence band is much smaller than the band gap.

Electrons from the valence band can, therefore, easily move into the acceptor level by being thermally agitated.

P – type semiconductor is called 'acceptor – type semiconductor'.

In a P – type semiconductor, holes are the majority charge carriers and the electrons are the minority charge carriers.

It can be shown that, $n\,p = n_i\,p_i = n_i^2$

Distinction between Intrinsic and Extrinsic Semiconductor:

S. No.	Intrinsic SC	Extrinsic SC
1	Pure Group IV elements.	Group III or Group V elements are introduced in Group IV elements.
2	Conductivity is only slight.	Conductivity is greatly increased.
3	Conductivity increases with rise in temperature.	Conductivity depends on the amount of impurity added.
4	The number of holes is always equal to the number of free electrons.	In N-type, the no. of electrons is greater than that of the holes and in P-type, the no. holes is greater than that of the electrons.

Distinction between Semiconductor and Metal:

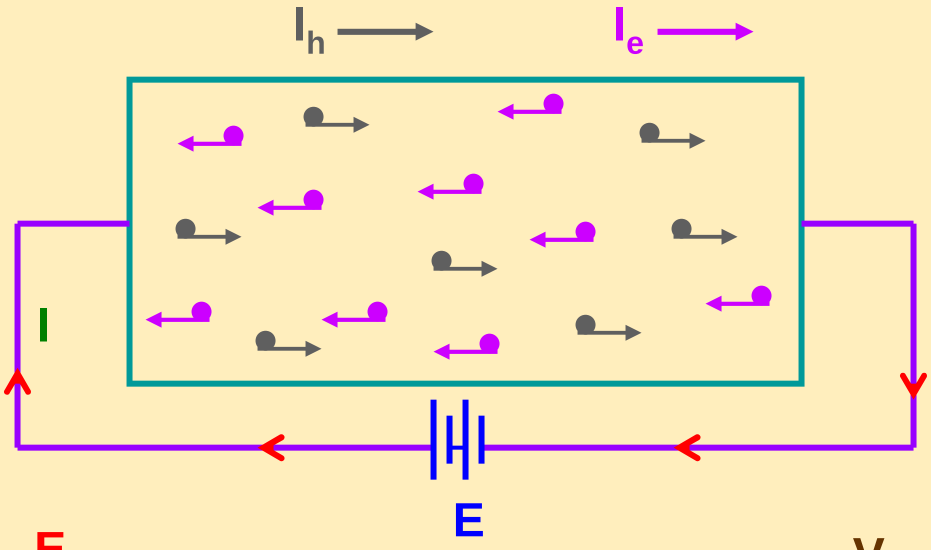
S. No.	Semiconductor	Metal
1	Semiconductor behaves like an insulator at 0 K. Its conductivity increases with rise in temperature.	Conductivity decreases with rise in temperature.
2	Conductivity increases with rise in potential difference applied.	Conductivity is an intrinsic property of a metal and is independent of applied potential difference.
3	Does not obey Ohm's law or only partially obeys.	Obeys Ohm's law.
4	Doping the semiconductors with impurities vastly increases the conductivity.	Making alloy with another metal decreases the conductivity.

Electrical Conductivity of Semiconductors:

$I = I_e + I_h$

$I_e = n_e e A v_e \qquad I_h = n_h e A v_h$

So, $I = n_e e A v_e + n_h e A v_h$



If the applied electric field is small, then semiconductor obeys Ohm's law.

$$\therefore \frac{V}{R} = n_e e A v_e + n_h e A v_h$$

$$= eA\,(n_e v_e + n_h v_h)$$

$$\text{Or} \quad \frac{V\,A}{\rho l} = eA\,(n_e v_e + n_h v_h)$$

$$\text{since} \quad R = \frac{\rho l}{A}$$

$$\frac{E}{\rho} = e\,(n_e v_e + n_h v_h) \qquad \text{since} \quad E = \frac{V}{l}$$

Mobility (μ) is defined as the drift velocity per unit electric field.

$$\therefore \frac{1}{\rho} = e\,(n_e \mu_e + n_h \mu_h)$$

$$\text{Or} \quad \sigma = e\,(n_e \mu_e + n_h \mu_h)$$

Note:

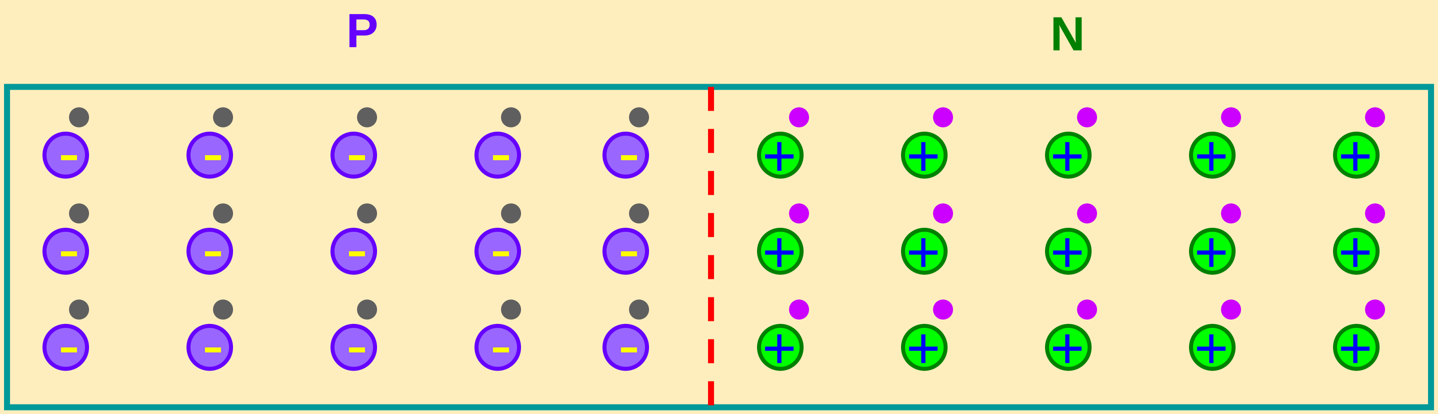
1. The electron mobility is higher than the hole mobility.
2. The resistivity / conductivity depends not only on the electron and hole densities but also on their mobilities.
3. The mobility depends relatively weakly on temperature.

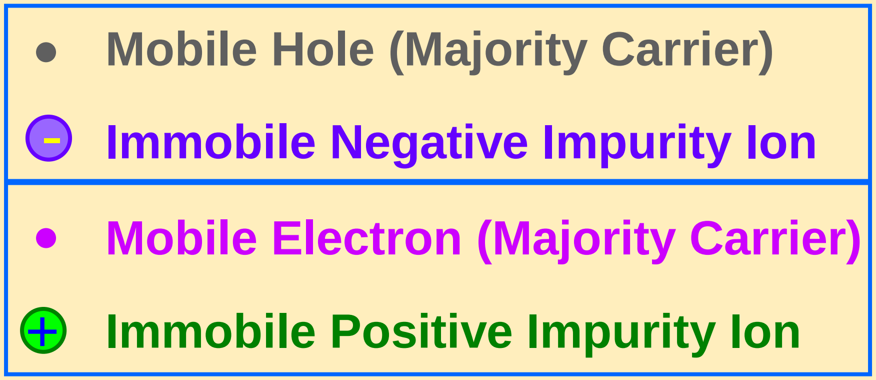
ELECTRONIC DEVICES - II

1. PN Junction Diode
2. Forward Bias of Junction Diode
3. Reverse Bias of Junction Diode
4. Diode Characteristics
5. Static and Dynamic Resistance of a Diode
6. Diode as a Half Wave Rectifier
7. Diode as a Full Wave Rectifier

PN Junction Diode:

When a P-type semiconductor is joined to a N-type semiconductor such that the crystal structure remains continuous at the boundary, the resulting arrangement is called a **PN junction diode** or a **semiconductor diode** or a **crystal diode**.





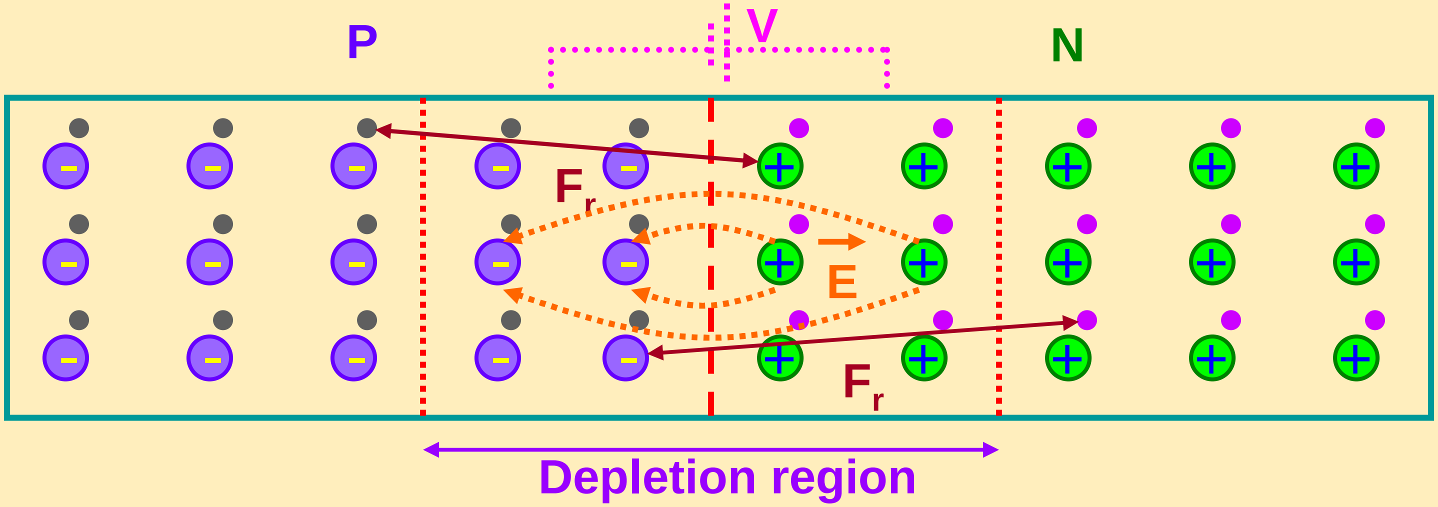
When a PN junction is formed, the P region has mobile holes (+) and immobile negatively charged ions.

N region has mobile electrons (-) and immobile positively charged ions.

The whole arrangement is electrically neutral.

For simplicity, the minority charge carriers are not shown in the figure.

PN Junction Diode immediately after it is formed :



After the PN junction diode is formed –

i) Holes from P region diffuse into N region due to difference in concentration.

ii) Free electrons from N region diffuse into P region due to the same reason.

iii) Holes and free electrons combine near the junction.

iv) Each recombination eliminates an electron and a hole.

v) The uncompensated negative immobile ions in the P region do not allow any more free electrons to diffuse from N region.

vi) The uncompensated positive immobile ions in the N region do not allow any more holes to diffuse from P region.

vii) The positive donor ions in the N region and the negative acceptor ions in the P region are left uncompensated.

viii) The region containing the uncompensated acceptor and donor ions is called 'depletion region' because this region is devoid of mobile charges.

Since the region is having only immobile charges, therefore, this region is also called 'space charge region'.

ix) The N region is having higher potential than P region.

x) So, an electric field is set up as shown in the figure.

xi) The difference in potential between P and N regions across the junction makes it difficult for the holes and electrons to move across the junction. This acts as a barrier and hence called 'potential barrier' or 'height of the barrier'.

xii) The physical distance between one side and the other side of the barrier is called 'width of the barrier'.

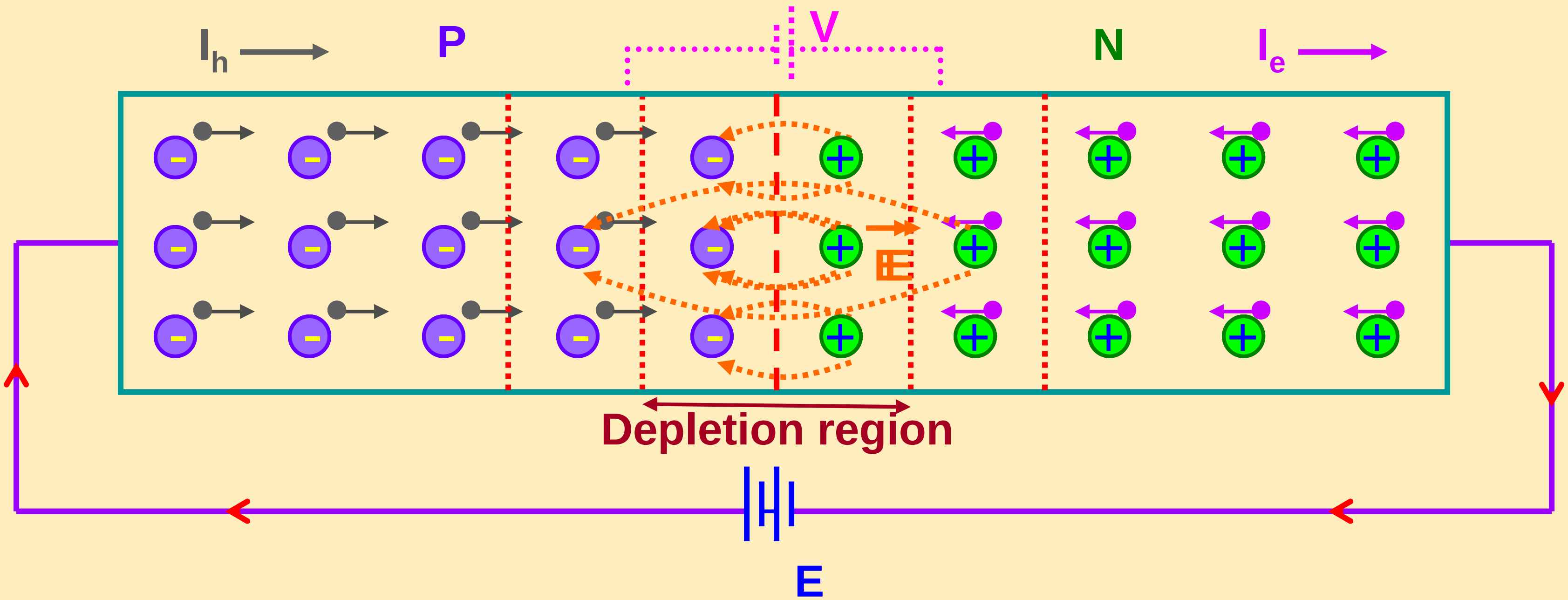
xiii) Potential barrier for Si is nearly 0.7 V and for Ge is 0.3 V.

xiv) The potential barrier opposes the motion of the majority carriers.

xv) However, a few majority carriers with high kinetic energy manage to overcome the barrier and cross the junction.

xvi) Potential barrier helps the movement of minority carriers.

Forward Bias:



When the positive terminal of the battery is connected to P-region and negative terminal is connected to N-region, then the PN junction diode is said to be forward-biased.

i) Holes in P-region are repelled by +ve terminal of the battery and the free electrons are repelled by –ve terminal of the battery.

ii) So, some holes and free electrons enter into the depletion region.

iii) The potential barrier and the width of the depletion region decrease.

iv) Therefore, a large number of majority carriers diffuse across the junction.

v) Hole current and electronic current are in the same direction and add up.

v) Once they cross the junction, the holes in N-region and the electrons in P-region become minority carriers of charge and constitute minority current.

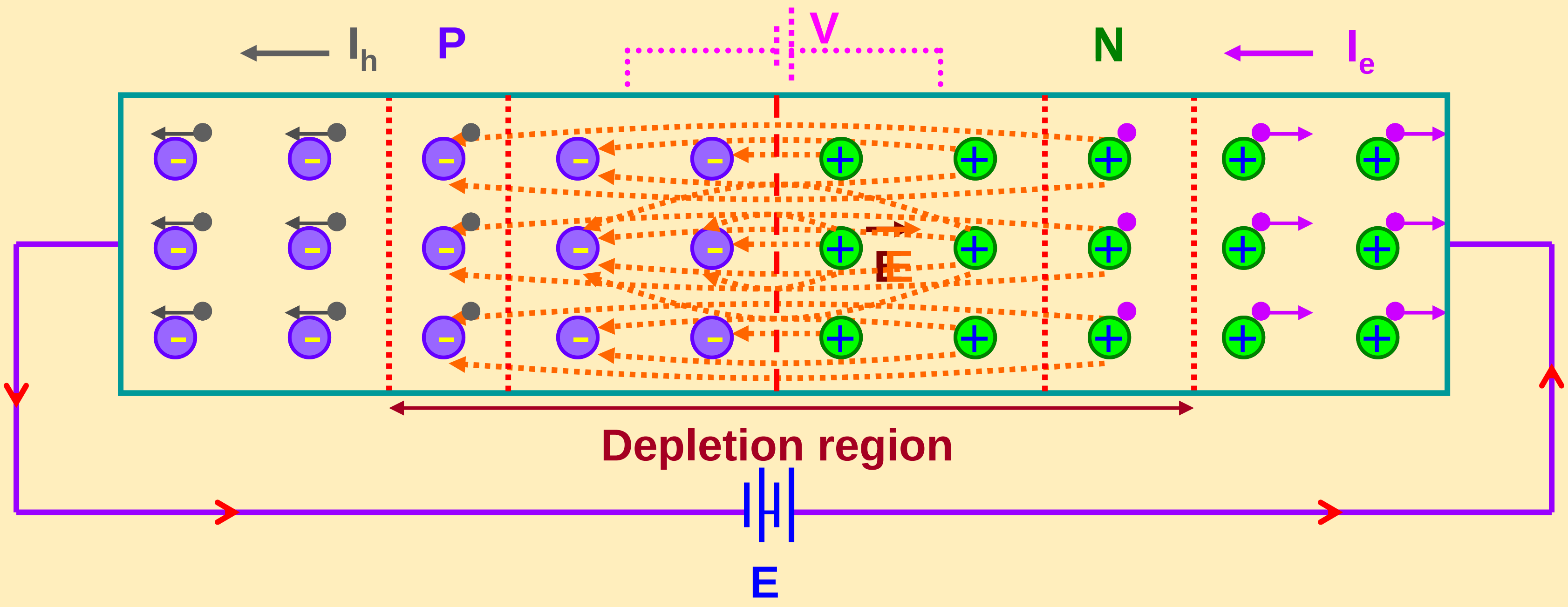
vi) For each electron – hole recombination, an electron from the negative terminal of the battery enters the N-region and then drifts towards the junction.

In the P-region, near the positive terminal of the battery, an electron breaks covalent bond in the crystal and thus a hole is created. The hole drifts towards the junction and the electron enters the positive terminal of the battery.

vii) Thus, the current in the external circuit is due to movement of electrons, current in P-region is due to movement of holes and current in N-region is due to movement of electrons.

viii) If the applied is increased, the potential barrier further decreases. As a result, a large number of majority carriers diffuse through the junction and a larger current flows.

Reverse Bias:



When the negative terminal of the battery is connected to P-region and positive terminal is connected to N-region, then the PN junction diode is said to be reverse-biased.

i) Holes in P-region are attracted by -ve terminal of the battery and the free electrons are attracted by +ve terminal of the battery.

ii) Thus, the majority carriers are pulled away from the junction.

iii) The potential barrier and the width of the depletion region increase.

iv) Therefore, it becomes more difficult for majority carriers diffuse across the junction.

v) But the potential barrier helps the movement of the minority carriers. As soon as the minority carriers are generated, they are swept away by the potential barrier.

vi) At a given temperature, the rate of generation of minority carriers is constant.

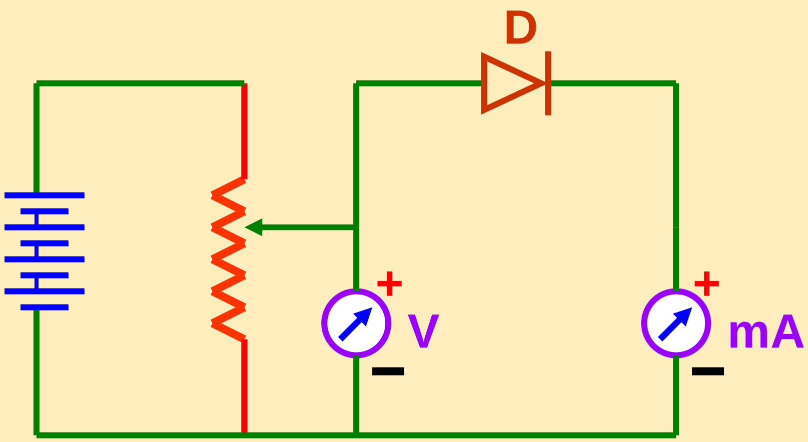
vii) So, the resulting current is constant irrespective of the applied voltage. For this reason, this current is called 'reverse saturation current'.

viii) Since the number of minority carriers is small, therefore, this current is small and is in the order of 10^{-9} A in silicon diode and 10^{-6} A in germanium diode.

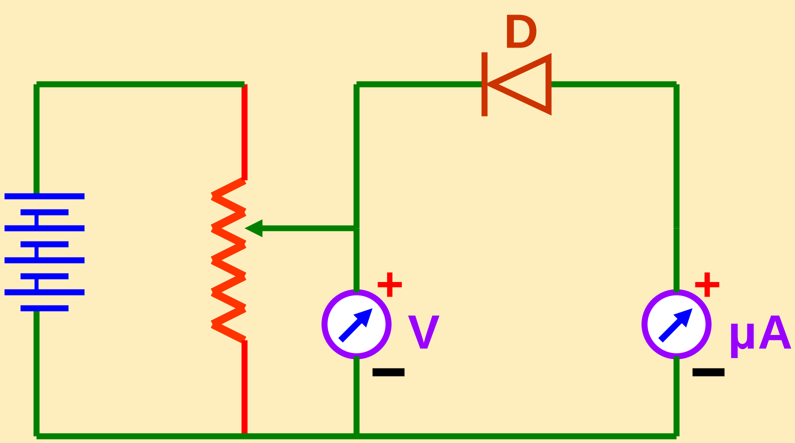
ix) The reverse – biased PN junction diode has an effective capacitance called 'transition or depletion capacitance'. P and N regions act as the plates of the capacitor and the depletion region acts as a dielectric medium.

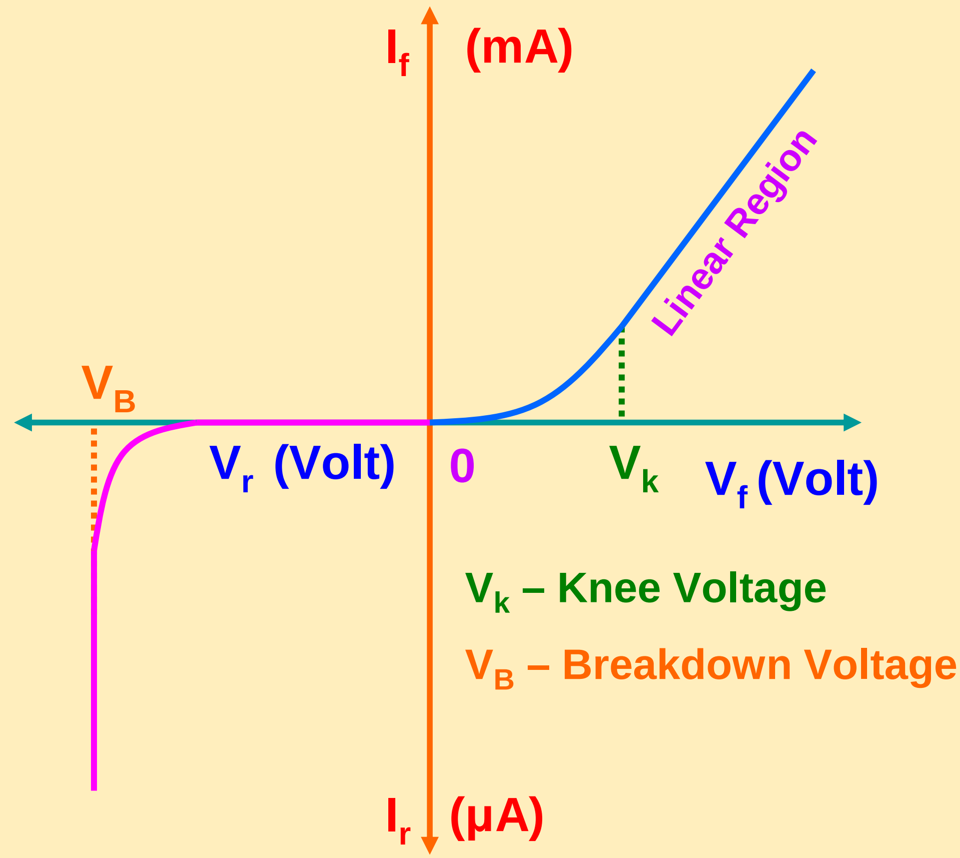
Diode Characteristics:

Forward Bias:



Reverse Bias:





Resistance of a Diode:

i) Static or DC Resistance $R_{d.c} = V / I$

ii) Dynamic or AC Resistance

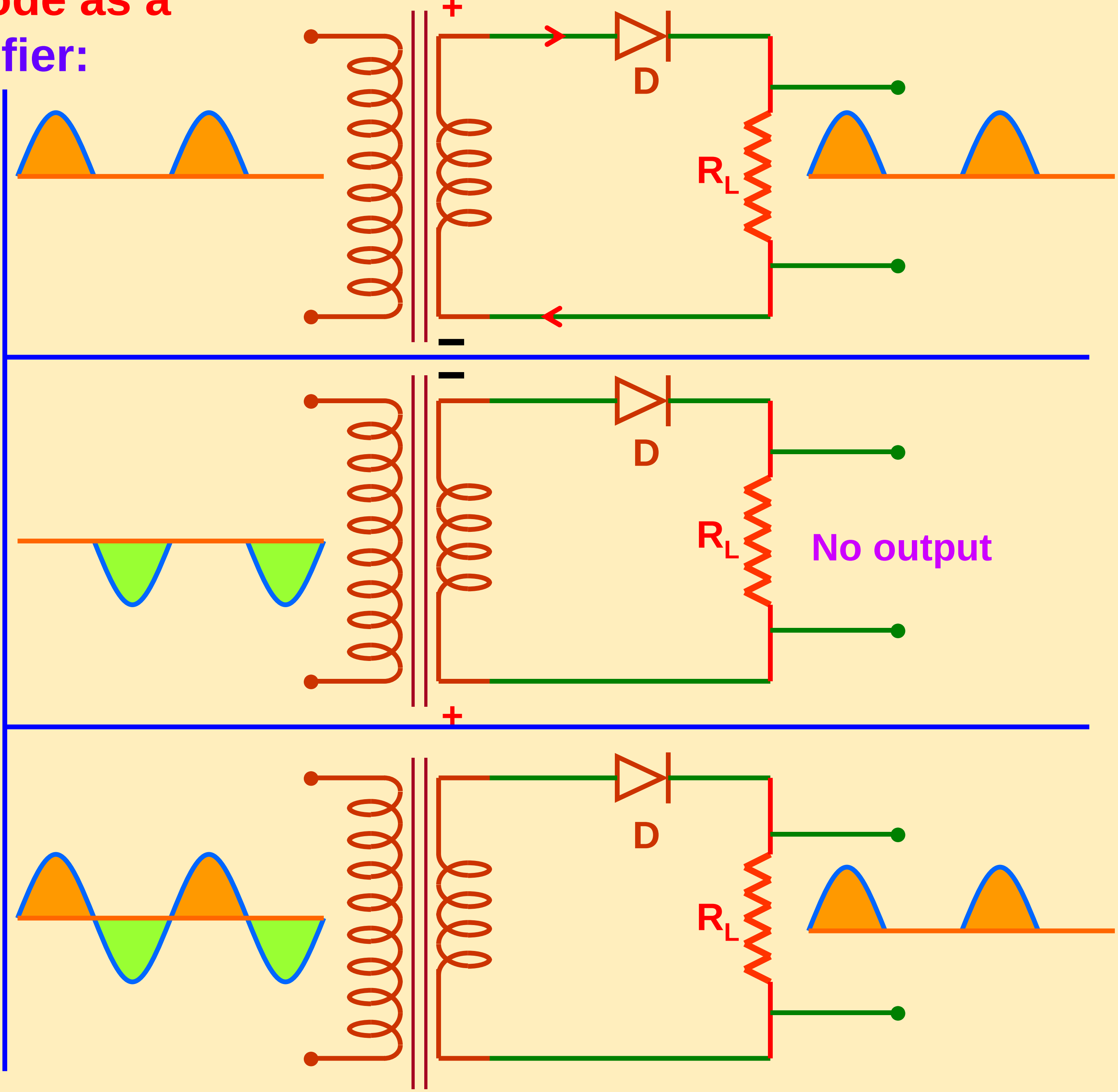
$$R_{a.c} = \Delta V / \Delta I$$

PN Junction Diode as a Half Wave Rectifier:

The process of converting alternating current into direct current is called 'rectification'.

The device used for rectification is called 'rectifier'.

The PN junction diode offers low resistance in forward bias and high resistance in reverse bias.

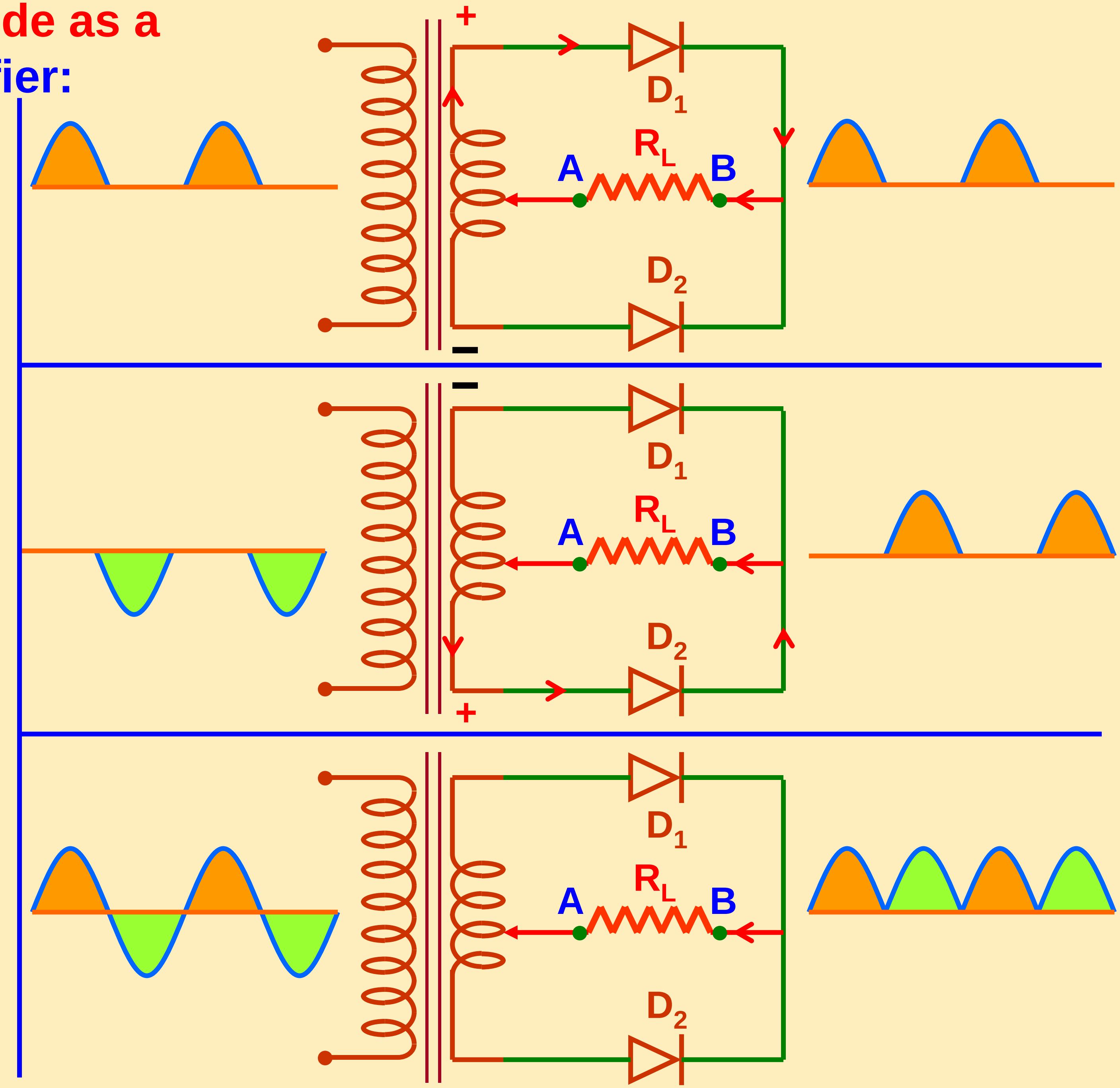


PN Junction Diode as a Full Wave Rectifier:

When the diode rectifies whole of the AC wave, it is called **'full wave rectifier'**.

During the positive half cycle of the input ac signal, the diode D_1 conducts and current is through **BA**.

During the negative half cycle, the diode D_2 conducts and current is through **BA**.



ELECTRONIC DEVICES - III

1. Junction Transistor
2. NPN and PNP Transistor Symbols
3. Action of NPN Transistor
4. Action of PNP Transistor
5. Transistor Characteristics in Common Base Configuration
6. Transistor Characteristics in Common Emitter Configuration
7. NPN Transistor Amplifier in Common Base Configuration
8. PNP Transistor Amplifier in Common Base Configuration
9. Various Gains in Common Base Amplifier
10. NPN Transistor Amplifier in Common Emitter Configuration
11. PNP Transistor Amplifier in Common Emitter Configuration
12. Various Gains in Common Emitter Amplifier
13. Transistor as an Oscillator

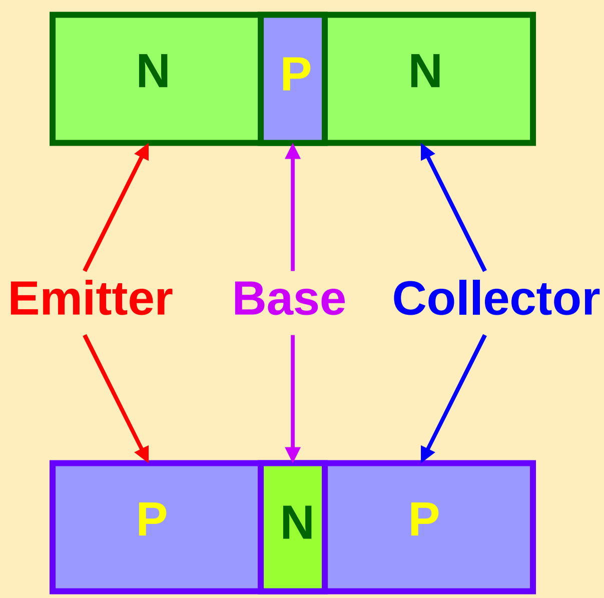
Junction Transistor:

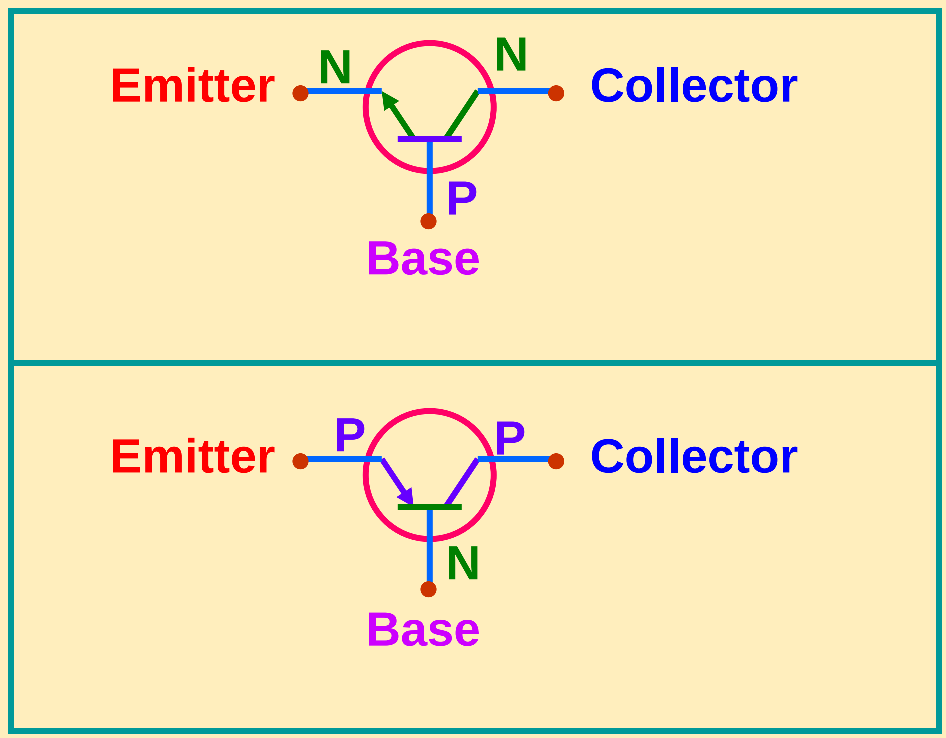
Transistor is a combination of two words 'transfer' and 'resistor' which means that transfer of resistance takes place from input to output section.

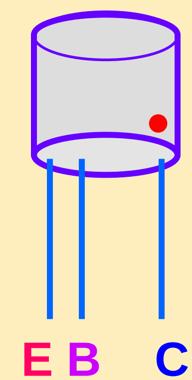
It is formed by sandwiching one type of extrinsic semiconductor between other type of extrinsic semiconductor.

NPN transistor contains P-type semiconductor sandwiched between two N-type semiconductors.

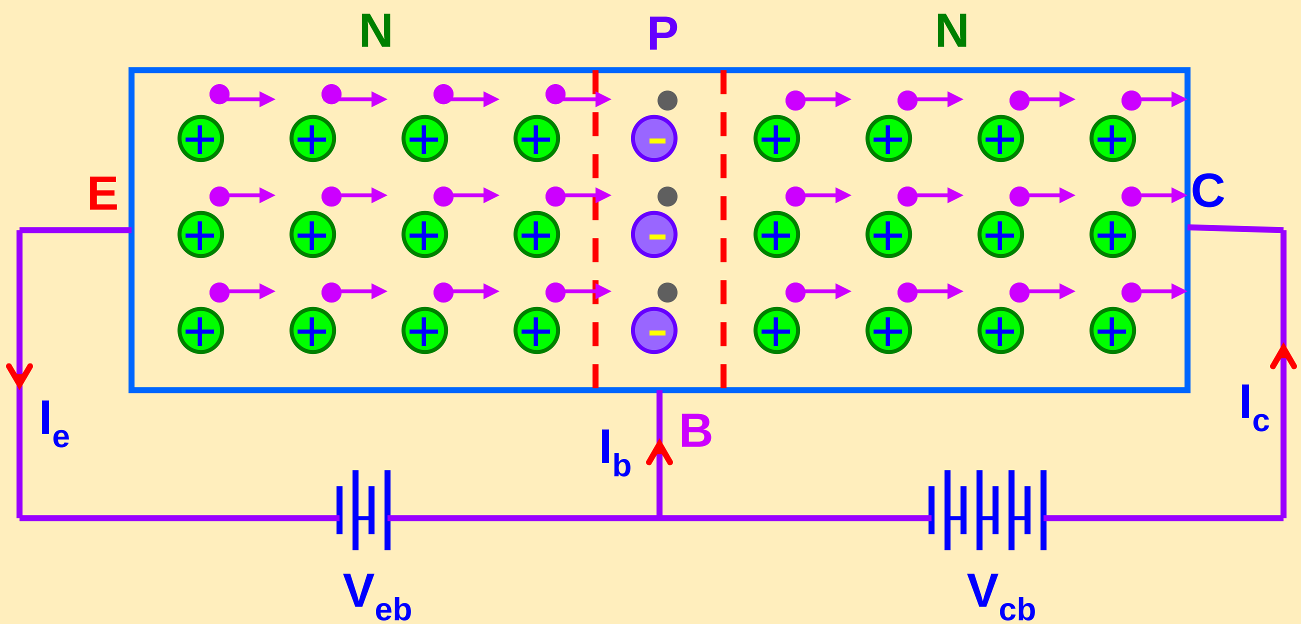
PNP transistor contains N-type semiconductor sandwiched between two P-type semiconductors.

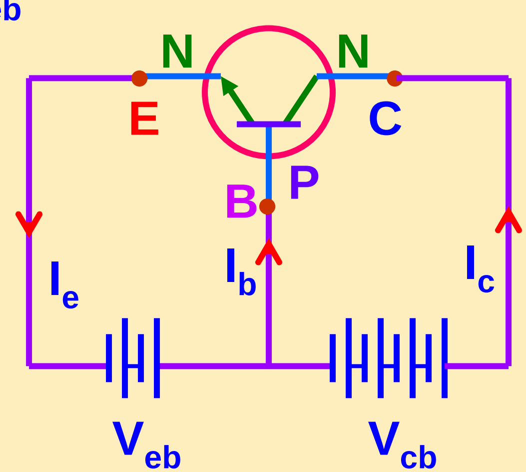






Action of NPN Transistor:





In NPN transistor, the arrow mark on the emitter is coming away from the base and represents the direction of flow of current. It is the direction opposite to the flow of electrons which are the main charge carriers in N-type crystal.

The emitter junction is forward-biased with emitter-base battery V_{eb}.
The collector junction is reverse biased with collector-base battery V_{cb}.

The forward bias of the emitter-base circuit helps the movement of electrons (majority carriers) in the emitter and holes (majority carriers) in the base towards the junction between the emitter and the base. This reduces the depletion region at this junction.

On the other hand, the reverse bias of the collector-base circuit forbids the movement of the majority carriers towards the collector-base junction and the depletion region increases.

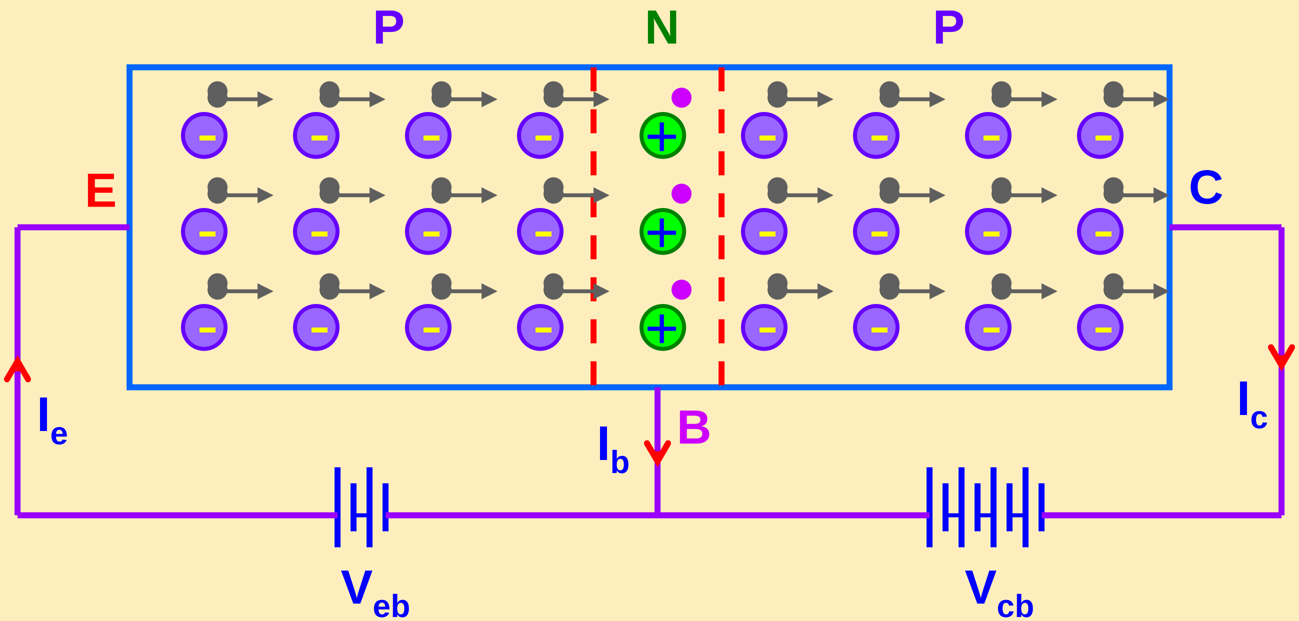
The electrons in the emitter are repelled by the –ve terminal of the emitter-base battery. Since the base is thin and lightly doped, therefore, only a very small fraction (say, 5%) of the incoming electrons combine with the holes. The remaining electrons rush through the collector and are swept away by the +ve terminal of the collector-base battery.

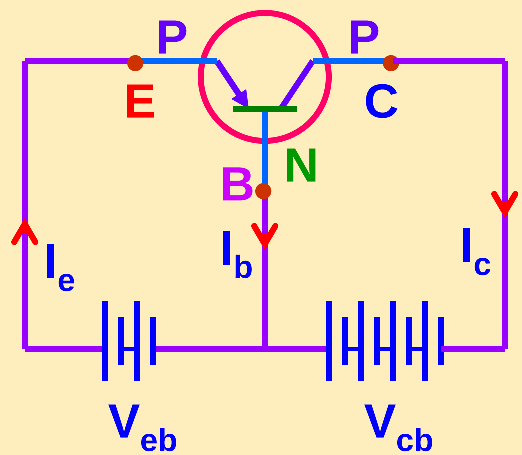
For every electron – hole recombination that takes place at the base region one electron is released into the emitter region by the –ve terminal of the emitter-base battery. The deficiency of the electrons caused due to their movement towards the collector is also compensated by the electrons released from the emitter-base battery.

The current is carried by the electrons both in the external as well as inside the transistor.

$$I_e = I_b + I_c$$

Action of PNP Transistor:





In PNP transistor, the arrow mark on the emitter is going into the base and represents the direction of flow of current. It is in the same direction as that of the movement of holes which are main charge carriers in P-type crystal.

The emitter junction is forward-biased with emitter-base battery V_{eb}.
The collector junction is reverse biased with collector-base battery V_{cb}.

The forward bias of the emitter-base circuit helps the movement of holes (majority carriers) in the emitter and electrons (majority carriers) in the base towards the junction between the emitter and the base. This reduces the depletion region at this junction.

On the other hand, the reverse bias of the collector-base circuit forbids the movement of the majority carriers towards the collector-base junction and the depletion region increases.

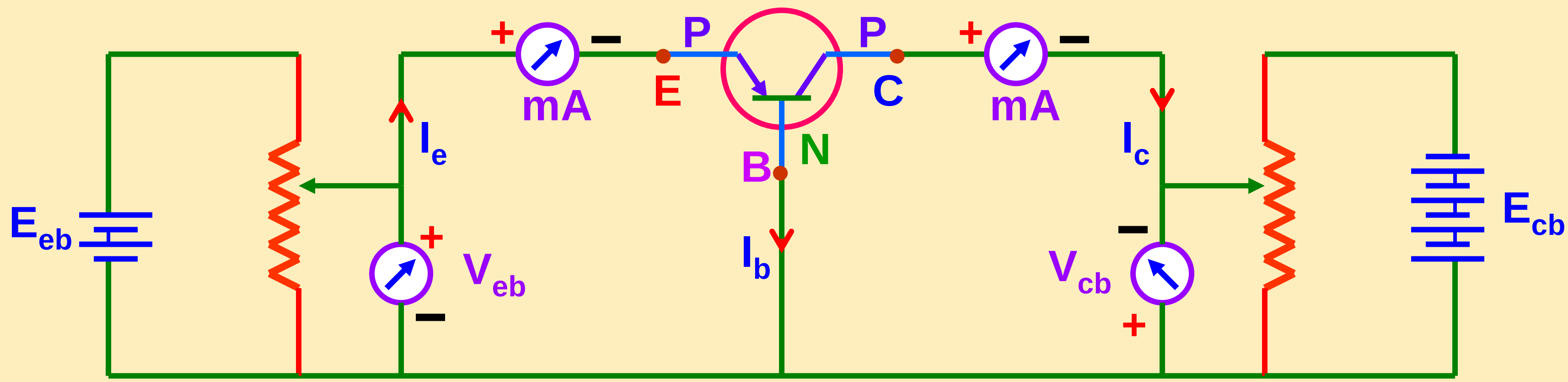
The holes in the emitter are repelled by the +ve terminal of the emitter-base battery. Since the base is thin and lightly doped, therefore, only a very small fraction (say, 5%) of the incoming holes combine with the electrons. The remaining holes rush through the collector and are swept away by the -ve terminal of the collector-base battery.

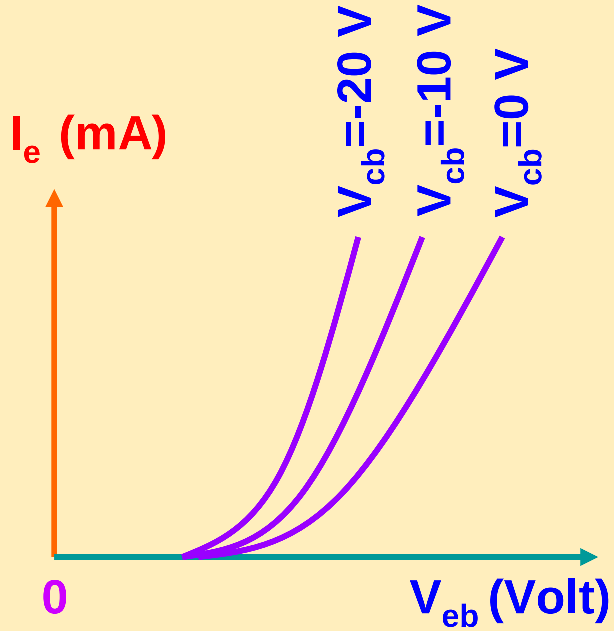
For every electron – hole recombination that takes place at the base region one electron is released into the emitter region by breaking the covalent bond and it enters the +ve terminal of the emitter-base battery. The holes reaching the collector are also compensated by the electrons released from the collector-base battery.

The current is carried by the electrons in the external circuit and by the holes inside the transistor.

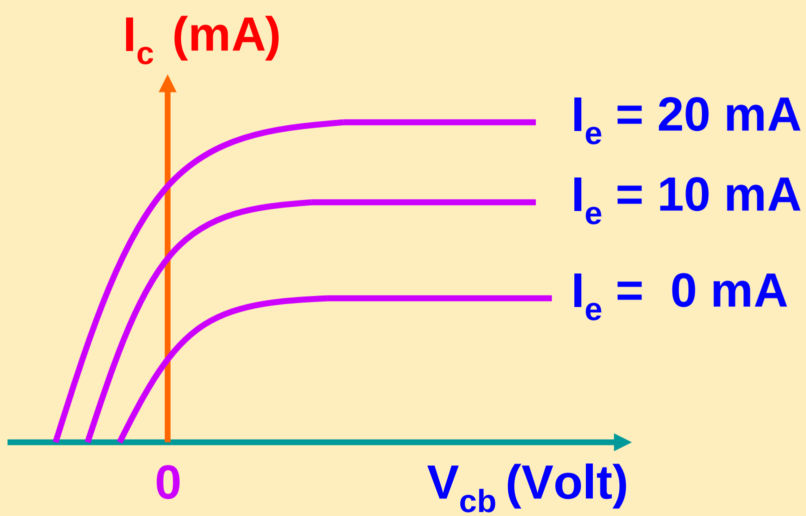
$$I_e = I_b + I_c$$

PNP Transistor Characteristics in Common Base Configuration:



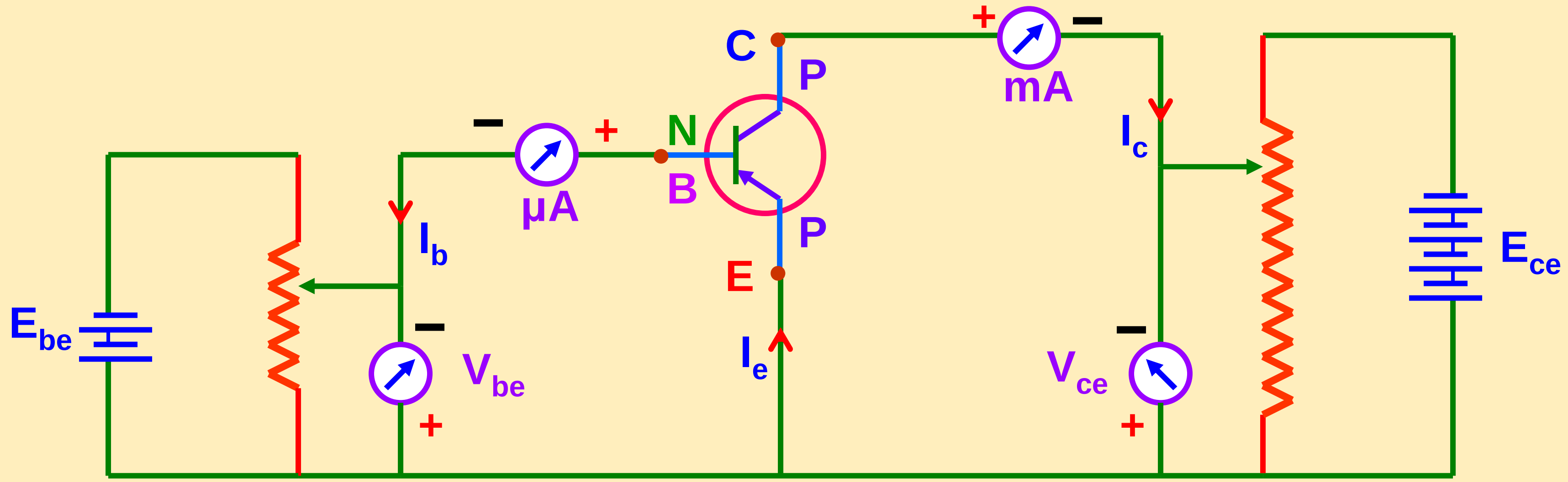


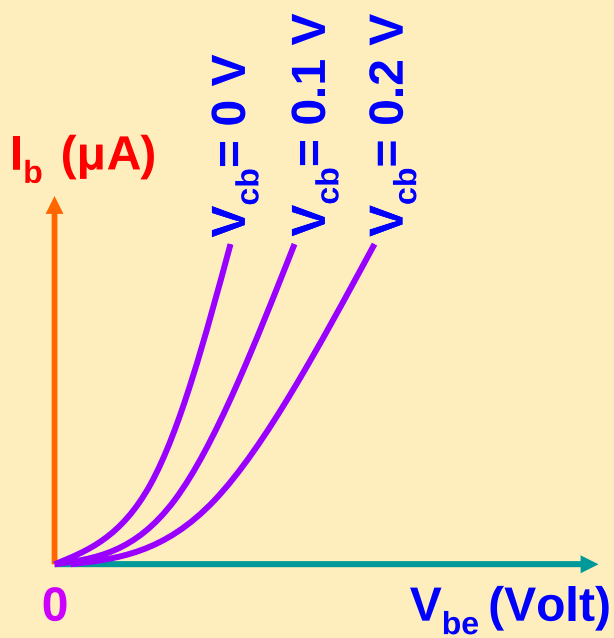
Input Characteristics



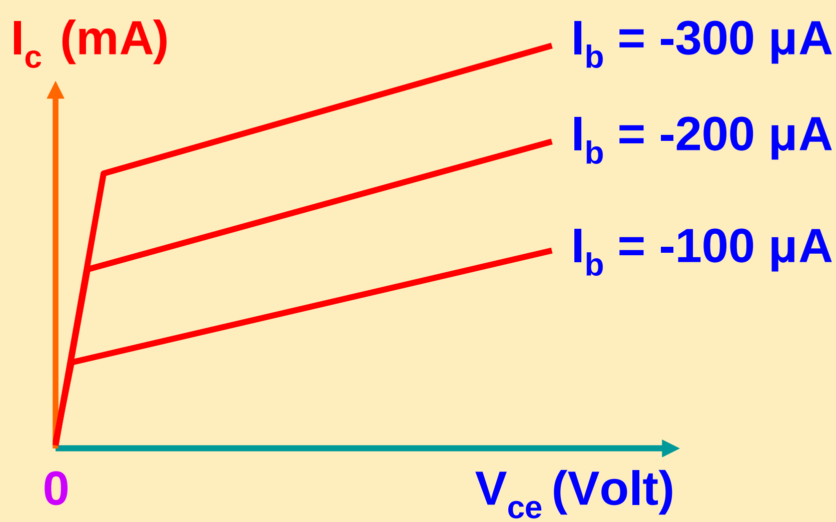
Output Characteristics

PNP Transistor Characteristics in Common Emitter Configuration:



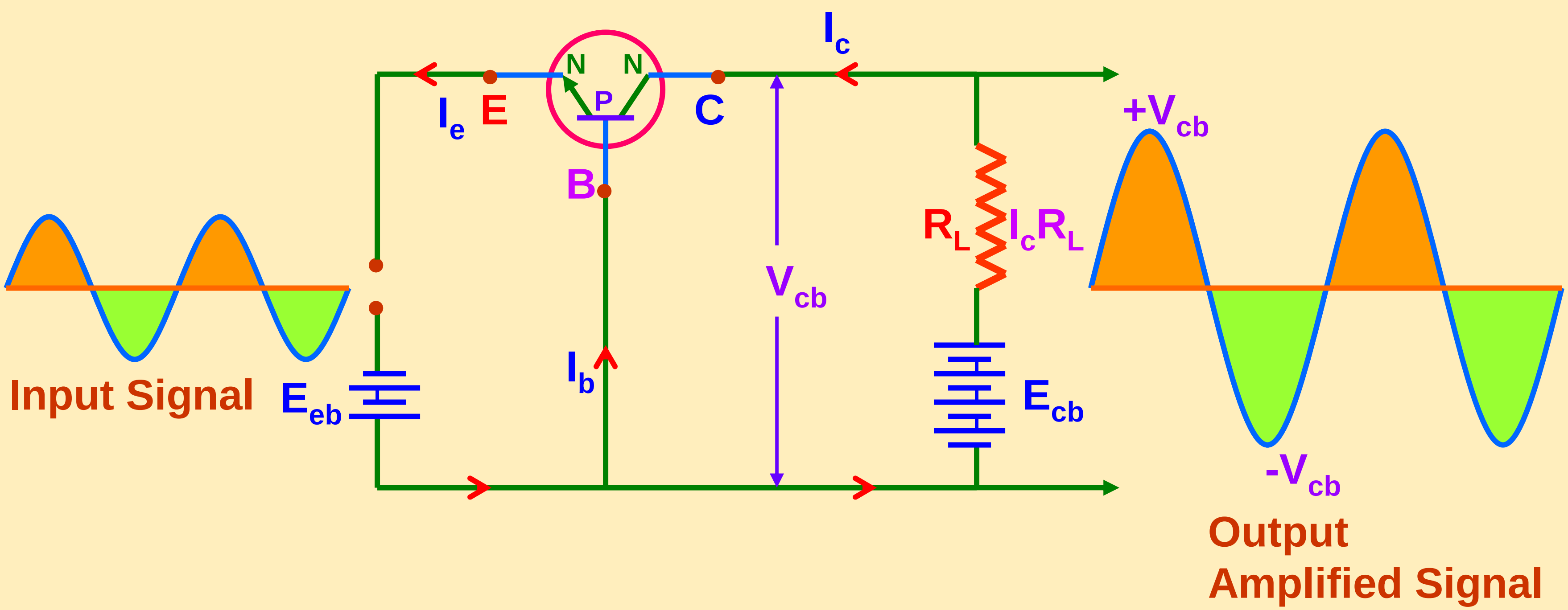


Input Characteristics



Output Characteristics

NPN Transistor as Common Base Amplifier:



Input section is forward biased and output section is reverse biased with biasing batteries E_{eb} and E_{cb}.

The currents I_e, I_b and I_c flow in the directions shown such that

$$I_e = I_b + I_c \quad \text{……….(1)}$$

I_cR_L is the potential drop across the load resistor R_L.

By Kirchhoff's rule,

$$V_{cb} = E_{cb} - I_c R_L \quad \text{……….(2)}$$

Phase Relation between the output and the input signal:

+ve Half cycle:

$$V_{cb} = E_{cb} - I_c R_L \quad \text{..........(2)}$$

During +ve half cycle of the input sinusoidal signal, forward-bias of N-type emitter decreases (since emitter is negatively biased).

This decreases the emitter current and hence the collector current.
Base current is very small (in the order of μA).

In consequence, the voltage drop across the load resistance R_L decreases.

From equation (2), it follows that V_{cb} increases above the normal value.

So, the output signal is +ve for +ve input signal.

-ve Half cycle:

During -ve half cycle of the input sinusoidal signal, forward-bias of N-type emitter increases (since emitter is negatively biased).

This increases the emitter current and hence the collector current.
Base current is very small (in the order of μA).

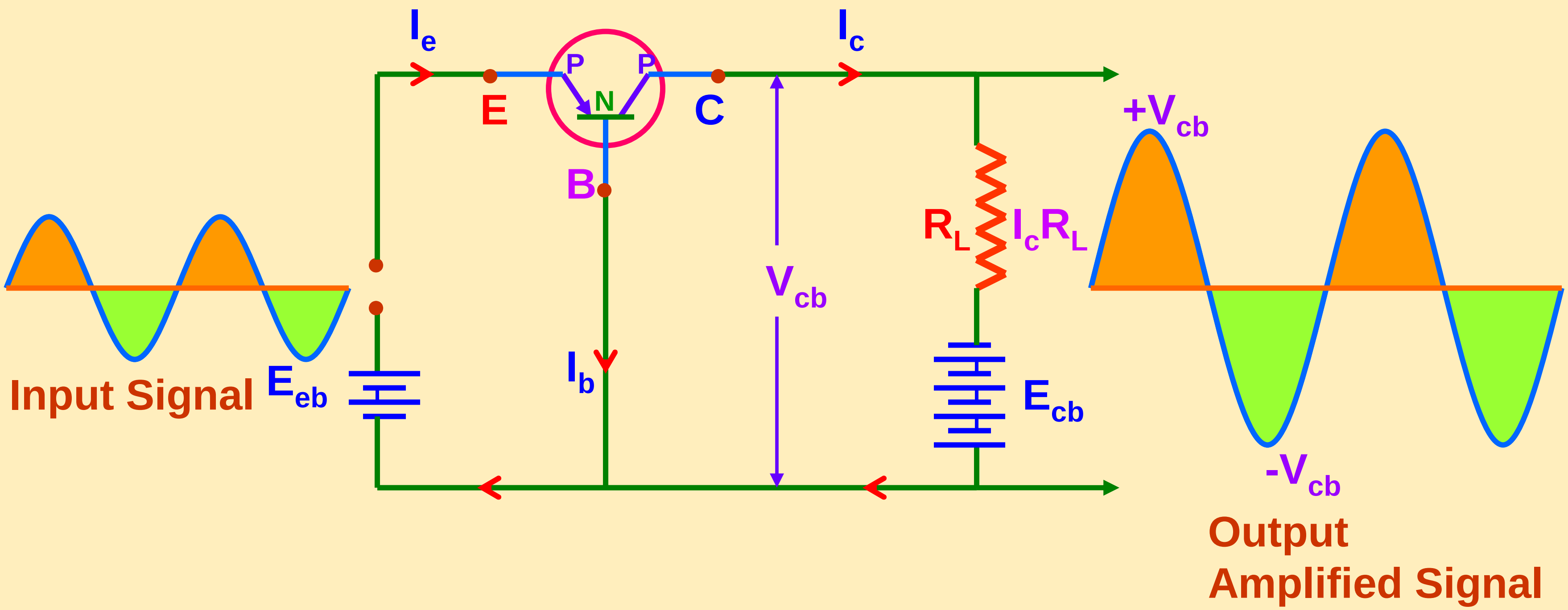
In consequence, the voltage drop across the load resistance R_L increases.

From equation (2), it follows that V_{cb} decreases below the normal value.

So, the output signal is -ve for -ve input signal.

Input and output are in same phase.

PNP Transistor as Common Base Amplifier:



Input section is forward biased and output section is reverse biased with biasing batteries E_{eb} and E_{cb}.

The currents I_e, I_b and I_c flow in the directions shown such that

$$I_e = I_b + I_c \quad \text{..........(1)}$$

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By Kirchhoff's rule,

$$V_{cb} = E_{cb} - I_c R_L \quad \text{..........(2)}$$

Phase Relation between the output and the input signal:

+ve Half cycle:

$V_{cb} = E_{cb} - I_c R_L$ ……….(2)

During +ve half cycle of the input sinusoidal signal, forward-bias of P-type emitter increases (since emitter is positively biased).

This increases the emitter current and hence the collector current.

Base current is very small (in the order of μA).

In consequence, the voltage drop across the load resistance R_L increases.

From equation (2), it follows that V_{cb} decreases. But, since the P-type collector is negatively biased, therefore, decrease means that the collector becomes less negative w.r.t. base and the output increases above the normal value (+ve output).

So, the output signal is +ve for +ve input signal.

-ve Half cycle:

During -ve half cycle of the input sinusoidal signal, forward-bias of P-type emitter decreases (since emitter is positively biased).

This decreases the emitter current and hence the collector current.

Base current is very small (in the order of μA).

In consequence, the voltage drop across the load resistance R_L decreases.

From equation (2), it follows that V_{cb} increases. But, since the P-type collector is negatively biased, therefore, increase means that the collector becomes more negative w.r.t. base and the output decreases below the normal value (-ve output).

So, the output signal is -ve for -ve input signal.

Input and output are in same phase.

Gains in Common Base Amplifier:

1) **Current Amplification Factor or Current Gain:**

(i) **DC current gain:** It is the ratio of the collector current (I_c) to the emitter current (I_e) at constant collector voltage.

$$\alpha_{dc} = \left(\frac{I_c}{I_e} \right)_{V_{cb}}$$

(ii) **AC current gain:** It is the ratio of change in collector current (ΔI_c) to the change in emitter current (ΔI_e) at constant collector voltage.

$$\alpha_{ac} = \left(\frac{\Delta I_c}{\Delta I_e} \right)_{V_{cb}}$$

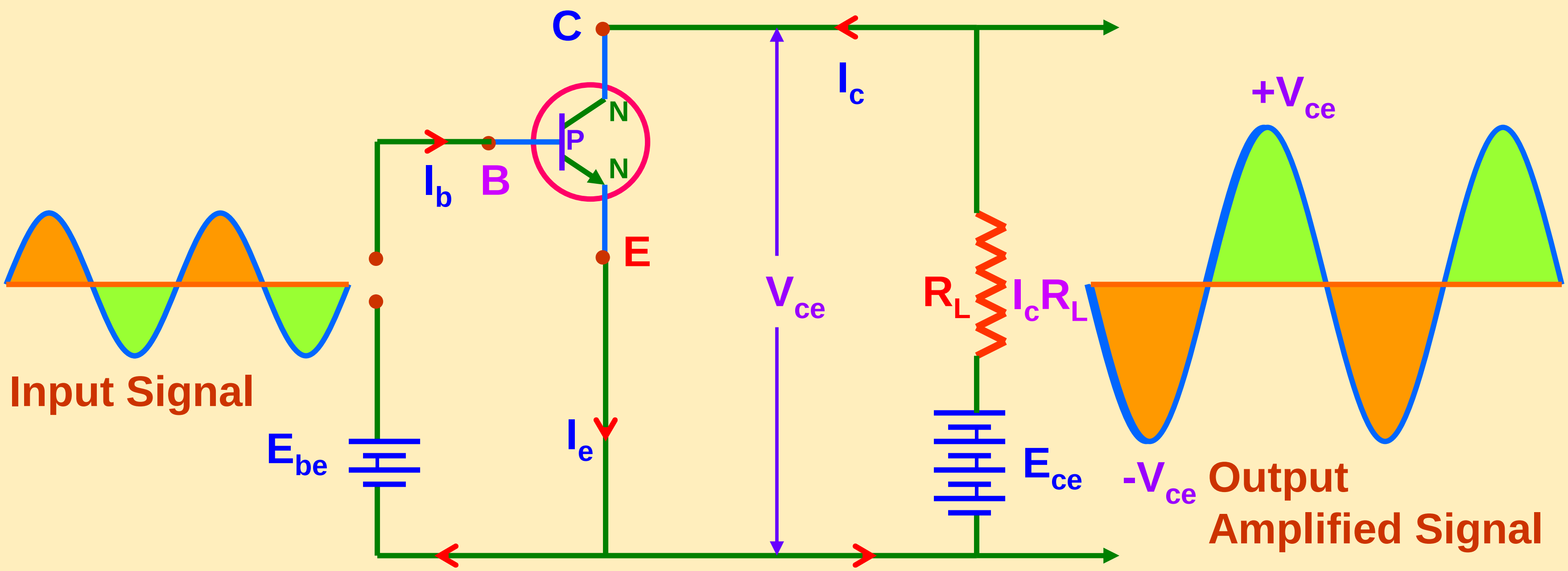
2) **AC voltage gain:** It is the ratio of change in output voltage (collector voltage ΔV_{cb}) to the change in input voltage (applied signal voltage ΔV_i).

$$A_{V\text{-}ac} = \left(\frac{\Delta V_{cb}}{\Delta V_i} \right) \quad \text{or} \quad A_{V\text{-}ac} = \left(\frac{\Delta I_c \times R_o}{\Delta I_e \times R_i} \right) \quad \text{or} \quad A_{V\text{-}ac} = \alpha_{ac} \times \text{ Resistance Gain}$$

3) **AC power gain:** It is the ratio of change in output power to the change in input power.

$$A_{P\text{-}ac} = \left(\frac{\Delta P_o}{\Delta P_i} \right) \quad \text{or} \quad A_{P\text{-}ac} = \left(\frac{\Delta V_{cb} \times \Delta I_c}{\Delta V_i \times \Delta I_e} \right) \quad \text{or} \quad A_{P\text{-}ac} = \alpha_{ac}^2 \times \text{ Resistance Gain}$$

NPN Transistor as Common Emitter Amplifier:



Input section is forward biased and output section is reverse biased with biasing batteries E_{be} and E_{ce}.

The currents I_e, I_b and I_c flow in the directions shown such that

$$I_e = I_b + I_c \quad \text{……….(1)}$$

I_cR_L is the potential drop across the load resistor R_L.

By Kirchhoff's rule,

$$V_{ce} = E_{ce} - I_c R_L \quad \text{……….(2)}$$

Phase Relation between the output and the input signal:

+ve Half cycle:

$$V_{ce} = E_{ce} - I_c R_L \quad \dots\dots\dots(2)$$

During +ve half cycle of the input sinusoidal signal, forward-bias of base and emitter increases (since P-type base becomes more positive and N-type emitter becomes more -ve).

This increases the emitter current and hence the collector current.
Base current is very small (in the order of μA).

In consequence, the voltage drop across the load resistance R_L increases.

From equation (2), it follows that V_{ce} decreases below the normal value.

So, the output signal is -ve for +ve input signal.

-ve Half cycle:

During -ve half cycle of the input sinusoidal signal, forward-bias of P-type base and N-type emitter decreases.

This decreases the emitter current and hence the collector current.
Base current is very small (in the order of μA).

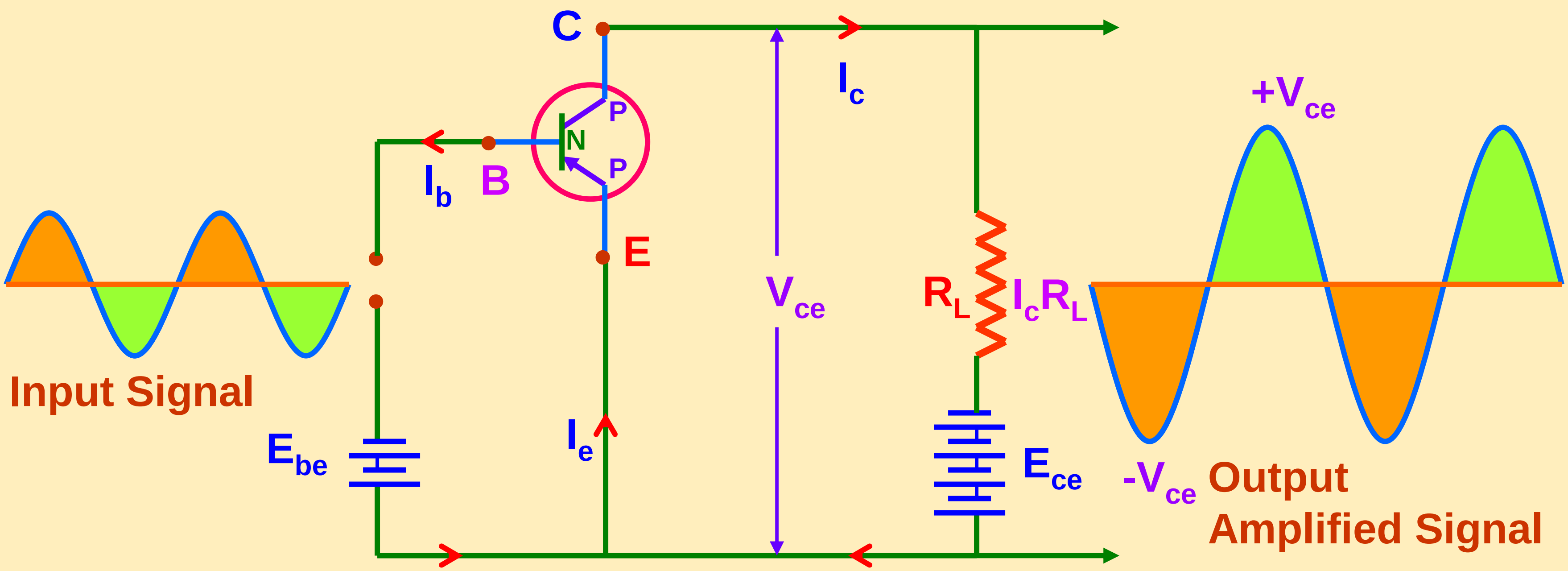
In consequence, the voltage drop across the load resistance R_L decreases.

From equation (2), it follows that V_{ce} increases above the normal value.

So, the output signal is +ve for -ve input signal.

Input and output are out of phase by 180°.

PNP Transistor as Common Emitter Amplifier:



Input section is forward biased and output section is reverse biased with biasing batteries E_{be} and E_{ce}.

The currents I_e, I_b and I_c flow in the directions shown such that

$$I_e = I_b + I_c \quad \text{……….(1)}$$

I_cR_L is the potential drop across the load resistor R_L.

By Kirchhoff's rule,

$$V_{ce} = E_{ce} - I_c R_L \quad \text{……….(2)}$$

Phase Relation between the output and the input signal:

+ve Half cycle:

$$V_{ce} = E_{ce} - I_c R_L \quad ……….(2)$$

During +ve half cycle of the input sinusoidal signal, forward-bias of base and emitter decreases (since N-type base becomes less negative and P-type emitter becomes less +ve).

This decreases the emitter current and hence the collector current.

Base current is very small (in the order of μA).

In consequence, the voltage drop across the load resistance R_L decreases.

From equation (2), it follows that V_{ce} increases. But, since P-type collector is negatively biased, therefore, increase means that the collector becomes more negative w.r.t. base and the output goes below the normal value.

So, the output signal is -ve for +ve input signal.

-ve Half cycle:

During -ve half cycle of the input sinusoidal signal, forward-bias of base and emitter increases.

This increases the emitter current and hence the collector current.

Base current is very small (in the order of μA).

In consequence, the voltage drop across the load resistance R_L increases.

From equation (2), it follows that V_{ce} decreases. But, since P-type collector is negatively biased, therefore, decrease means that the collector becomes less negative w.r.t. base and the output goes above the normal value.

So, the output signal is +ve for -ve input signal.

Input and output are out of phase by 180°.

Gains in Common Emitter Amplifier:

1) Current Amplification Factor or Current Gain:

(i) DC current gain: It is the ratio of the collector current (I_c) to the base current (I_b) at constant collector voltage.

$$\beta_{dc} = \left(\frac{I_c}{I_b}\right)_{V_{ce}}$$

(ii) AC current gain: It is the ratio of change in collector current (ΔI_c) to the change in base current (ΔI_b) at constant collector voltage.

$$\beta_{ac} = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{ce}}$$

2) AC voltage gain: It is the ratio of change in output voltage (collector voltage ΔV_{ce}) to the change in input voltage (applied signal voltage ΔV_i).

$$A_{V\text{-}ac} = \left(\frac{\Delta V_{ce}}{\Delta V_i}\right) \quad \text{or} \quad A_{V\text{-}ac} = \left(\frac{\Delta I_c \times R_o}{\Delta I_b \times R_i}\right) \quad \text{or} \quad A_{V\text{-}ac} = \beta_{ac} \times \text{Resistance Gain}$$

Also $A_V = g_m R_L$

3) AC power gain: It is the ratio of change in output power to the change in input power.

$$A_{P\text{-}ac} = \left(\frac{\Delta P_o}{\Delta P_i}\right) \quad \text{or} \quad A_{P\text{-}ac} = \left(\frac{\Delta V_{ce} \times \Delta I_c}{\Delta V_i \times \Delta I_b}\right) \quad \text{or} \quad A_{P\text{-}ac} = \beta_{ac}^2 \times \text{Resistance Gain}$$

4) **Transconductance:** It is the ratio of the small change in collector current (ΔI_c) to the corresponding change in the input voltage (base voltage (ΔV_b) at constant collector voltage.

$$g_m = \left[\frac{\Delta I_c}{\Delta V_b}\right]_{V_{ce}} \quad \text{or} \quad g_m = \frac{\beta_{ac}}{R_i}$$

Relation between α and β:

$$I_e = I_b + I_c$$

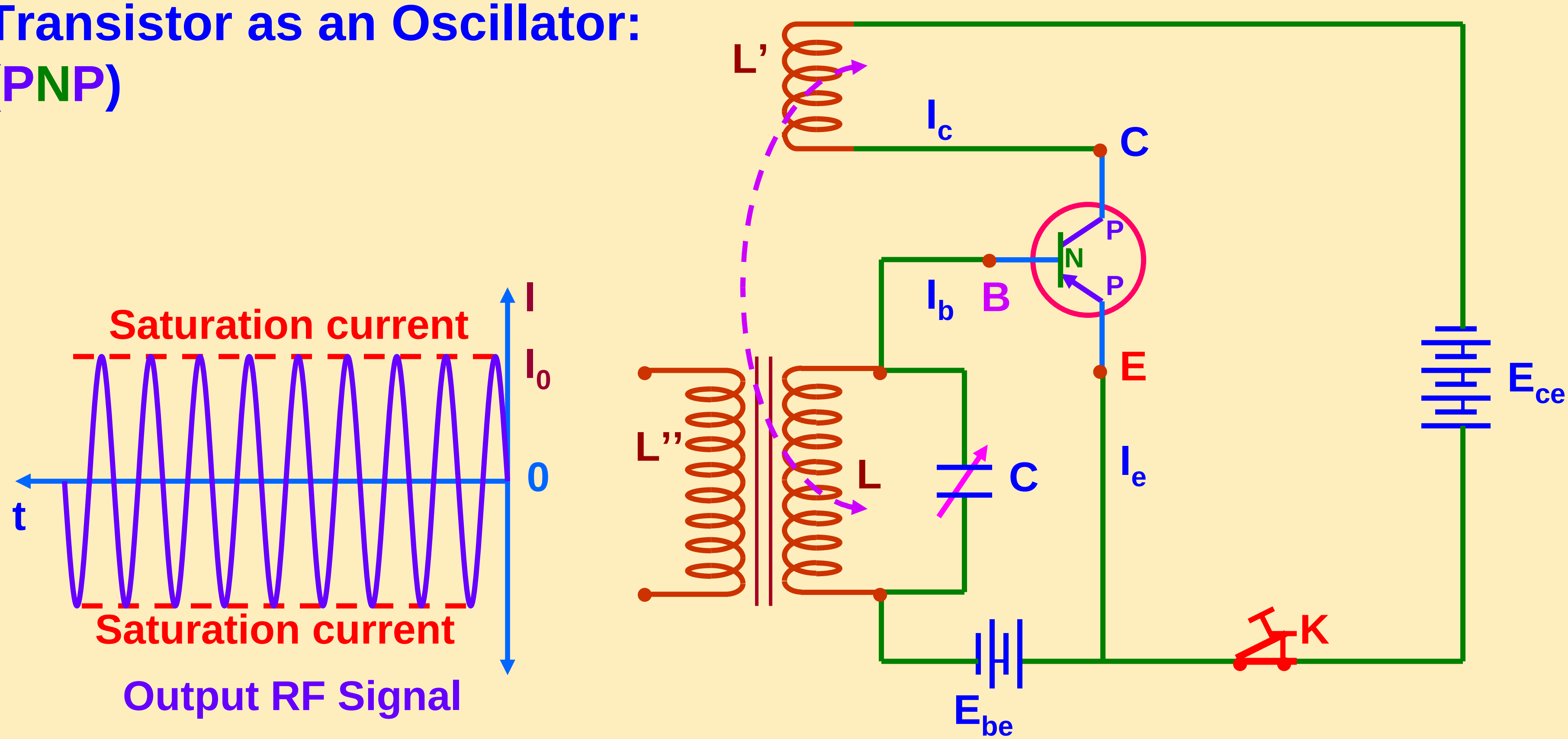
Dividing the equation by I_c, we get

$$\frac{I_e}{I_c} = \frac{I_b}{I_c} + 1$$

But $$\alpha = \left[\frac{I_c}{I_e}\right] \quad \text{and} \quad \beta = \left[\frac{I_c}{I_b}\right]$$

$$\therefore \quad \frac{1}{\alpha} = \frac{1}{\beta} + 1 \quad \text{or} \quad \boxed{\beta = \frac{\alpha}{1 - \alpha}} \quad \text{and} \quad \boxed{\alpha = \frac{\beta}{1 + \beta}}$$

Transistor as an Oscillator:
(PNP)



An oscillator is a device which can produce undamped electromagnetic oscillations of desired frequency and amplitude.

It is a device which delivers a.c. output waveform of desired frequency from d.c. power even without input signal excitation.

Tank circuit containing an inductance L and a capacitance C connected in parallel can oscillate the energy given to it between electrostatic and magnetic energies. However, the oscillations die away since the amplitude decreases rapidly due to inherent electrical resistance in the circuit.

In order to obtain undamped oscillations of constant amplitude, transistor can be used to give regenerative or positive feedback from the output circuit to the input circuit so that the circuit losses can be compensated.

When key K is closed, collector current begins to grow through the tickler coil L' . Magnetic flux linked with L' as well as L increases as they are inductively coupled. Due to change in magnetic flux, induced emf is set up in such a direction that the emitter – base junction is forward biased. This increases the emitter current and hence the collector current.

With the increase in collector current , the magnetic flux across L' and L increases. The process continues till the collector current reaches the saturation value. During this process the upper plate of the capacitor C gets positively charged.

At this stage, induced emf in L becomes zero. The capacitor C starts discharging through the inductor L.

The emitter current starts decreasing resulting in the decrease in collector current. Again the magnetic flux changes in L' and L but it induces emf in such a direction that it decreases the forward bias of emitter – base junction.

As a result, emitter current further decreases and hence collector current also decreases. This continues till the collector current becomes zero. At this stage, the magnetic flux linked with the coils become zero and hence no induced emf across L.

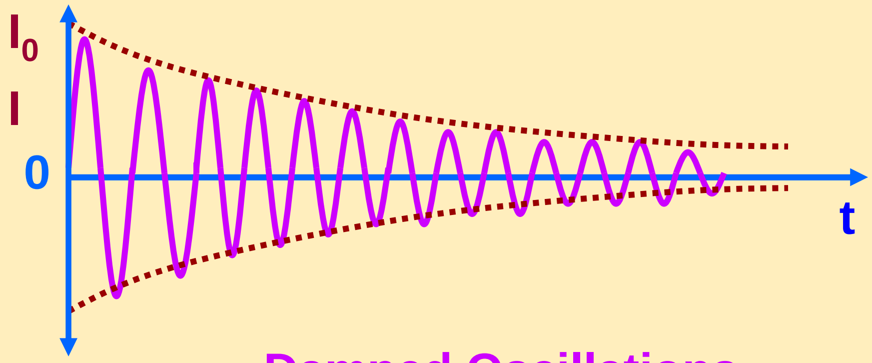
However, the decreasing current after reaching zero value overshoots (goes below zero) and hence the current starts increasing but in the opposite direction. During this period, the lower plate of the capacitor C gets +vely charged.

This process continues till the current reaches the saturation value in the negative direction. At this stage, the capacitor starts discharging but in the opposite direction (giving positive feedback) and the current reaches zero value from –ve value.

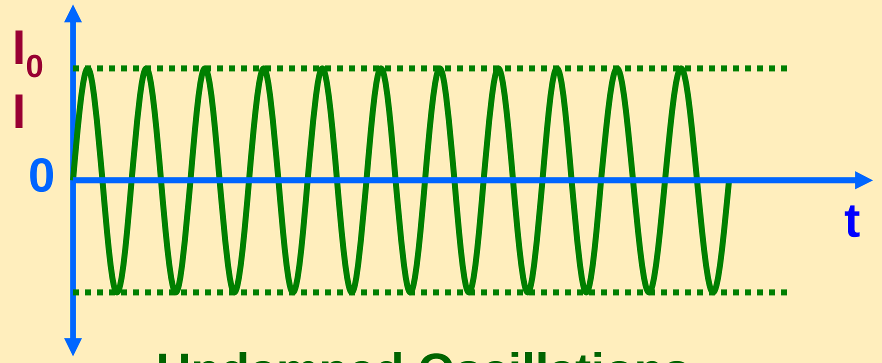
The cycle again repeats and hence the oscillations are produced. The output is obtained across L''.

The frequency of oscillations is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$



Damped Oscillations

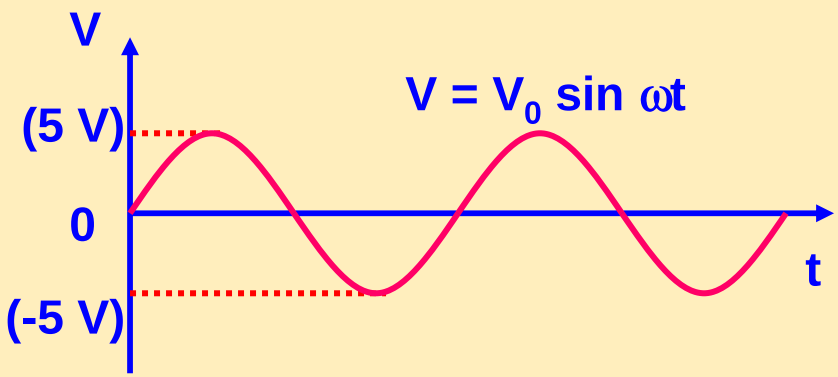


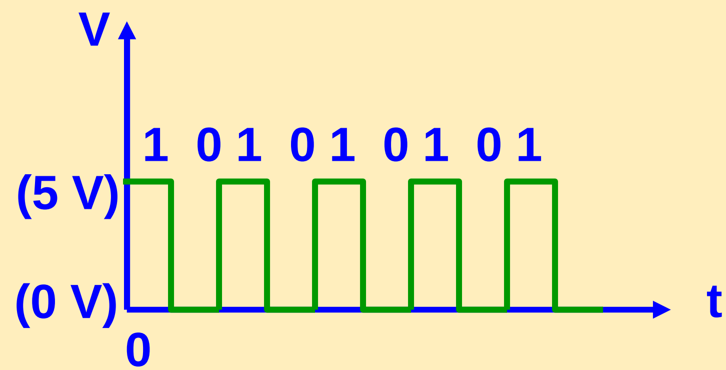
Undamped Oscillations

ELECTRONIC DEVICES - IV

1. Analog and Digital Signal
2. Binary Number System
3. Binary Equivalence of Decimal Numbers
4. Boolean Algebra
5. Logic Operations: OR, AND and NOT
6. Electrical Circuits for OR, AND and NOT Operations
7. Logic Gates and Truth Table
8. Fundamental Logic Gates: OR, AND and NOT (Digital Circuits)
9. NOR and NAND Gates
10. NOR Gate as a Building Block
11. NAND Gate as a Building Block
12. XOR Gate

Analogue signal	Digital signal
A continuous signal value which at any instant lies within the range of a maximum and a minimum value.	A discontinuous signal value which appears in steps in pre-determined levels rather than having the continuous change.





Digital Circuit:

An electrical or electronic circuit which operates only in two states (binary mode) namely ON and OFF is called a Digital Circuit.

In digital system, high value of voltage such as +10 V or +5 V is represented by ON state or 1 (state) whereas low value of voltage such as 0 V or -5V or -10 V is represented by OFF state or 0 (state).

Binary Number System:

A number system which has only two digits i.e. 0 and 1 is known as binary number system or binary system.

The states ON and OFF are represented by the digits 1 and 0 respectively in the binary number system.

Binary Equivalence of Decimal Numbers:

Decimal number system has base (or radix) 10 because of 10 digits viz. 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 used in the system.

Binary number system has base (or radix) 2 because of 2 digits viz. 0 and 2 used in the system.

D	0	1	2	3	4	5	6	7	8	9
B	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

D	10	11	12	13	14	15
B	1010	1011	1100	1101	1110	1111

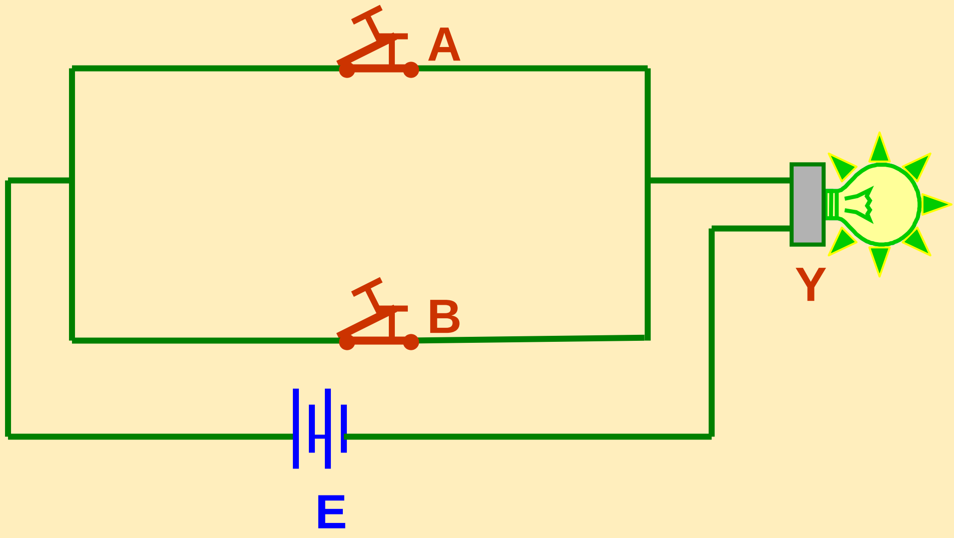
Boolean Algebra:

George Boole developed an algebra called Boolean Algebra to solve logical problems. In this, 3 logical operations viz. OR, AND and NOT are performed on the variables.

The two values or states represent either 'TRUE' or 'FALSE'; 'ON' or 'OFF'; 'HIGH' or 'LOW'; 'CLOSED' or 'OPEN'; 1 or 0 respectively.

OR Operation:

OR operation is represented by '+'.
Its boolean expression is Y = A + B
It is read as "Y equals A OR B".
It means that "if A is true OR B is true, then Y will be true".



Truth Table

Switch A	Switch B	Bulb Y
OFF	OFF	OFF
OFF	ON	ON
ON	OFF	ON
ON	ON	ON

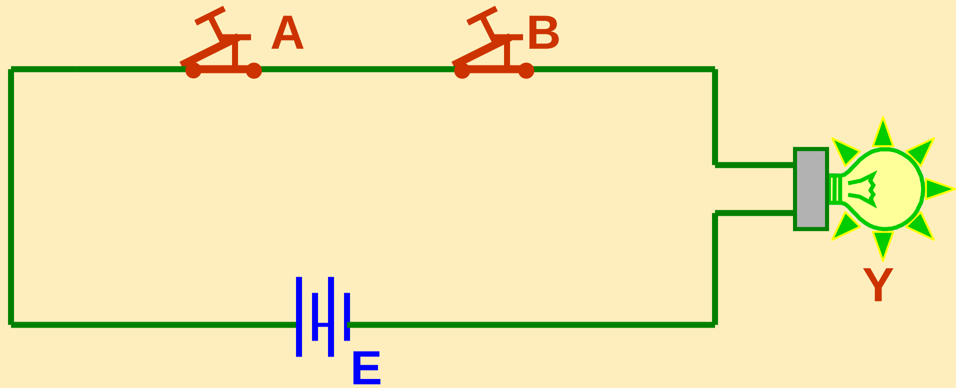
AND Operation:

AND operation is represented by '.'

Its boolean expression is Y = A . B

It is read as "Y equals A AND B".

It means that "if both A and B are true, then Y will be true".



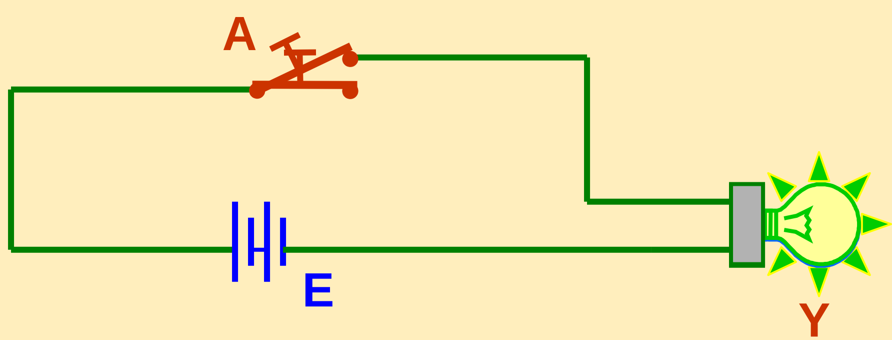
Truth Table

Switch A	Switch B	Bulb Y
OFF	OFF	OFF
OFF	ON	OFF
ON	OFF	OFF
ON	ON	ON

NOT Operation:

NOT operation is represented by ′ or ¯ .Its boolean expression is Y = A′ or $\bar{A}$

It is read as "Y equals NOT A". It means that "if A is true, then Y will be false".



Truth Table

Switch A	Bulb Y
OFF	ON
ON	OFF

Logic Gates:

The digital circuit that can be analysed with the help of Boolean Algebra is called logic gate or logic circuit.

A logic gate can have two or more inputs but only one output.

There are 3 fundamental logic gates namely OR gate, AND gate and NOT gate.

Truth Table:

The operation of a logic gate or circuit can be represented in a table which contains all possible inputs and their corresponding outputs is called a truth table.

If there are n inputs in any logic gate, then there will be n^2 possible input combinations.

0 and 1 inputs are taken in the order of ascending binary numbers for easy understanding and analysis.

Eg. for 4 input gate

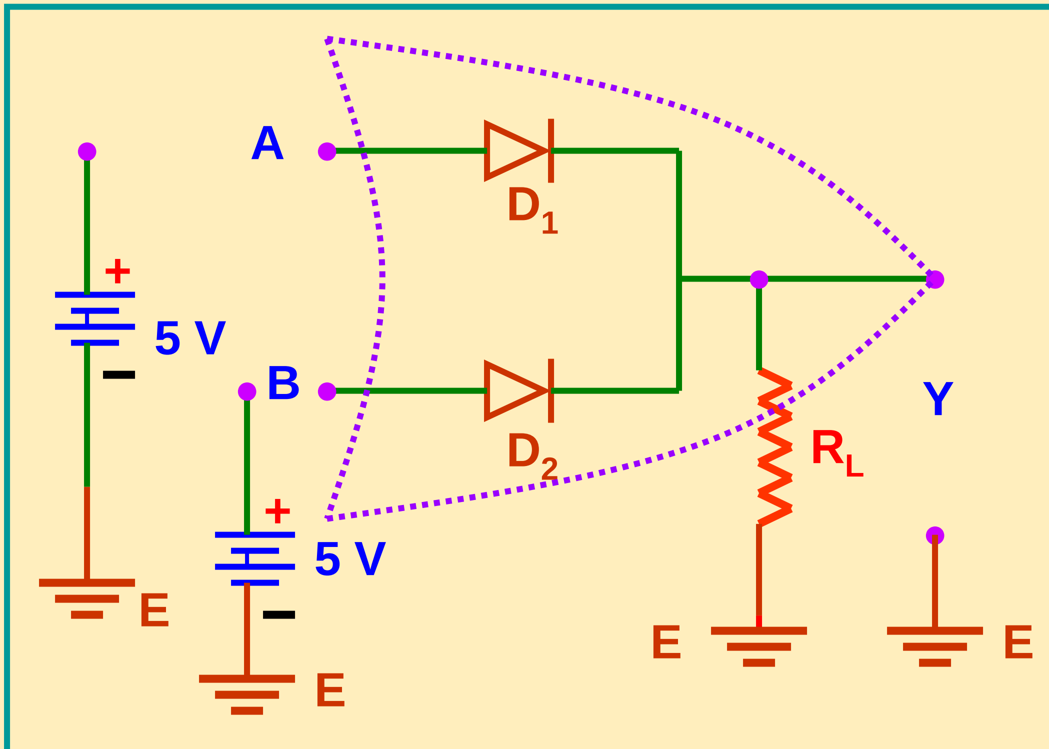
A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Digital OR Gate:

The positive voltage (+5 V) corresponds to high input i.e. 1 (state).

The negative terminal of the battery is grounded and corresponds to low input i.e. 0 (state).

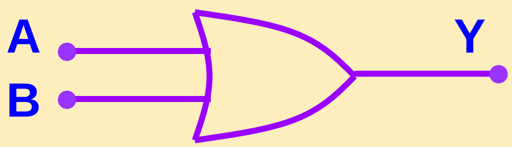
Case 1: Both A and B are given 0 input and the diodes do not conduct current. Hence no output is across R_L. i.e. Y = 0



Case 2: A is given 0 and B is given 1. Diode D_1 does not conduct current (cut-off) but D_2 conducts. Hence output (5 V) is available across R_L. i.e. Y = 1

Case 3: A is given 1 and B is given 0. Diode D_1 conducts current but D_2 does not conduct. Hence output (5 V) is available across R_L. i.e. Y = 1

Case 4: A and B are given 1. Both the diodes conduct current. However output (only 5 V) is available across R_L. i.e. Y = 1



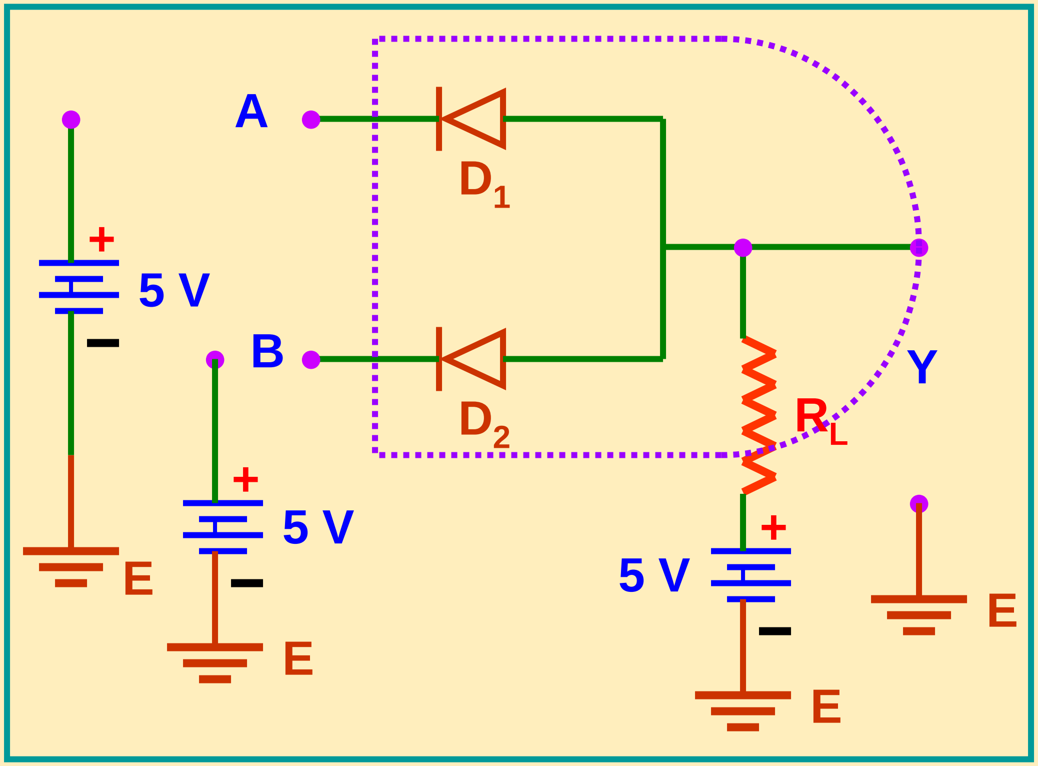
Truth Table

A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1

Digital AND Gate:

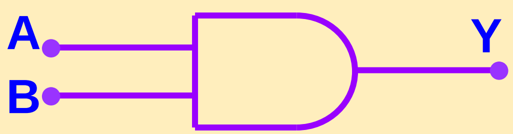
Case 1: Both A and B are given 0 input and the diodes conduct current (Forward biased). Since the current is drained to the earth, hence, no output across R_L.
i.e. $Y = 0$

Case 2: A is given 0 and B is given 1. Diode D_1 being forward biased conducts current but D_2 does not conduct. However, the current from the output battery is drained through D_1. So, $Y = 0$



Case 3: A is given 1 and B is given 0. Diode D_1 does not conduct current but D_2 being forward biased conducts . However, the current from the output battery is drained through D_2. Hence, no output is available across R_L. i.e. $Y = 0$

Case 4: A and B are given 1. Both the diodes do not conduct current. The current from the output battery is available across R_L and output circuit. Hence, there is voltage drop (5 V) across R_L. i.e. $Y = 1$



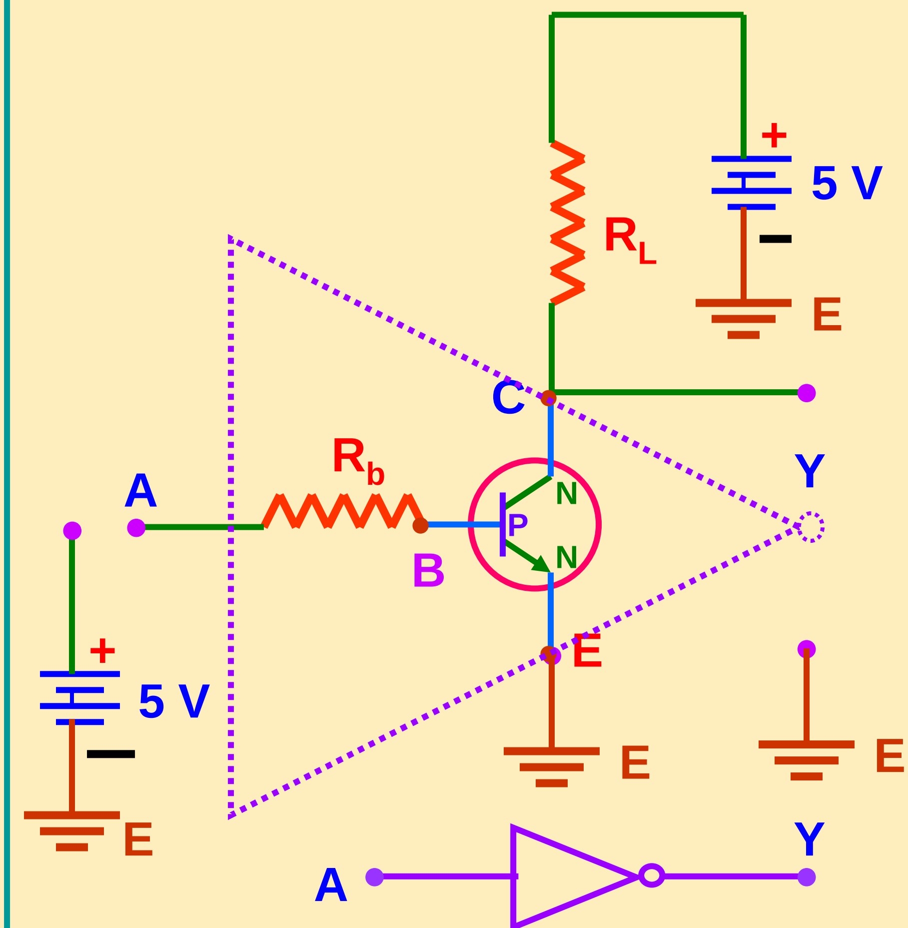
Truth Table

A	B	Y = A . B
0	0	0
0	1	0
1	0	0
1	1	1

Digital NOT Gate:

NPN transistor is connected to biasing batteries through Base resistor (R_b) and Collector resistor (R_L). Emitter is directly earthed. Input is given through the base and the output is tapped across the collector.

Case 1: A is given 0 input. In the absence of forward bias to the P-type base and N-type emitter, the transistor is in cut-off mode (does not conduct current). Hence, the current from the collector battery is available across the output unit. Therefore, voltage drop of 5 V is available across Y. i.e. Y= 1



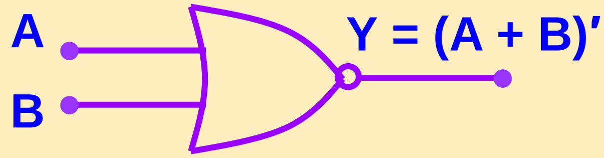
Case 2: A is given 1 input by connecting the +ve terminal of the input battery. P-type base being forward biased makes the transistor in conduction mode. The current supplied by the collector battery is drained through the transistor to the earth. Therefore, no output is available across Y. i.e. Y = 0

Truth Table

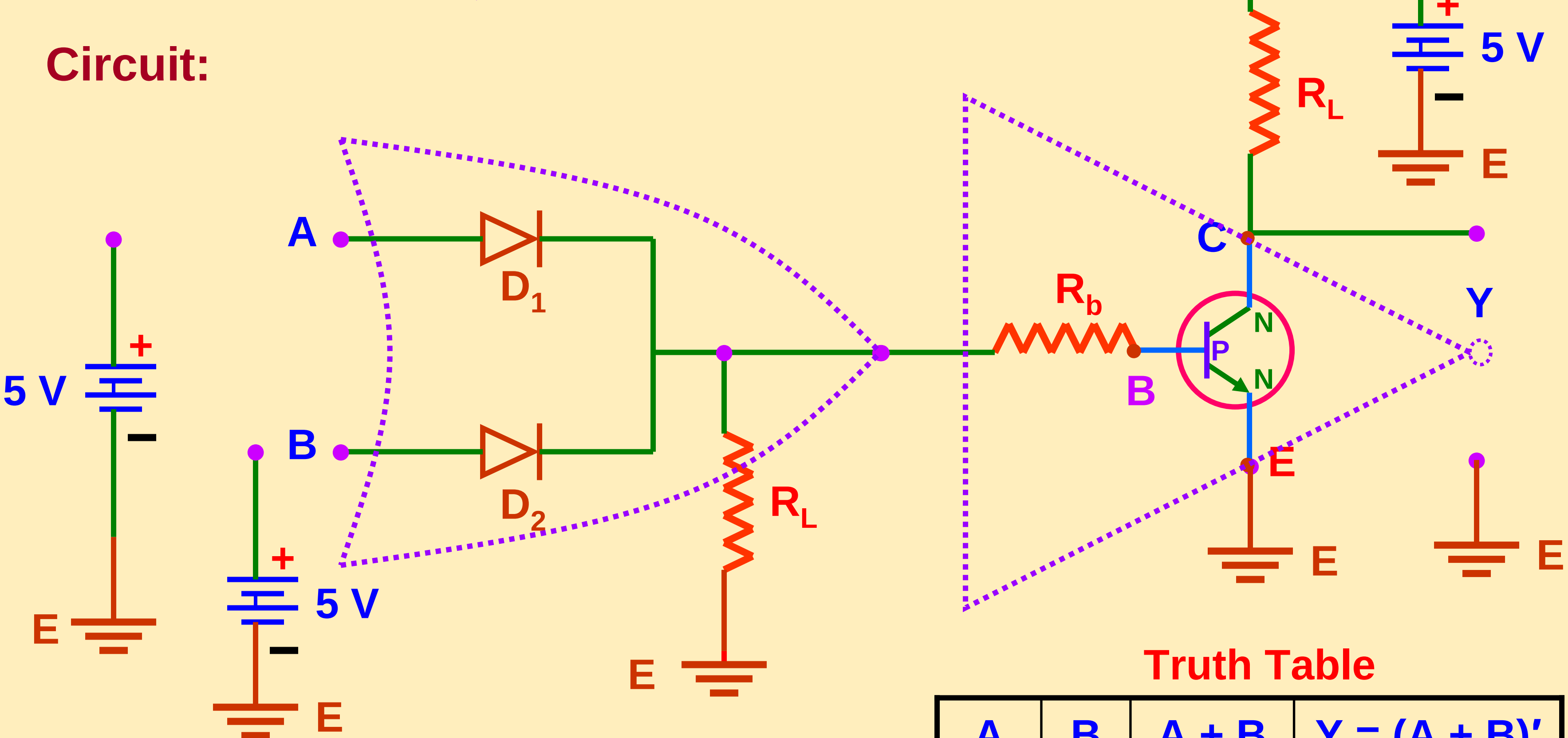
A	Y=A′
0	1
1	0

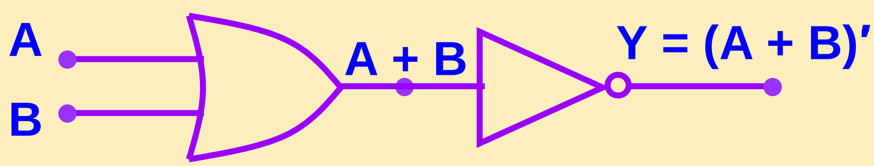
NOR Gate:

Symbol:



Circuit:



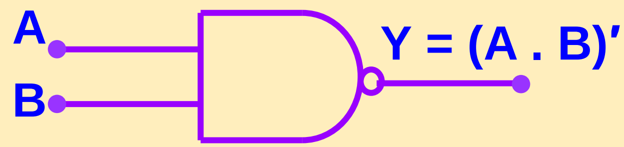


Truth Table

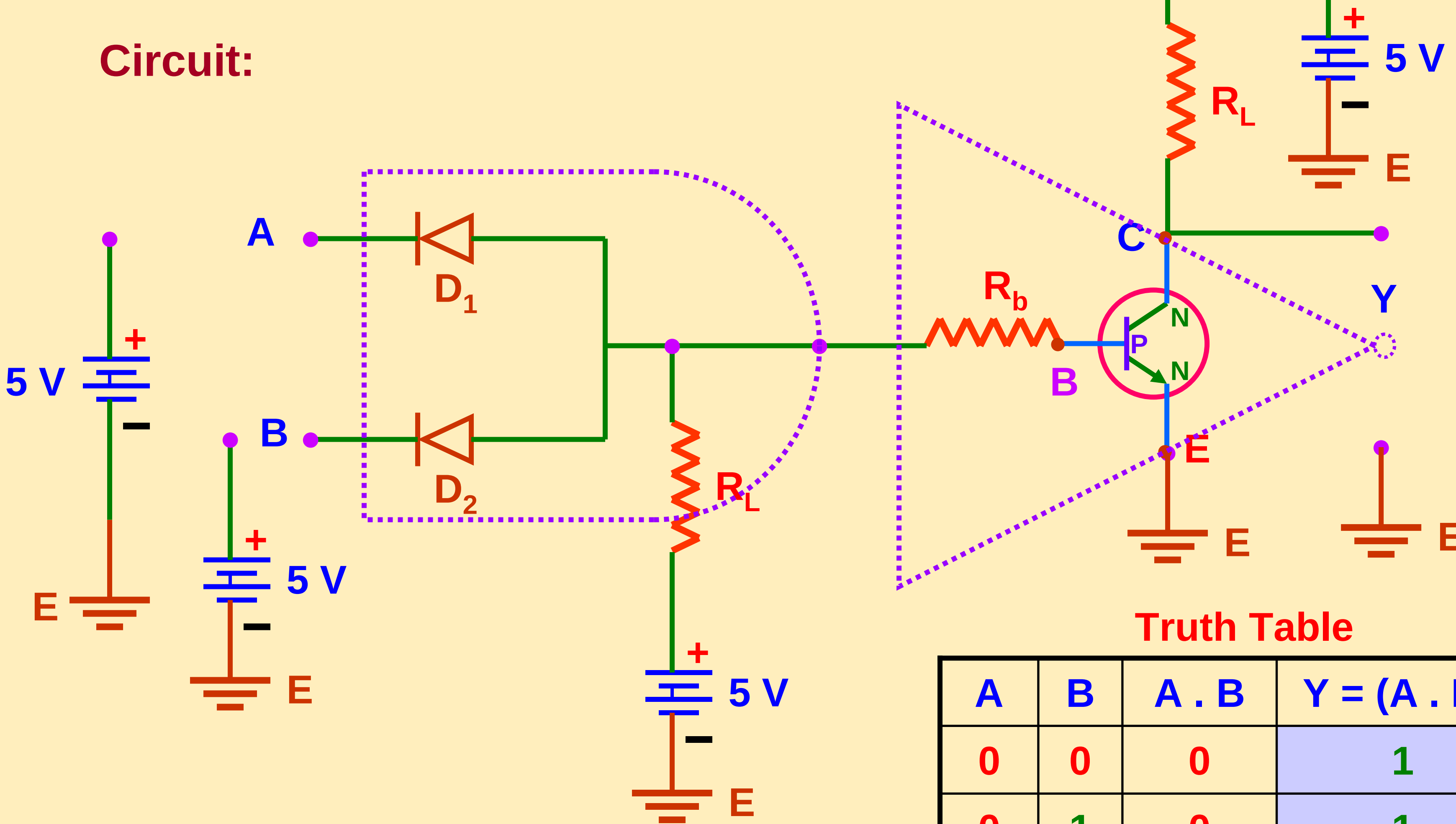
A	B	A + B	Y = (A + B)′
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

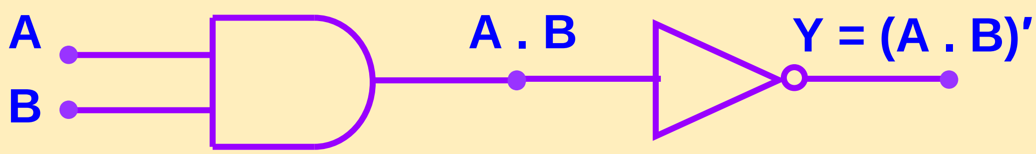
NAND Gate:

Symbol:



Circuit:



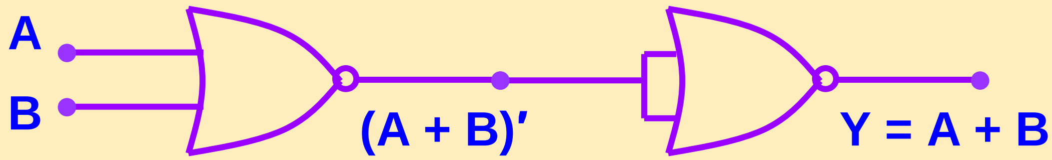


Truth Table

A	B	A . B	Y = (A . B)′
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

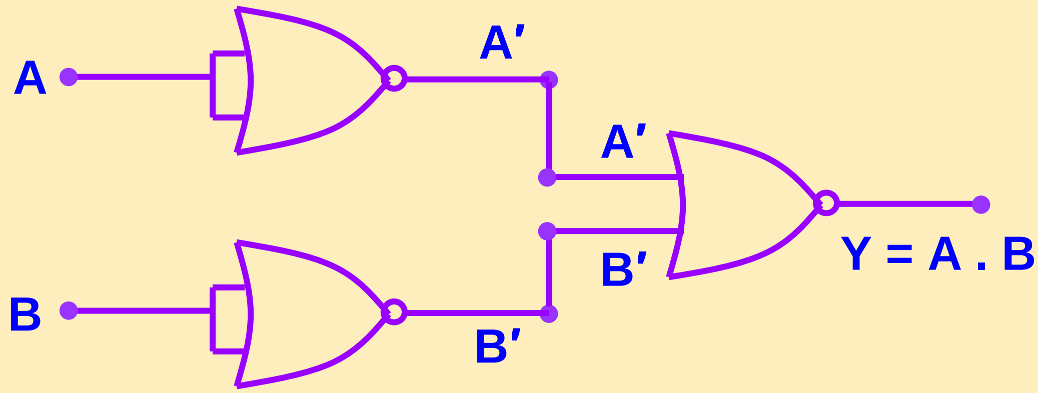
NOR Gate as a Building Block:

OR Gate:



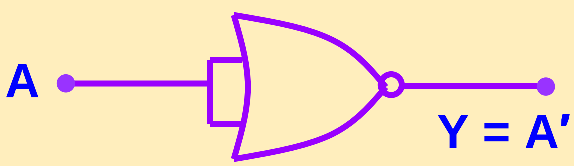
A	B	(A + B)′	A + B
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

AND Gate:



A	B	A′	B′	A′+B′	(A′+B′)′
0	0	1	1	1	0
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	0	1

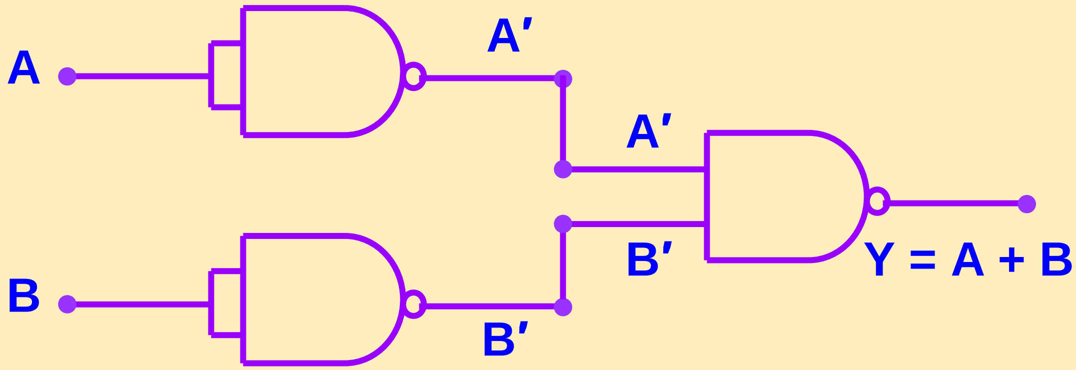
NOT Gate:



A	A′
0	1
1	0

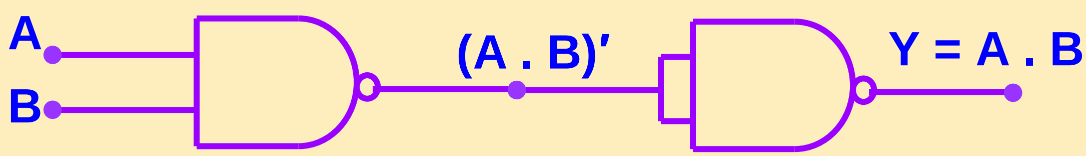
NAND Gate as a Building Block:

OR Gate:



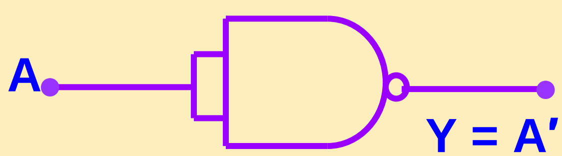
A	B	A′	B′	A′.B′	(A′ . B′)′
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

AND Gate:



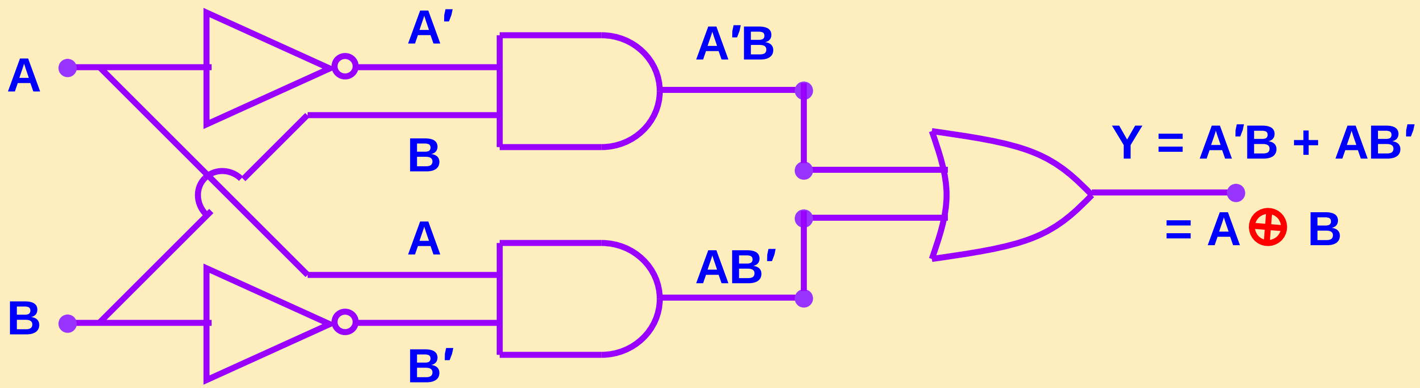
A	B	(A . B)′	A . B
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

NOT Gate:

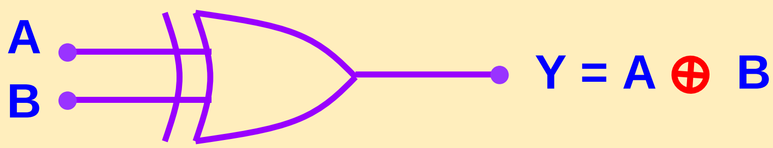


A	A′
0	1
1	0

XOR Gate:



A	B	A′	B′	A′B	AB′	Y = A′B + AB′ = A ⊕ B
0	0	1	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	0	0	0



COMMUNICATION SYSTEMS

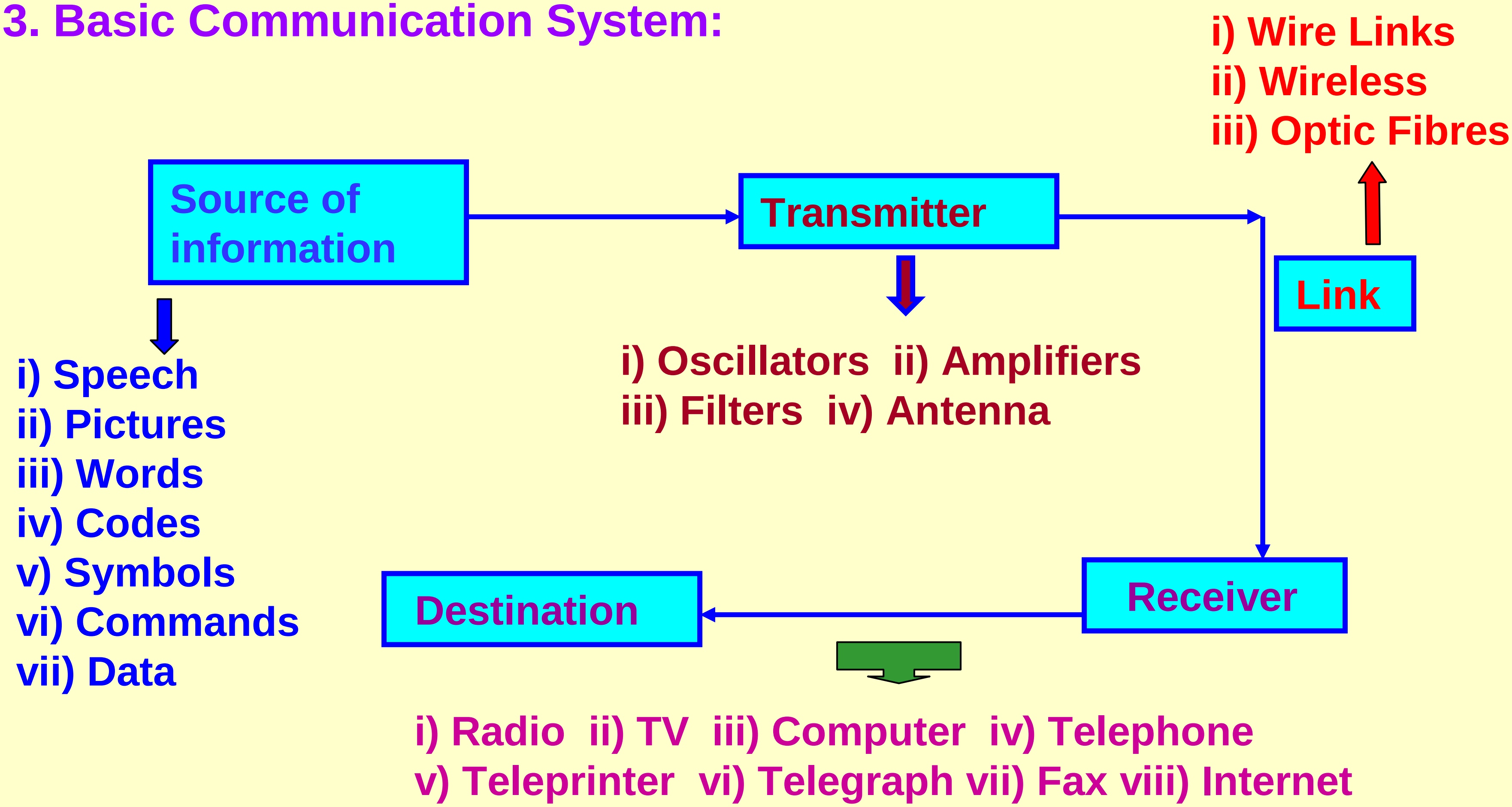
1. BASICS OF COMMUNICATION
2. AMPLITUDE MODULATION

BASICS OF COMMUNICATION

1. Communication: Processing, sending and receiving of information

2. Information: Intelligence, signal, data or any measurable physical quantity

3. Basic Communication System:



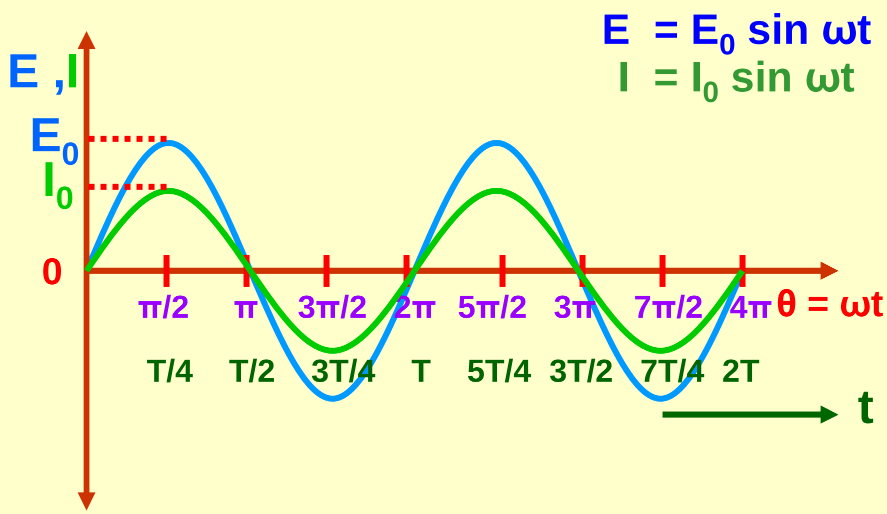
Forms of Communication:

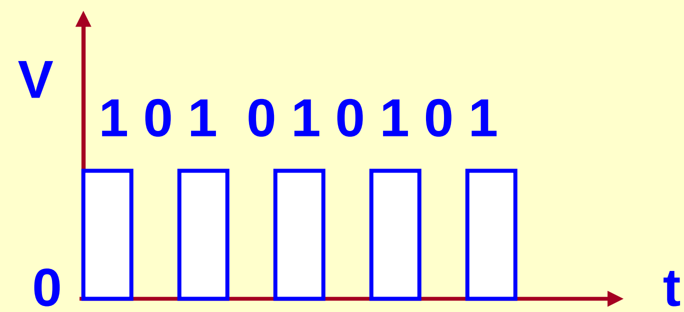
1. Radio Broadcast
2. Television Broadcast
3. Telephony
4. Telegraphy
5. Radar
6. Sonar
7. Fax (Facsimile Telegraphy)
8. E-mail
9. Teleprinting
10. Telemetering
11. Mobile Phones
12. Internet

Types of communication:

1. Cable communication
2. Ground wave communication
3. Sky wave communication
4. Satellite communication
5. Optic fibre communication

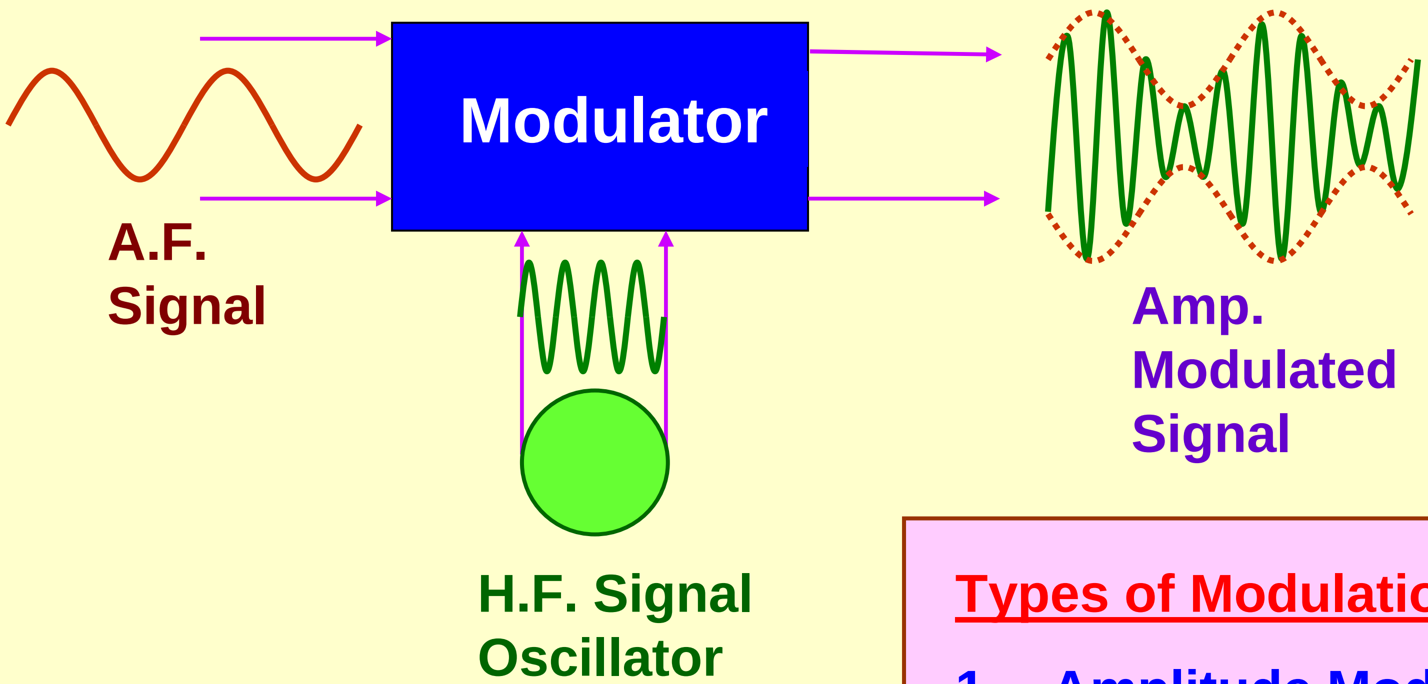
Analogue signal	Digital signal
A continuous signal value which at any instant lies within the range of a maximum and a minimum value.	A discontinuous signal value which appears in steps in pre-determined levels rather than having the continuous change.





MODULATION:

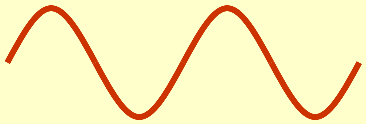
Modulation is the process of variation of some characteristic of a high frequency wave (carrier wave) in accordance with the instantaneous value of a modulating signal.



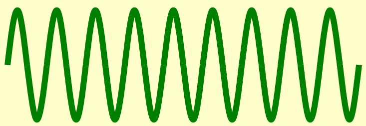
Types of Modulation:

1. **Amplitude Modulation**
2. **Frequency Modulation**
3. **Pulse Modulation**
4. **Phase Modulation**

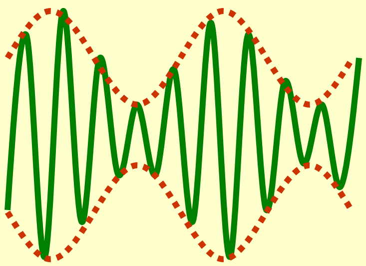
AMPLITUDE MODULATION (AM):



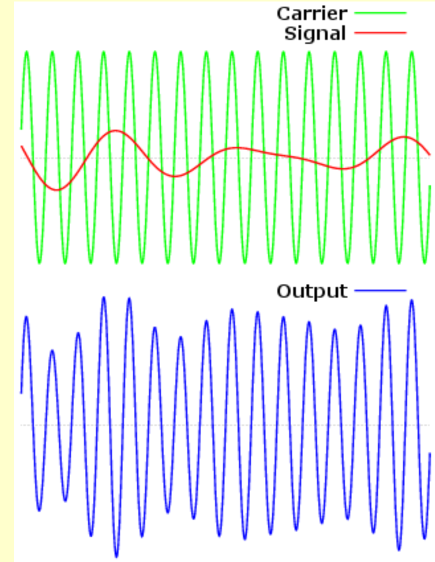
$e_m = E_m \sin \omega_m t$



$e_c = E_c \sin \omega_c t$



$e = (E_c + E_m \sin \omega_m t) \sin \omega_c t$



(Courtesy: Internet)

$e = E_c \sin \omega_c t + (m_a E_c/2) \cos (\omega_c - \omega_m)t - (m_a E_c/2) \cos (\omega_c + \omega_m)t$

Modulation Index $(m_a) = k_a E_m / E_c$

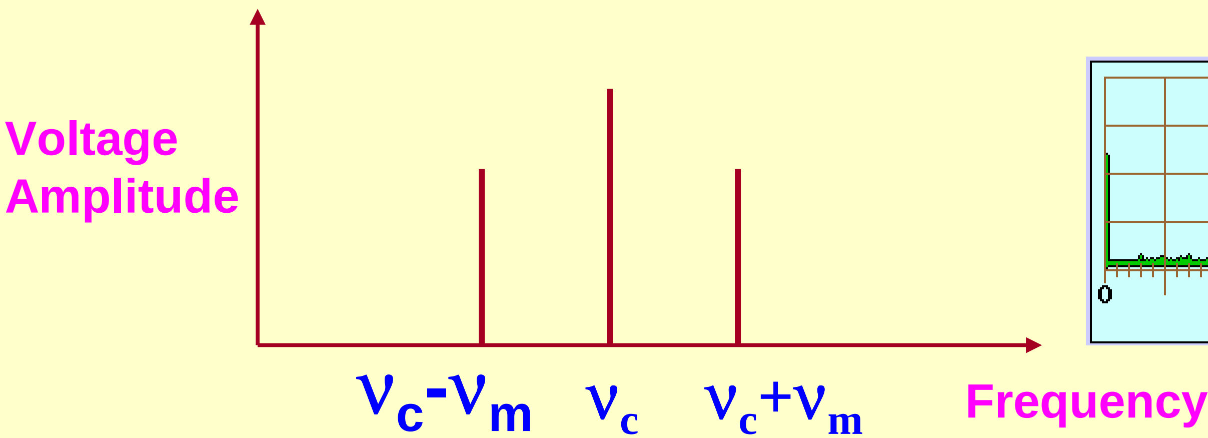
If $k_a = 1$, then $m_a = E_m / E_c$

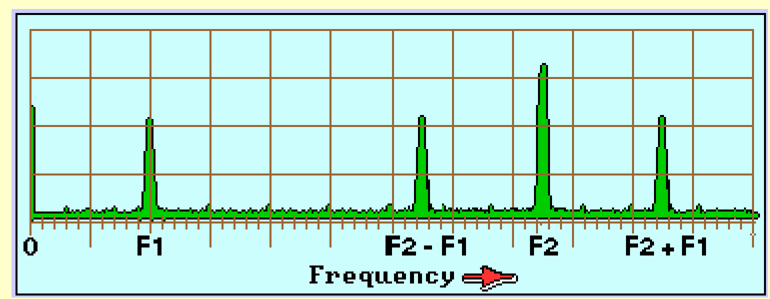
Inferences from equation for e:

1. The **Amplitude Modulated wave** is the summation of three sinusoidal waves with the frequencies ν_c, $\nu_c - \nu_m$ and $\nu_c + \nu_m$ namely Original frequency, Lower Side Band frequency and Upper Side Band frequency respectively.

2. The Bandwidth required for AM, $BW = 2\ \nu_m$

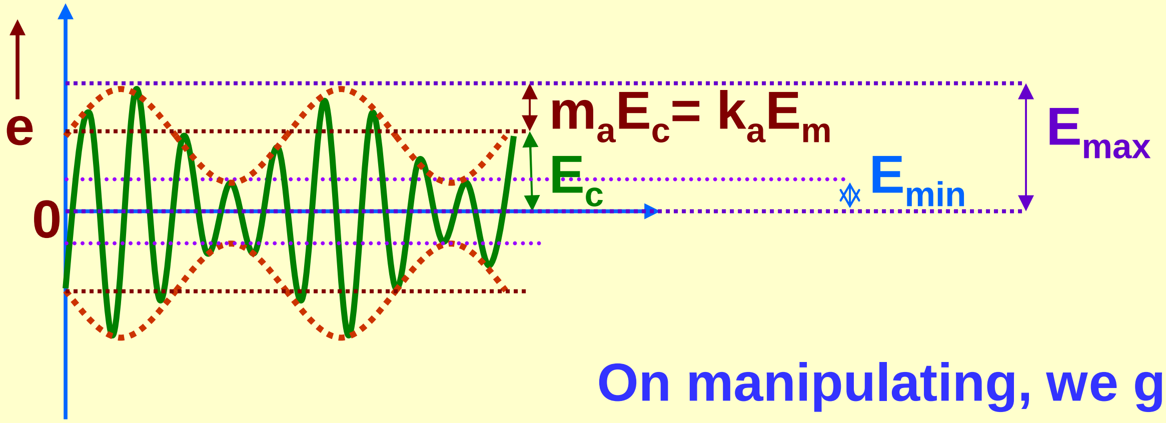
3. The amplitude E_c of the unmodulated carrier wave is made proportional to the instantaneous voltage ($e_m = E_m \sin \omega_m t$) of the modulating wave.





(Courtesy: Internet)

Significance of Modulation Index:

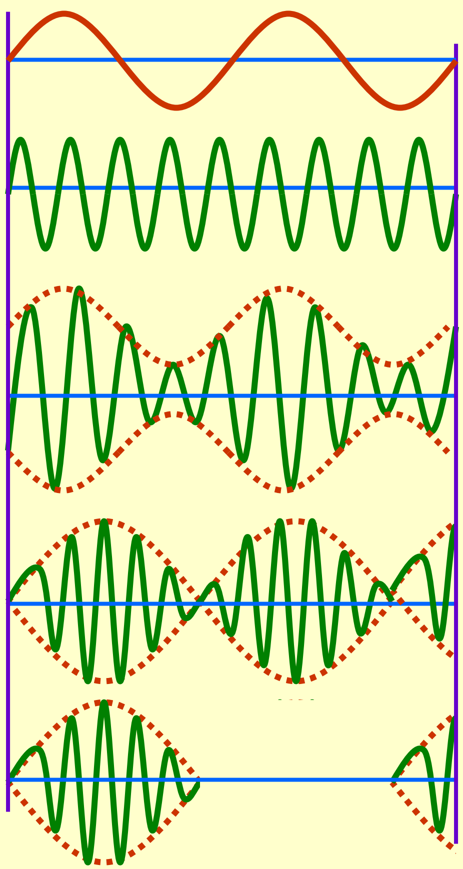


$$E_{max} = E_c + m_aE_c$$

$$E_{min} = E_c - m_aE_c$$

On manipulating, we get

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$



AF signal

$m_a = 0$ (No modulation)

$m_a = 0.5$ or 50%

$m_a = 1$ or 100%

$m_a > 1$ or 100%

Generally,

$0 < m_a < 1$

Power Relation in the AM wave:

If the modulated wave is applied to a resistor of resistance R (say antenna circuit), then the r.m.s. power dissipated in the form of heat is,

$$P_{r.m.s} = (1/R)[\{E_c /2\sqrt{2}\}^2 + \{m_a E_c /2\sqrt{2}\}^2 + \{m_a E_c /2\sqrt{2}\}^2]$$

$$P_{rms} = (E_c^{\,2} /2R)\,[1 + (m_a^2 /2)] = P_c[1 + (m_a^2 /2)]$$

(where P_c is power dissipated by unmodulated carrier wave)

If $m_a = 1$, then $P_{rms} \rightarrow P_{max}$ and $P_{max} = 3\,P_c /2$

Similarly, Power carried by both side bands $P_{SB} = P_{rms} / 3$ which is wasted.

Advantages:

1. AM is an easier method of transmitting and receiving speech signals.
2. It requires simple and inexpensive receivers.
3. It is a fairly efficient system of modulation.

Drawbacks:

1. AM is more likely to suffer from noise.
2. Appreciable energy is contained by three components of AM wave. Sufficient energy can be saved by suppressing carrier wave and one of the side bands. This process makes the equipment complex.
3. Cost of such transmitters and receivers becomes practically more.

Space Communication

This Chapter includes:

1. Space Communication
2. Power Density, Attenuation
3. Range of Electromagnetic Waves
4. Ground Wave Propagation
5. Sky Wave Propagation
6. Space Wave Propagation
7. TV Transmission and Height of TV Antenna
8. Satellite Communication
9. Remote Sensing Satellites

Space Communication:

Space Communication means free space communication.

A free space does not have solid particles or ionised particles and it has no gravitational or other fields of its own. When the frequency of transmitted wave is very high the actual space is considered nearly a free space.

Power Density:

Power density is radiated power per unit area and is inversely proportional to the square of distance from the source.

Antenna:

Antenna is a device which acts as an emitter of electromagnetic waves and it also acts as a first receiver of energy.

Attenuation:

Attenuation is the loss of power of radiation due to absorption of energy in space and power density goes on decreasing as the electromagnetic waves go away from their source.

It is proportional to the square of the distance travelled and is generally measured in decibel (dB).

Range of Electromagnetic Waves:

S. No.	Name of the frequency range (Band)	Short Form	Frequency Range
1	Very Low Frequency	VLF	3 kHz to 30 kHz
2	Low Frequency	LF	30 kHz to 300 kHz
3	Medium Frequency or Medium Wave	MF or MW	300 kHz to 3 MHz
4	High Frequency or Short Wave	HF or SW	3 MHz to 30 MHz
5	Very High Frequency	VHF	30 MHz to 300 MHz
6	Ultra High Frequency	UHF	300 MHz to 3,000 MHz
7	Super High Frequency or Micro Waves	SHF	3,000 MHz to 30,000 MHz (3 GHz to 30 GHz)
8	Extremely High Frequency	EHF	30 GHz to 300 GHz

Propagation of Electromagnetic Waves:

Depending on the frequency, radio waves and micro waves travel in space in different ways depending on the behaviour of these waves w.r.t. the earth and the atmosphere. They are:

1. Ground wave propagation
2. Sky (or ionospheric) wave propagation
3. Space (or tropospheric) wave propagation

1. Ground wave propagation: (AM Radio waves)

In ground wave propagation, the radio waves (AM) travel along the surface of the earth. These waves are called ground waves or surface waves.

In fact, these waves are not confined to surface of the earth but are guided along the earth's surface and they follow the curvature of the earth.

The energy of the radio waves decreases as they travel over the surface of the earth due to the conductivity and permittivity of the earth's surface.

Attenuation increases with the increase in frequency.

Therefore, the ground waves are limited to frequency of 1.5 MHz (1500 kHZ) or wavelength of 200 m.

Ground waves progress along the surface of the earth and must be vertically polarised to prevent from short-circuiting the electric component.

A wave induces currents in the earth over which it passes and thus loses some energy by absorption. This is made up by energy diffracted downward from the upper portions of the wavefront.

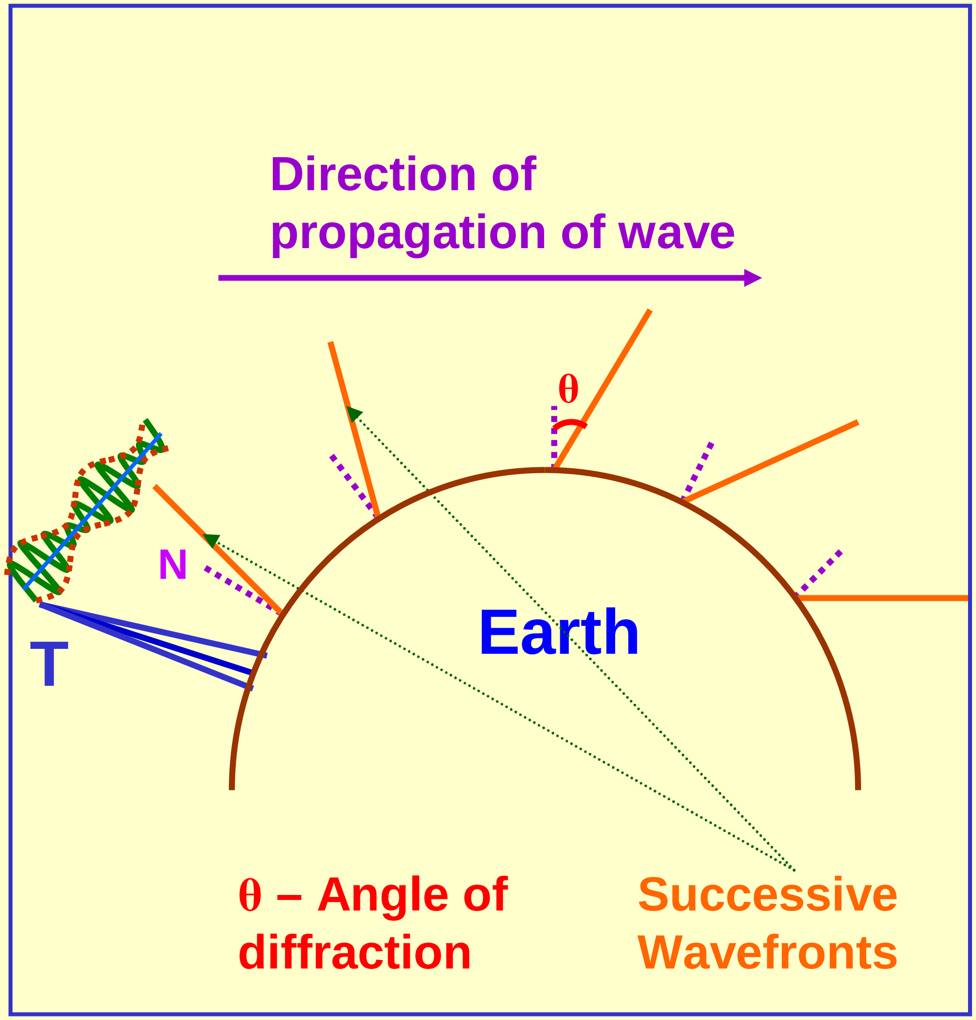
Another way of attenuation is due to diffraction and gradual tilting of the wavefront.

The increasing tilt of the wavefront causes greater short-circuiting of electric field components of the wave.

Eventually, at some distance from the antenna, the wave “lies down and dies”.

The maximum range of a transmitter depends on its frequency as well as its power.

In MF band, the range can not be increased only by increasing its power because propagation is definitely limited by its tilt.



2. Sky wave propagation or Ionospheric wave propagation: (AM Radio waves)

Sky waves are the AM radio waves which are received after being reflected from ionosphere. The propagation of radio wave signals from one point to another via reflection from ionosphere is known as sky wave propagation.

The sky wave propagation is a consequence of the total internal reflection of radiowaves. Higher we go in the ionosphere, free electron density increases and refractive index decreases.

The UV and high energy radiations from the Sun are absorbed by the air molecules and they get ionised to form the ionised layer or electrons and ions. Ionosphere extends from 80 km to 300 km in the atmosphere above the earth's surface.

The oscillating electric field of electromagnetic wave (frequency ω) does not affect the velocity of the ions (negligible change because the em wave field is weak) in the ionosphere but changes the velocity of the electrons.

This changes the effective dielectric constant ε' and hence the refractive index n' as compared to the free space values ε_0 and n_0.

ε' and n' are related to ε_0 and n_0 as

$$n' = \sqrt{(\varepsilon' n_0)} \quad \text{or} \quad n' = n_0 \left[1 - (Ne^2 / \varepsilon_0 m\omega^2)\right]^{1/2}$$

where e is the electronic charge, m is the mass of the electron and N is the electron density in the ionosphere.

It is clear that the refractive index of ionosphere n' is less than its free space value n_0. So, it acts as rarer medium. Therefore, for the angle of incidence above the critical angle, the electromagnetic waves undergo total internal reflection and reach the earth back.

Since n' depends on ω and N, the waves of different frequencies will be reflected back from the different depths of ionosphere depending on electron density N in that region.

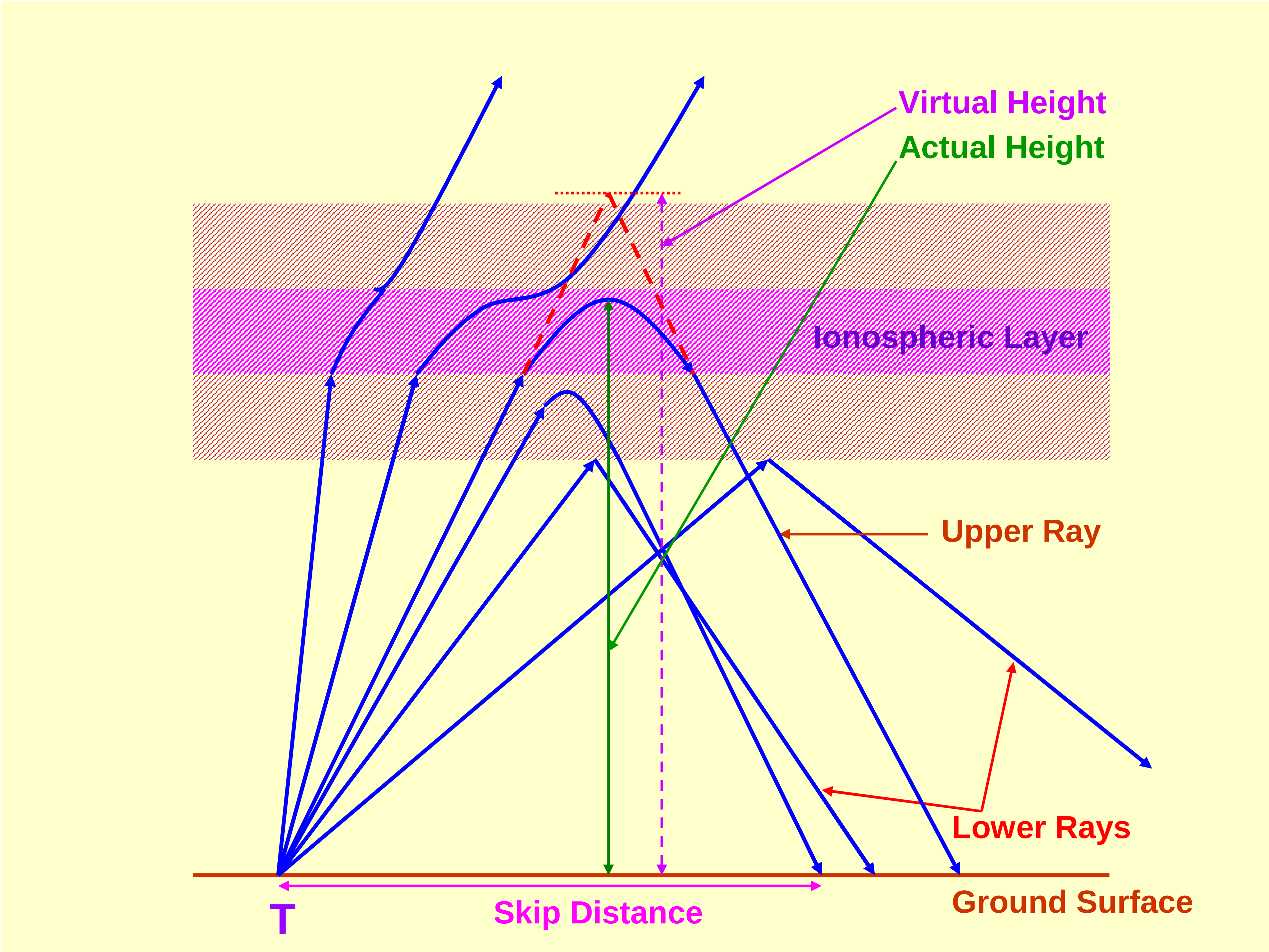
If the frequency ω is too high, then the electron density N may never be so high as to produce total internal reflection. This frequency is called 'critical frequency' (f_c). If the maximum electron density of the ionosphere is N_{max} per m^3, then the critical frequency is given by:

$$f_c \approx 9(N_{max})^{1/2}$$

The critical frequency ranges approximately from 5 to 10 MHz.
The frequencies higher than this cross the ionosphere and do not return back to the earth.

The sky wave propagation is limited to the range of 2 MHz to 30 MHz. This region is called 'short wave band'.

The communication in AM band below 200 m wavelength is via the sky wave only.


Virtual Height
Actual Height
Ionospheric Layer
Upper Ray
Lower Rays
Ground Surface
T
Skip Distance

Important Terms used in Sky wave propagation:

Critical Frequency (f_c):

It is the highest frequency for a given ionospheric layer that can be returned down to the earth by that layer after having been beamed straight up at it.

$$f_c \approx 9(N_{max})^{1/2}$$

Maximum Usable Frequency (MUF):

It is the limiting frequency but for some specific angle of incidence other than the normal.

$$MUF = \frac{\text{Critical Frequency}}{\cos\theta} = f_c \sec\theta$$

This is called 'secant law' and is very useful in making preliminary calculations for a specific MUF. Strictly speaking, it applies only to the flat earth and the flat reflecting layer.

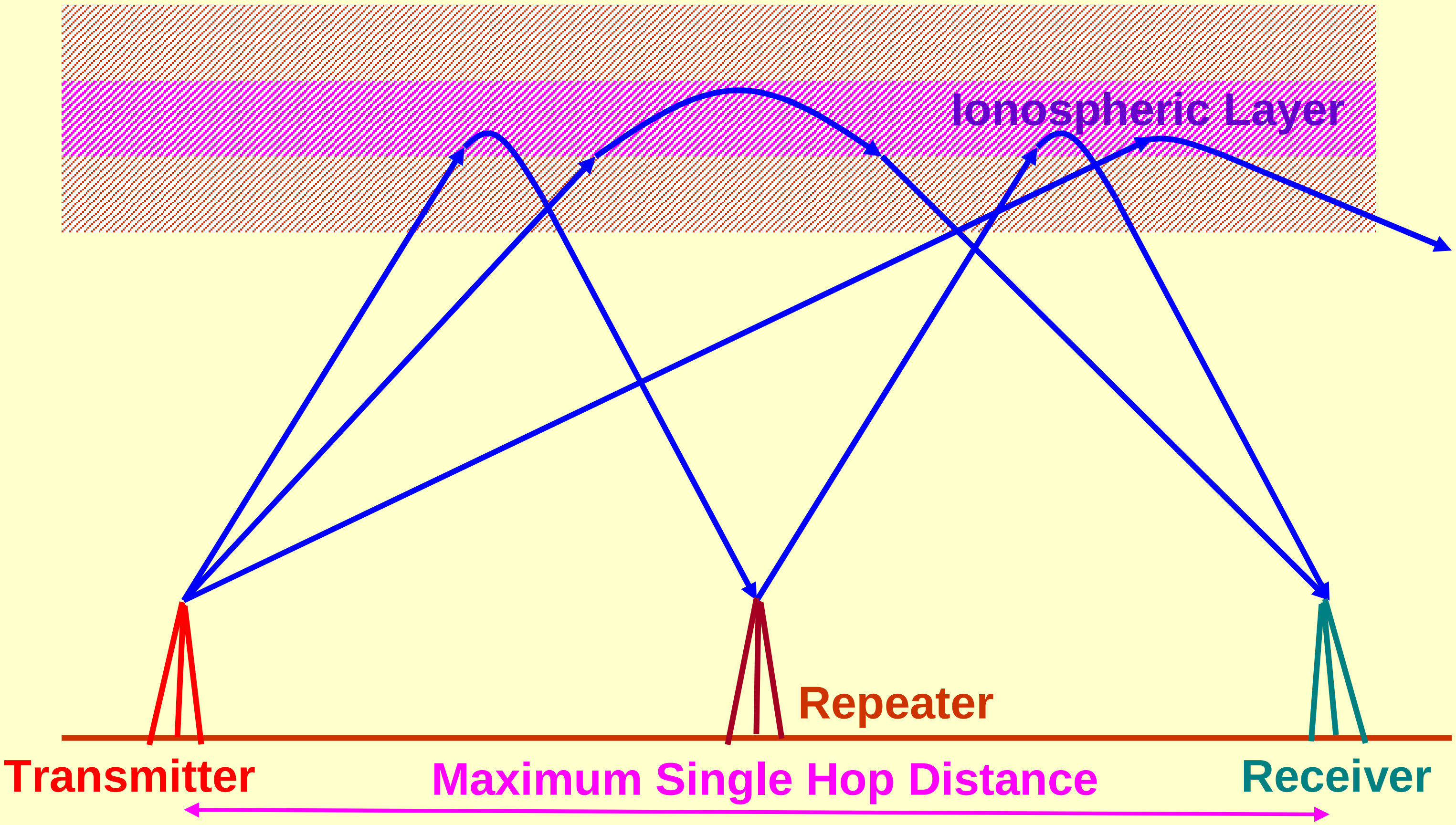
Skip Distance:

It is the shortest distance from a transmitter, measured along the surface of the earth, at which a sky wave of fixed frequency (more than f_c) will be returned to earth, but nevertheless a definite minimum also exists for any fixed transmitting frequency.

At the skip distance, only the normal or lower ray can reach the destination, whereas at greater distances, the upper ray can be received as well, causing interference. This is a reason why frequencies not much below the MUF are used for transmission.

Another reason is the lack of directionality of high-frequency antennas.

If the frequency used is low enough, it is possible to receive lower rays by two different paths after either one or two hops. But this will result in interference again.



3. Space wave propagation or Tropospheric wave propagation: (AM Radio waves)

Space waves travel in (more or less) straight lines. But they depend on line-of-sight conditions. So, they are limited in their propagation by the curvature of the earth.

They propagate very much like electromagnetic waves in free space.

This mode is forced on the waves because their wavelengths are too short for reflection from the ionosphere, and because the ground wave disappears very close to the transmitter, owing to tilt.

Radio Horizon:

The radio horizon for space waves is about four-thirds as far as the optical horizon. This beneficial effect is caused by the varying density of the atmosphere, and because of diffraction around the curvature of the earth.

It is given with good approximation, by the empirical formula

$$d_t = 4\sqrt{h_t}$$

where d_t = distance (in km) from the transmitting antenna,

h_t = height (in m) of transmitting antenna above the ground

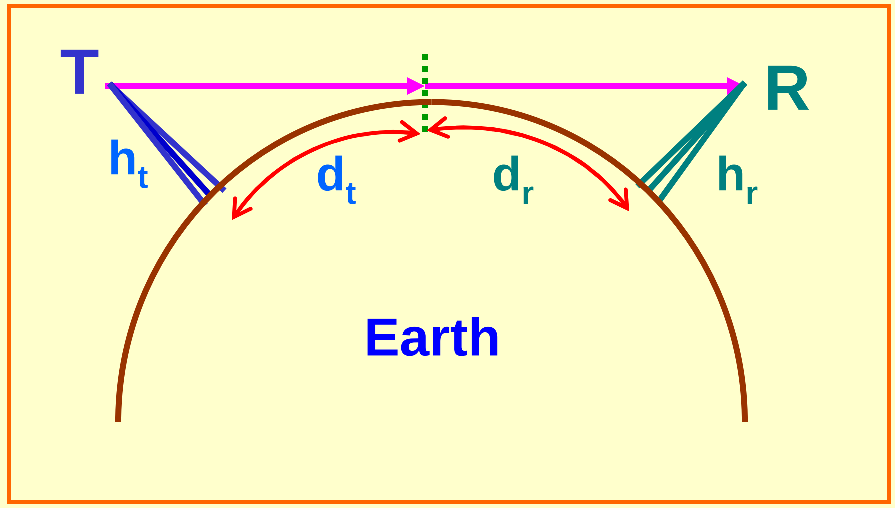
The same formula applies to the receiving antenna.

The distance between the Transmitter and the Receiver is

$$d = d_t + d_r = 4\sqrt{h_t} + 4\sqrt{h_r}$$

If the transmitting and receiving antennas are 225 m and 16 m above the ground, then the distance between them can be 76 km (= 60 + 16).

Commercially, links more than 100 km are hardly used.



Frequency Modulated Communication (TV Signals):

The TV signals are frequency – modulated. They employ frequency greater than 80 MHz.

They can not be propagated by ground wave because the signals get absorbed by ground due to their high frequency.

The propagation by sky wave is also not possible because the ionosphere can not reflect the frequencies higher than 40 MHz.

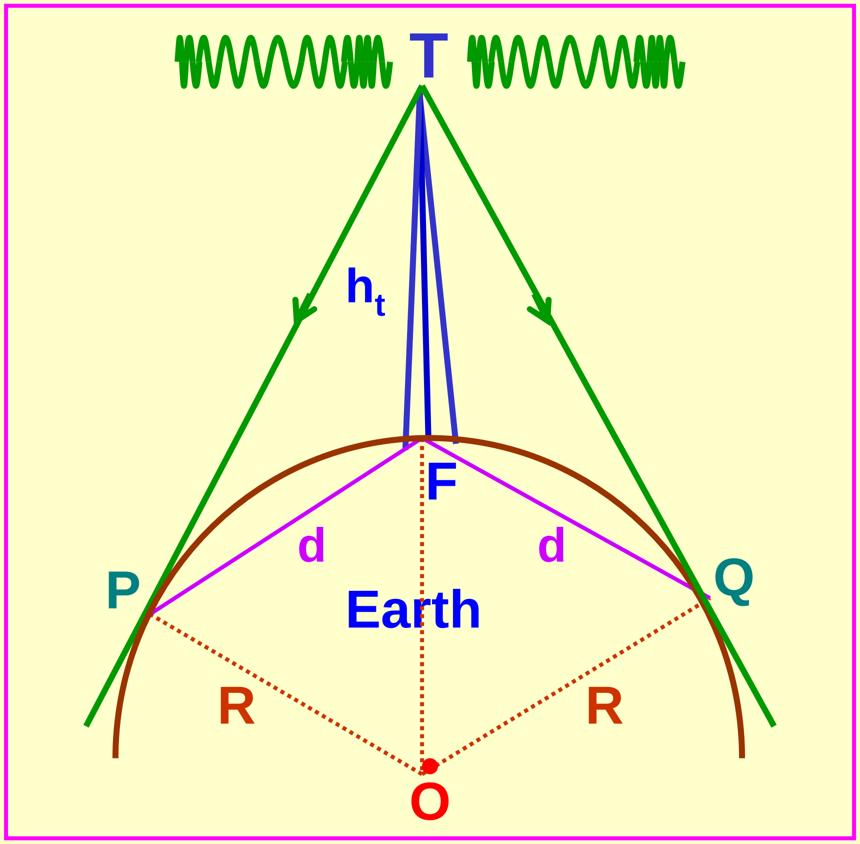
The only way for the transmission of TV signals is that the receiving antenna should directly intercept the signal from the transmitting antenna. (Space-wave or line of sight propagation)

Height of TV Transmitting Antenna:

The TV signals (frequency modulated electromagnetic waves) travelling in a straight line directly reach the receiver end and are then picked up by the receiving antenna.

Due to the finite curvature of the earth, the waves cannot be seen beyond the tangent points P and Q.

The effective range of reception of the broadcast is essentially the region from P to Q which is covered by the line of sight.



Let h be the height of the transmitting antenna, d be the distance (radius) of coverage from the foot of the tower and R be the radius of the earth.

$$OT^2 = OQ^2 + QT^2$$

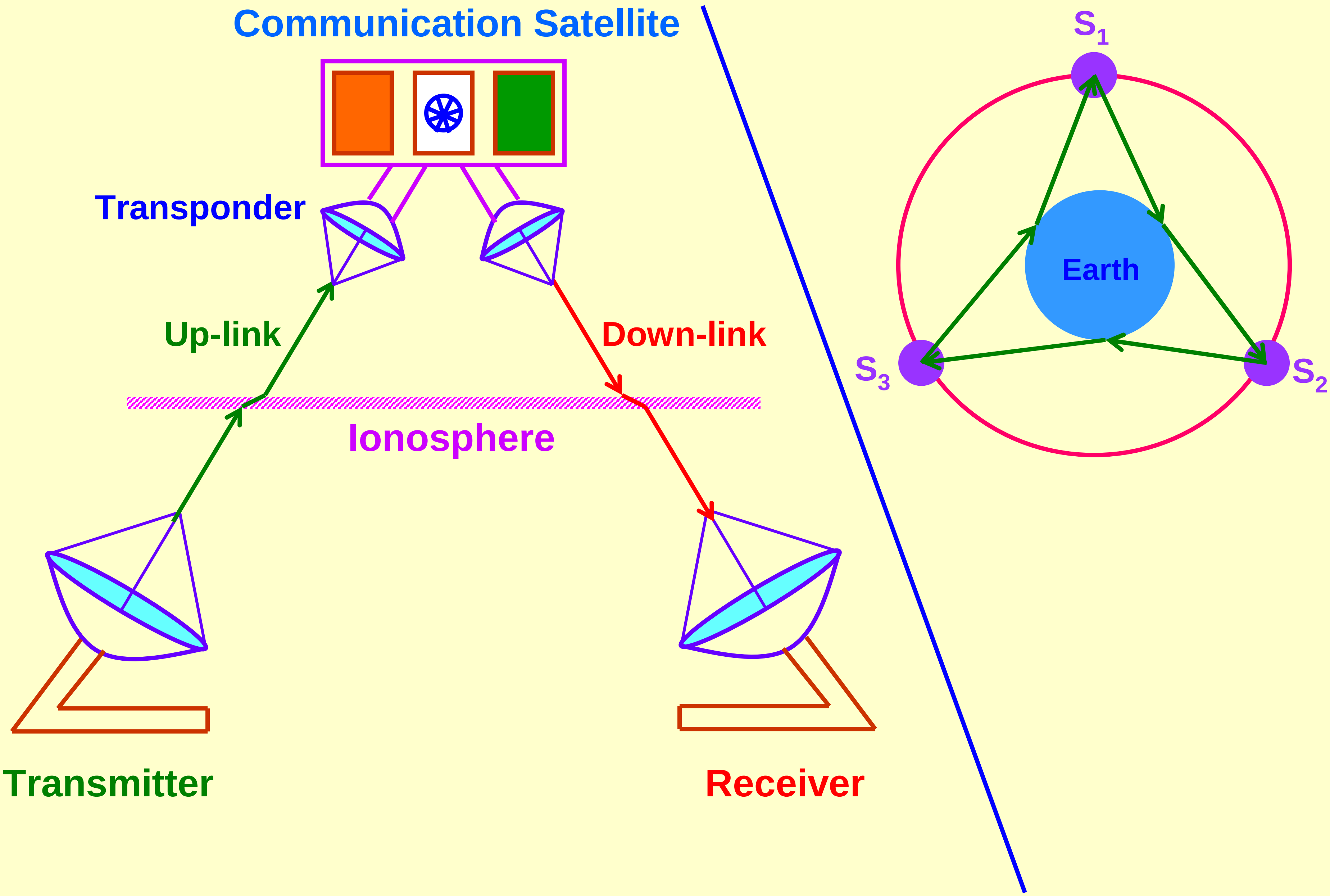
$$(R + h)^2 = R^2 + d^2 \qquad \text{(Note: } QT \approx FQ = d\text{)}$$

or $$d^2 = h^2 + 2hR$$

$$d = \sqrt{(h^2 + 2hR)} \qquad \text{or} \qquad \boxed{d \approx \sqrt{(2hR)}}$$

The antenna of height 80 m can transmit the signal with coverage radius of 32 km and area of 3217 sq. km.

Satellite Communication:



Satellite communication uses UHF / Microwave regions. Microwaves carrying audio, video, telephone, telex, FAX signals, etc. are transmitted from the earth to the satellites orbiting in the space and retransmitted from the satellites to different parts of the earth (world).

The special devices used for this purpose in satellites are called 'transponders'.

Satellite communication is mainly done through 'geostationary satellites'. Three geostationary satellites placed in equatorial orbits at 120° from one another can cover practically the whole populated land area of the world.

Frequency modulation is used for both 'up channel' and 'down channel' transmission. Though FM needs a larger bandwidth, it offers good immunity from interference and requires less power in the satellite transmitter.

Orbit of Communication Satellite:

For global communication, a satellite should move uniformly round a circular orbit with a period of $84.4[r / R]^{3/2}$ minutes, where r is the radius of the orbit of the satellite and R is the radius of the earth.

The circular orbit of the communication satellite is specified in terms of:

(i) The orbit radius (ii) The angle of inclination of the orbit's plane to the Earth's equatorial plane (iii) The position of the ascending mode
(iv) The phase angle of the satellite.

Height of Communication Satellite:

The area of the earth from which a satellite is visible increases with the altitude.

At altitudes below 10,000 km, the number of satellites required for global coverage would be excessive.

At altitudes above 20,000 km, the time taken by signals may be large enough to cause confusion in telephonic conversation.

If time-delay difficulties are ignored, then a synchronous satellite at 36,000 km height can be advantageously used.

Earth-Track Integral System for Communication Satellites:

If several satellites are spaced around the same orbit in space, the tracks of the satellites will be different due to Earth's rotation about its own axis.

If four satellites are placed into different orbits with their ascending modes displaced successively by 30° intervals to the east direction, the difference, in effects of Earth's rotation, can be counteracted and the paths of all the satellites relative to the Earth will be the same.

Such Earth-Track integral systems can be arranged to have the satellite period an integral factor of the sidereal day in order to have the same track repeated day after day.

Remote Sensing Satellites:

'Remote Sensing' is obtaining information about an object by observing it from a distance and without coming into actual contact with it.

The orbit of a remote sensing satellite is such that the satellite passes over a particular latitude at approximately the same local time. i.e. the position of the Sun with respect to a point on the Earth remains approximately the same as the satellite passes over it. Such orbits are called Sun-synchronous orbits.

A remote sensing satellite takes photographs of a particular region with nearly the same illumination every time it passes through that region.

Applications:

1. In Geology
2. In Agriculture
3. In Forestry
4. In Land Mapping
5. In Ocean and Coastal Data
6. In Monitoring Environmental Conditions
7. In Biodiversity
8. In Ground Water Management
9. In Flood Damage Assessment
10. In the Field of Defence
11. In Mapping Wastelands
12. In Early Warning Systems (Natural Calamities)
13. In Management of Water Resources
14. In Fisheries Sectors
15. In Tourism Industry
16. In Planning Pipeline Routs, Ring Roads and Urban Settlements